Last Name: First Name:

Physics 102 Spring 2007: Exam 1—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and 15 multiple-choice questions.
- The test is graded on a scale of 100 points; free-response question I counts for 30 points, free-response II counts for 25 points, and the multiple-choice questions account for 45 points (three points each).
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

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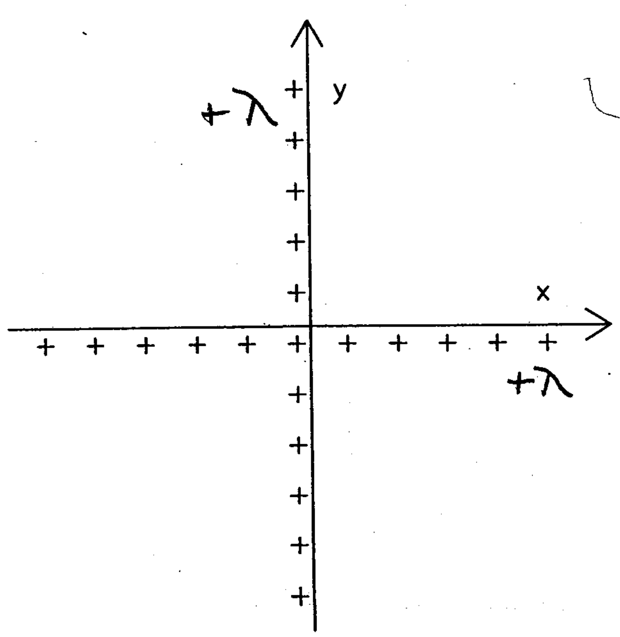
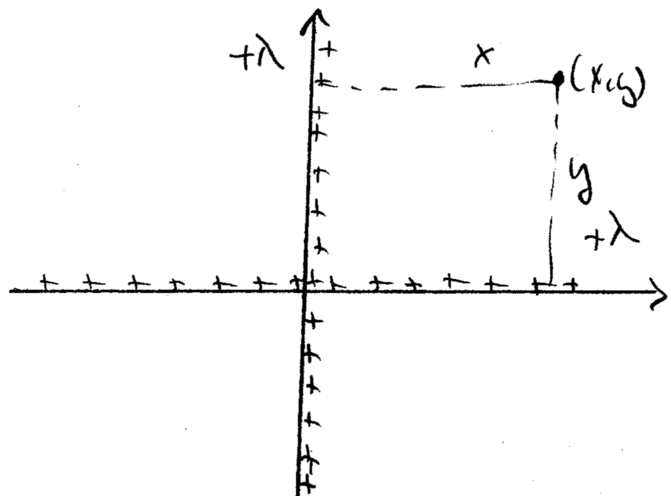
30 I. A very long, uniform line of charge with positive linear charge density $+\lambda$ lies along the x-axis. An identical line of charge lies along the y-axis.

8 (a) Determine the electric field $\vec{E}(x, y)$ for all points in the $x-y$ plane.
6 (b) Determine the change in electrostatic potential ΔV between the points $x = a, y = a$ and $x = a, y = 3a$.
6 (c) Determine ΔV between the points $x = a, y = a$ and $x = 3a, y = a$.
5 (d) How much work must be done to move a small negative charge $-q$ from the point $x = 3a, y = 3a$ to the point $x = a, y = a$?
5 (e) For a very long linear charge distribution, we do not define the zero of electrostatic potential to be an infinity. Why not?



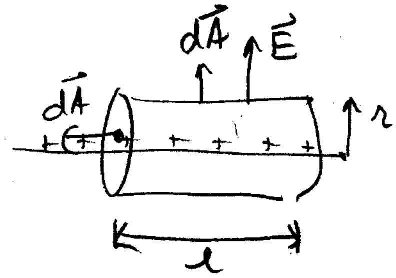
Physics 102 - Exam 1

I.



(a) By Gauss' law, the field due to a long line of charge is

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$



$$2\pi r \ell E = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

– direction is radially outward from the wire. r measures the distance from the wire.

The line of charge on the x-axis produces a field

$$\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 y}\hat{y}$$

– correct for all values of y since $\vec{E}_1$ changes direction when y changes direction.

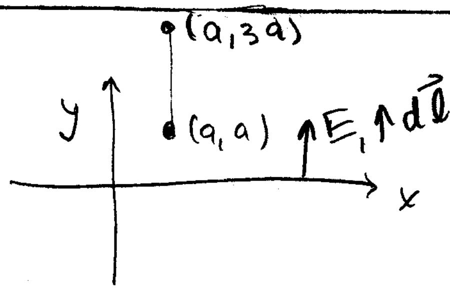
The field due to the line of charge on the y-axis is

$$\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 x}\,\hat{\imath}$$

The total field is the sum

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 x}\,\hat{\imath} + \frac{\lambda}{2\pi\varepsilon_0 y}\,\hat{\jmath}}$$

(b)



In moving from (a, a) to (a, 3a), we are moving perpendicular to $\vec{E}_2$, so it does not contribute to ΔV

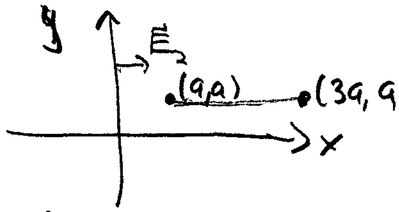
$$\Delta V = -\int \vec{E}\cdot d\vec{l} \qquad d\vec{l} = d\vec{y}$$

$$\Delta V = -\int_a^{3a} \frac{\lambda}{2\pi\varepsilon_0 y}\,dy = \frac{-\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3a}{a}\right)$$

$$\boxed{\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\ln 3}$$

The potential is decreasing, as expected since we are moving in the direction of $\vec{E}_1$

(c)

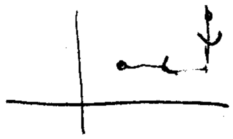


Now we are moving perpendicular to $\vec{E}_1$, so only $\vec{E}_2$ contributes.

$$\Delta V = -\int_a^{3a} \frac{\lambda}{2\pi\varepsilon_0 x}\,dx = \boxed{\frac{-\lambda}{2\pi\varepsilon_0}\ln 3 = \Delta V}$$

(d) In going from (3a, 3a) to (a, a) we can consider the path (3a, 3a) → (3a, a) → (a, a). Then

$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{+\lambda}{2\pi\epsilon_0}\ln 3 + \frac{\lambda}{2\pi\epsilon_0}\ln 3$$



The sign changes because we are moving in the opposite direction as was considered in (b) & (c).

Then $\boxed{W = -q\Delta V = -\frac{q\lambda}{\pi\epsilon_0}\ln 3}$

This is the work done by an external agent.
The field does positive work.

(e) Unlike a spherical charge distribution, for a line of charge $E \sim 1/r$. Then

$$\Delta V = -\int_{r_0}^{\infty}\vec{E}\cdot d\vec{r} = -\int_{r_0}^{\infty}\frac{\lambda}{2\pi\epsilon_0 r}\,dr = \frac{\lambda}{2\pi\epsilon_0}\ln(r)\Big|_{r_0}^{\infty} \to \infty$$

The integral defining ΔV diverges if we take the limit $\to \infty$. So instead we have to select finite limits to define ΔV.

Phys 102 Exam 1
Grading criteria

I. 30 pts

(a) 8 pts

Gauss law 2 pts

field for a single line 4 pts

Correctly adding the two contributions 2 pts

(b) 6 pts

$\Delta V = -\int \vec{E} \cdot d\vec{l}$ 2 pts

Correctly setting up integral 3 pts

Correct answer 1

-1 for wrong sign

(c) 6 pts

$\Delta V = -\int \vec{E} \cdot d\vec{l}$ 2 pts

Correct integral 3 pts

Correct answer 1 pt -1 for wrong sign.

(d) 5 pts

$W = q\,\Delta V$ 2 pts

Use ΔV from (c) & (b) 2 pts

Correct answer 1 pt

(e) 5 pts

Give full credit if they have the general idea that $\Delta V \to \infty$ as $r \to \Delta$.

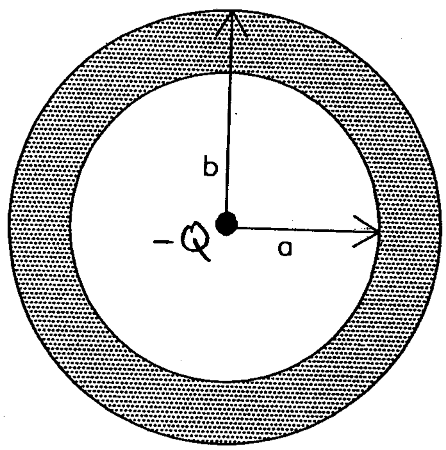
25 II. A spherical shell made of conducting material has an inner radius a and an outer radius b. The shell carries no net charge. Inside the shell, located at the center, is a negative point charge of $-Q$. The point charge is suspended from a thin thread and does not touch the shell. The coordinate r measures the distance from the center of the sphere.

2 (a) Determine the charges on the inner and outer surfaces of the spherical shell.

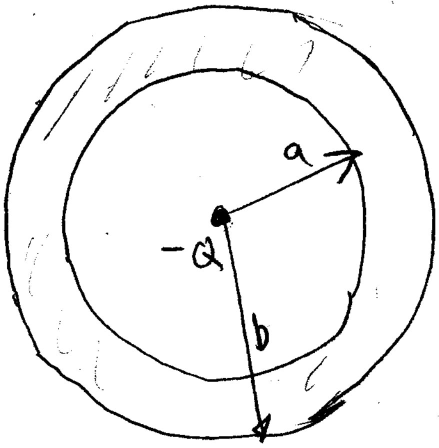
10 (b) Determine the electric field everywhere in space. Sketch $\vec{E}(r)$ vs. r.

10 (c) Determine the electrostatic potential $V(r)$ everywhere in space. Sketch $V(r)$ vs. r.

3 (d) Show that the relation $E_r = -\frac{dV}{dr}$ is satisfied.



II.



(a) A total charge $+Q$ must be on the inner surface, so that inside the field inside the conductor $= 0$.

Since the net charge on the conductor $= 0$, the outer surface must have charge $-Q$

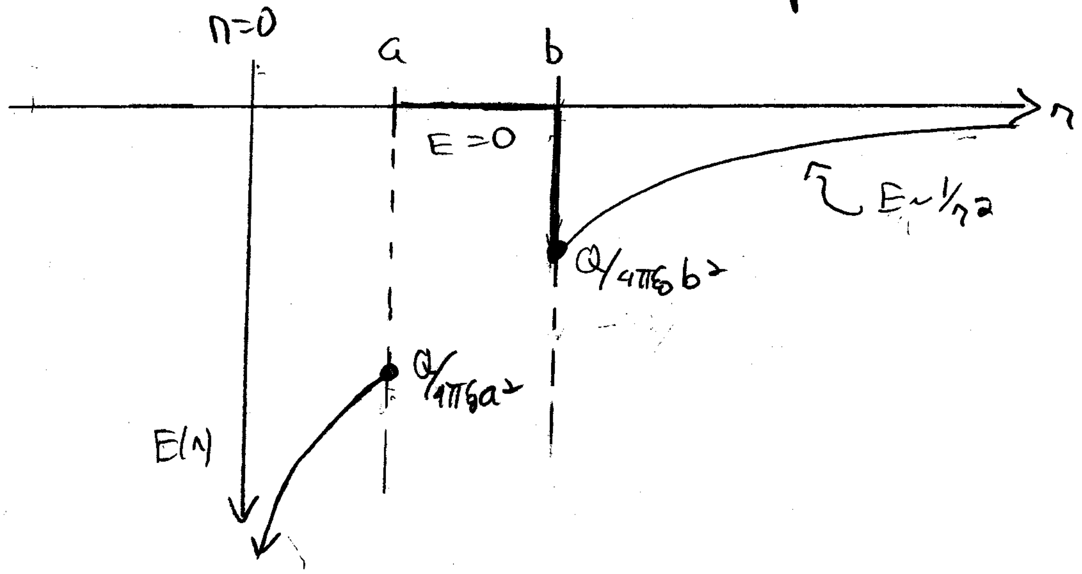
(b) For $r > b$, $\vec{E} = \frac{-Q}{4\pi\epsilon_0 r^2}\hat{r}$

Outside a spherical charge distribution, $\vec{E}$ is the same as a point charge.

For $a < r < b$ $\vec{E} = 0$

For $r < a$, $\vec{E} = -\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

from Gauss' law, only the charge enclosed contributes to $\vec{E}$, for a spherically symmetric distribution.



(c) for $r>b$ $V(r)$ is the same as for a point charge:

$$V(r) = -\frac{Q}{4\pi\varepsilon_0 r} \qquad r>b$$

for $a<r<b$, $\vec{E}=0$ and $V(r) =$ constant

$$V(r) = \frac{-Q}{4\pi\varepsilon_0 b} \qquad a<r<b$$

for $r<a$ we can calculate ΔV & add it to $V(r=b)$

$$\Delta V = -\int_r^a \vec{E}\cdot d\vec{l}$$

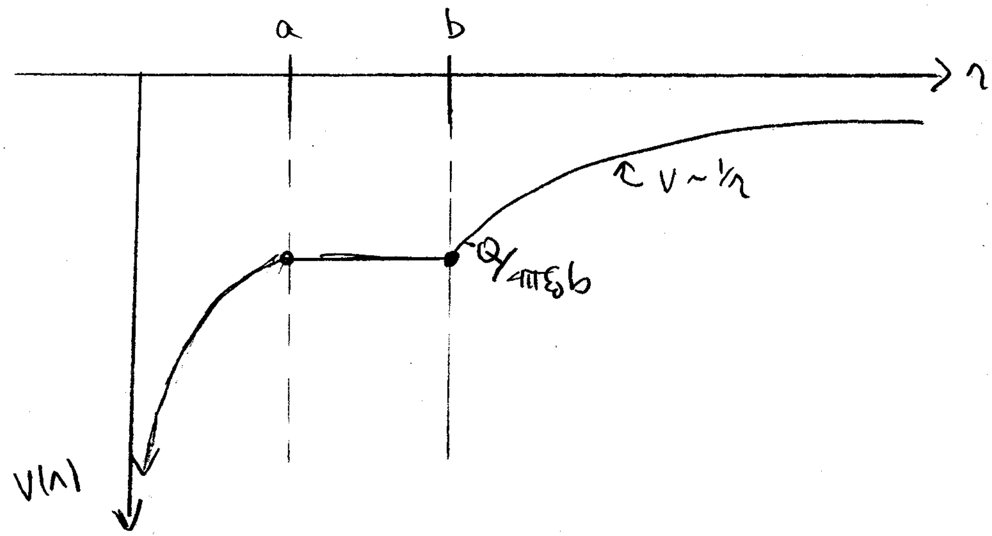
$\vec{E}$ $d\vec{l}$ $\vec{E}$ and $d\vec{l}$ are parallel

$$\Delta V = \frac{-Q}{4\pi\varepsilon_0}\int_r^a \frac{dr'}{r'^2} = \frac{Q}{4\pi\varepsilon_0 r'}\Bigg|_r^a = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right) = \Delta V$$

$$\boxed{V(r) = V(r=b)+\Delta V = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{r}\right) \qquad r<a}$$

Check limits $V(r\to a) = -Q/4\pi\varepsilon_0 b$ ✓

$V(r\to 0) \to -\infty$ ✓



(d) for $r > b$,

$$-\frac{dV}{dr} = \frac{-Q}{4\pi\varepsilon_0 r^2} = E_r \quad \text{from (b)}$$

for $a < r < b$, $V(r)$ is constant so $-\frac{dV}{dr} = 0$

for $r < a$, $-\frac{dV}{dr} = \frac{-Q}{4\pi\varepsilon_0 r^2} = E_r$ from (b)

Physics 102 - Grading Criteria

II. 25 pts

(a) 2 pts

one point for each surface

(b) 10 pts

E for $r > b$ 2 pts

E for $a < r < b$ 3 pts

E for $r < a$ 3 pts

Sketch 2 pts

(c) 10 pts

V for $r > b$ 2 pts

V for $a < r < b$ 3 pts

V for $r < a$ 3 pts

sketch 2 pts

(d) 3 pts

1 point for each region

$r > b$

$a < r < b$

$r < a$

Last Name: First Name:

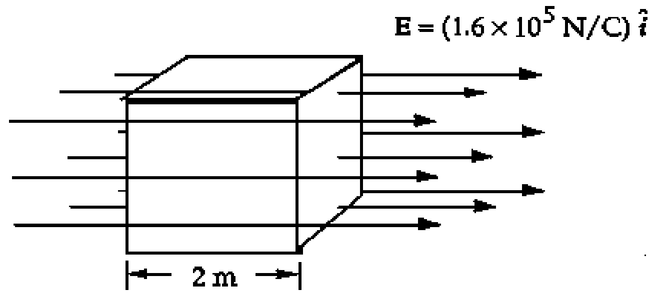
Physics 102 Spring 2007: Exam #1 —Multiple-Choice Questions

1. Two small conducting spheres attract one another electrostatically. This can occur for a variety of reasons. Which of the following statements MUST be true?

   (A) At least one sphere is charged.
   (B) Neither sphere is charged.
   (C) Both are charged.
   (D) Both have the same sign of charge.
   (E) None of these is correct.

2. A cubical surface with sides 2.0 m long is oriented with its right and left faces perpendicular to a uniform electric field $\vec{\mathbf{E}} = 1.6 \times 10^5\,\hat{\imath}$ N/C. The net charge enclosed by this surface is approximately

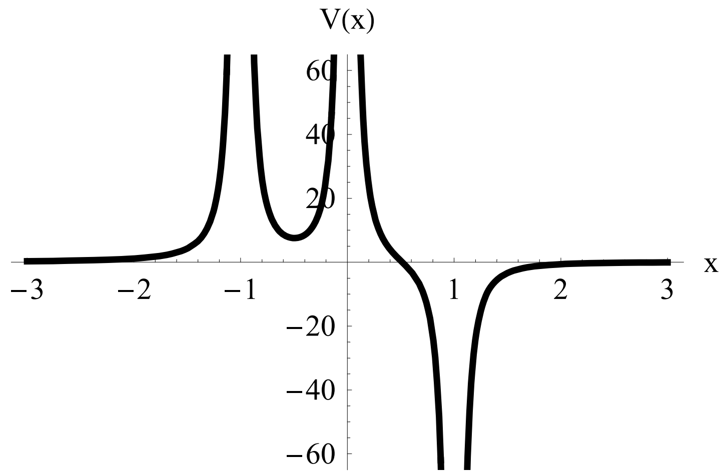
   (A) $8.0 \times 10^4\,\varepsilon_0$.
   (B) $1.6 \times 10^5\,\varepsilon_0$.
   (C) $3.2 \times 10^5\,\varepsilon_0$.
   (D) $6.4 \times 10^5\,\varepsilon_0$.
   (E) 0.



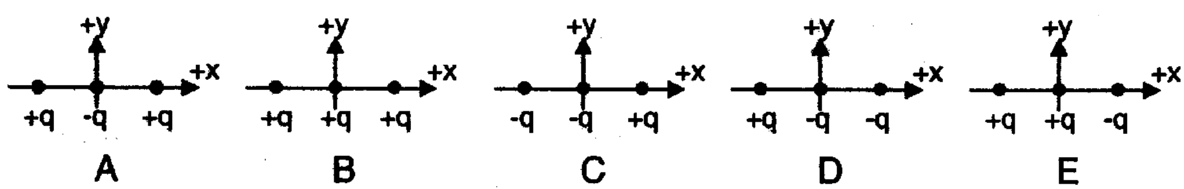
3. A surface is constructed so that at all points on the surface the electric field, $\vec{\mathbf{E}}$, points inward. It can be concluded that

   (A) the surface encloses a net positive charge.
   (B) the surface encloses a net negative charge.
   (C) the surface encloses no net charge.
   (D) the infinitesimal surface vector, $d\vec{\mathbf{A}}$ at all points on the surface is necessarily parallel to the electric field vector, $\vec{\mathbf{E}}$.
   (E) the infinitesimal surface vector, $d\vec{\mathbf{A}}$ at all points on the surface is necessarily perpendicular to the electric field vector, $\vec{\mathbf{E}}$.

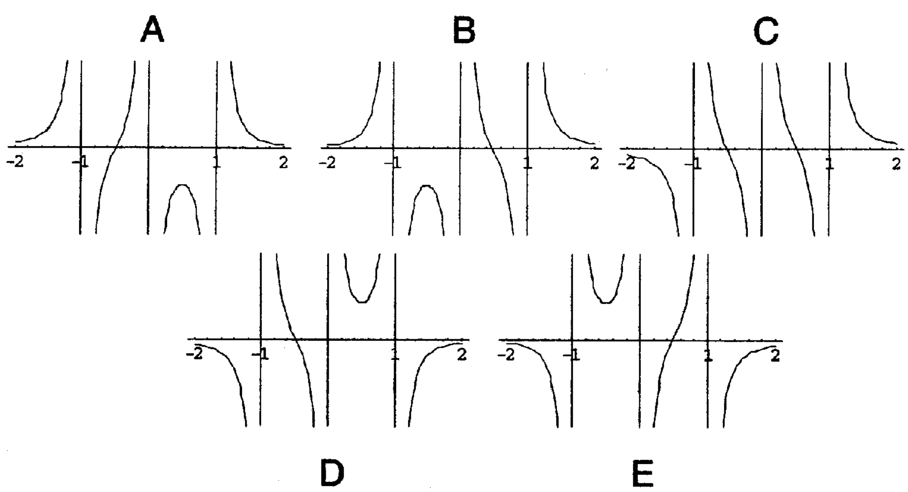
For questions 4 and 5, refer to the graph below, depicting the potential on the x-axis as a function of x



4. Which of the charge configurations depicted below would give rise to the potential shown above?

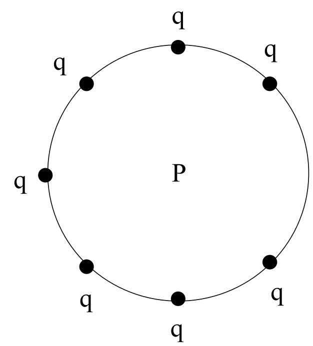


5. What is the x component of the electric field for points on the x-axis given by the potential depicted above?



6. Originally there was a ring of eight equal charges equally spaced in a circle of radius R as shown in the figure below. One charge was removed. What is the magnitude of the electric potential at point P located in the center of the circle after the charge is removed?

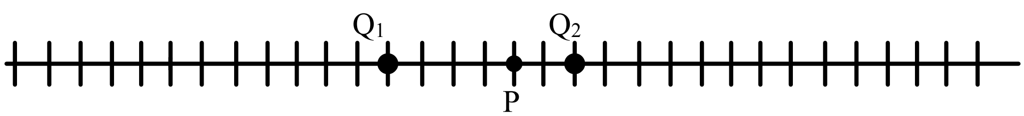
   (A) 0.
   (B) $k\,q/R$
   (C) $7\,(k\,q/R)$
   (D) $(\frac{1}{8})\,(k\,q/R)$
   (E) $(\frac{7}{8})\,(k\,q/R)$



7. A solid, conducting sphere of radius R is positively charged. Of the following distances from the center of the sphere, which location will have the greatest electric potential? *(Take V=0 at $r \to \infty$)*

   (A) 0 (center of the sphere).
   (B) 1.1 R.
   (C) 1.25 R.
   (D) 2 R.
   (E) None of the above because the potential is constant.

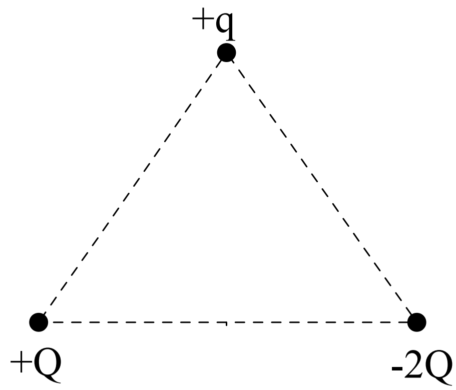
8. Charges Q_1 and Q_2 are situated as shown below. The electric field is zero at point P. What conclusions can be drawn about the charges Q_1 and Q_2?



   I. The magnitudes of Q_1 and Q_2 are equal.
   II. The magnitude of Q_1 is greater than the magnitude of Q_2.
   III. The magnitude of Q_1 is less than the magnitude of Q_2.
   IV. Q_1 and Q_2 have the same sign.
   V. Q_1 and Q_2 have different signs.

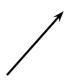
   (A) I and IV are correct.
   (B) II and IV are correct.
   (C) II and V are correct.
   (D) III and IV are correct.
   (E) III and V are correct.

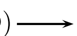
9. Three charges $+q$, $+Q$, and $-2Q$ are placed at the corners of an equilateral triangle as shown below. The net force on charge $+q$ due to the other two charges is best represented by which of the following arrows?

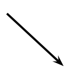


(A)

(B)

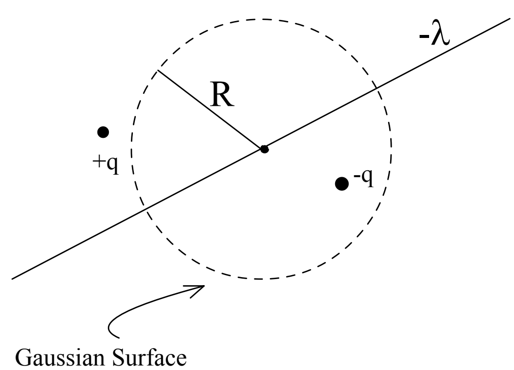
(C) 

(D) 

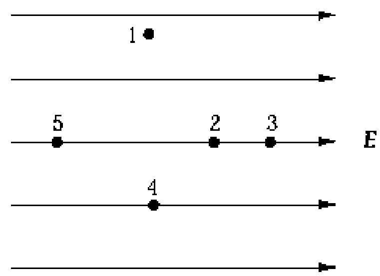
(E) 

10. A very long wire contains a uniform negative charge density $-\lambda$. A charge $-q$ rests inside a spherical Gaussian surface of radius R. The sphere's center lies on the wire. Located outside the sphere is another charge $+q$. What is the total electric flux through the sphere?

(A) $\Phi = 0$
(B) $\Phi = \frac{-q}{\varepsilon_0}$
(C) $\Phi = \frac{-k\,q}{R^2}$
(D) $\Phi = \frac{-2\,\lambda\,R - q}{\varepsilon_0}$
(E) $\Phi = \frac{-2\,\lambda\,R + q}{\varepsilon_0}$

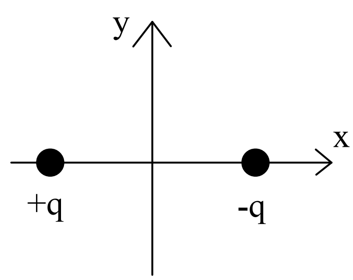


11. Which of the points shown in the figure below are at the same potential?

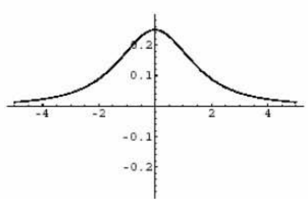


(A) 2 and 5.
(B) 2, 3, and 5.
(C) 1 and 4.
(D) 1 and 5.
(E) 2 and 4.

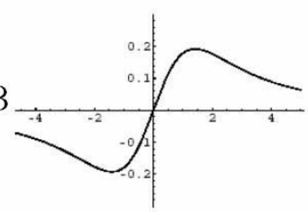
12. Two charges are assembled as shown below, which graph correctly depicts values of E_x for points along the y-axis?



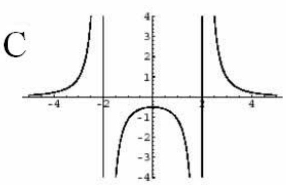
A



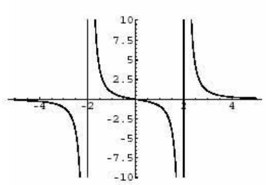
B



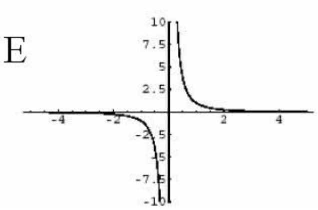
C



D

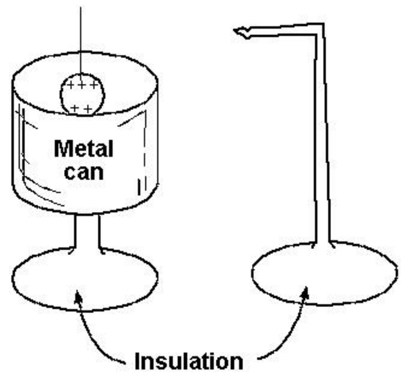


E



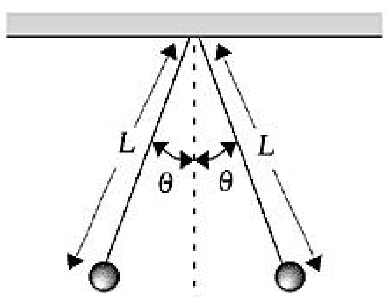
13. A charged metal ball is lowered into an insulated metal can and permitted to touch the inside of the can. If the ball is withdrawn after a few minutes, which of the following statement(s) is (are) correct?

    I. The inside of the can is charged.
    II. The outside of the can is charged.
    III. The metal ball is charged.
    IV. A new uncharged metal ball will be attracted to the metal ball, and the outside of the can.



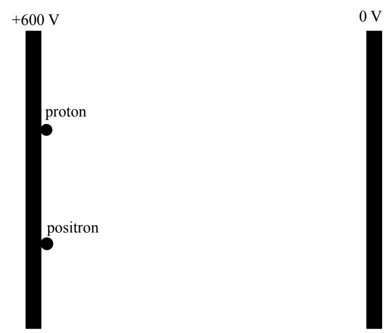
    (A) I only.
    (B) II only.
    (C) III only.
    (D) II and III only.
    (E) II and III and IV only.

14. Two small conducting spheres, each with mass m and charge $+q$, are suspended from a point by threads of length L. The horizontal distance between the two spheres is R. Which of the following statements about the magnitude of the tension in the string is correct?



    (A) $T = m\,g$.
    (B) $T < m\,g$.
    (C) $T > m\,g$.
    (D) $T = m\,g\ (L/R)$.
    (E) $T = m\,g\ \left(L/R^2\right)$.

15. Two very flat,metallic plates separated a distance d have a potential difference of 600 Volts between them as shown below. A proton and positron (which is a positively charged electron) are released from rest at the surface of the metal plate held at 600 Volts. Which of the following statement(s) is (are) correct?



I. The positron's acceleration is larger than the proton's acceleration.
II. The positron's acceleration is the same as the proton's acceleration.
III. The positron's acceleration is smaller than the proton's acceleration.
IV. The positron's kinetic energy will be larger than the proton's kinetic energy right before the particles hit the 0 Volt plate.
V. The positron's kinetic energy will be the same as the proton's kinetic energy right before the particles hit the 0 Volt plate.
VI. The positron's kinetic energy will be smaller than the proton's kinetic energy right before the particles hit the 0 Volt plate.

(A) I and IV only.
(B) II and VI only.
(C) III and VI only.
(D) I and V only.
(E) II and V only.

Last Name: KEY First Name: KEY

Physics 102 Spring 2007: Exam #1 —Multiple-Choice Answers

	A	B	C	D	E
1	X				
2					X
3		X			
4					X
5				X	
6			X		
7	X				
8		X			
9					X
10				X	
11			X		
12	X				
13		X			
14			X		
15				X	