Information Processing & Management Vol. 18, No. 2, pp. 43–53, 1982
Printed in Great Britain.

0306–4573/82/060043–11$03.00/0
Pergamon Press Ltd.

UNTANGLING CITATION NETWORKS†

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(*Received for publication* 10 *December* 1981)

Abstract—The pattern of citation links between articles permits a placement in a space of points representing these articles. Using the rule that articles are placed at the centroid of the articles to which they are linked, pictures can be drawn revealing key papers and their descendants. This simple rule for placing papers by their citation pattern may also be used to scale other citation-based data. The example used in this paper is the placement of overlapping groups of articles linked by shared articles.

INTRODUCTION

Pictures of articles joined by citations may reveal classic papers and assist in identifying their ancestors and descendants. In some pictures, articles are represented as points on a line with similar articles close to each other. In other cases, the points represent articles placed in a 2-dimensional plane or 3-dimensional box. These points are not placed at random, but according to the structure dictated by the citations linking the articles. One may view the goal of the article placement procedure as an untangling of the citation network that reveals the structure of the cited and citing article. One method for producing these pictures is centroid scaling[1, 2] which locates cited and citing articles in a multidimensional space. Once this space is generated by the centroid scaling, it may be plotted either as a single multidimensional space containing all articles or as overlaying multidimensional spaces, one for each year of published articles. One may then use these maps to examine the citation history of a set of articles over time or to view a static placement of articles within a given year or range of years. This paper shows how the centroid method may be used to generate these pictures for a variety of different citation-based data.

The similarity between an article and the articles on its reference list is one of the cornerstones of citation analysis[3]. Many different techniques have been used to graphically display the pattern of citation linkages as exemplified by Garfield's[4] diagonal display historiographs. Lacking a systematic way of placing the citation network articles in the display, however, has lead to the more recent concentration on the linkages between cited articles or between citing articles. In neither case is the full complexity of the cited and citing sides of the citation network considered. When scaling only cited articles, the citation of two articles by a single article is used to indicate the similarity between cited–cited article pairs. By counting the number of articles that cited both articles one can therefore measure the similarity between the cited articles. Similiar articles should in general be highly co-cited and should have a similar co-citation pattern with respect to all other cited articles. This justifies the scaling and clustering of articles using their degree of co-citation as a measure of article to article similarity. Likewise, the degree of overlap of reference lists of two articles measures the degree of bibliographic coupling which may be used to cluster and map the citing articles. Therefore, using co-citation and bibliographic coupling one can generate separate maps of the cited and citing articles, but there is currently no way to overlay the two maps.

THE CENTROID METHOD

The centroid method[2] used in this paper overcomes this need to concentrate either on cited or cited articles. By using only the similarity between each article and the articles it cites or is cited by both cited and citing articles may be located in a common space. For purposes of graphic display, articles may subsequently be classified by year of publication separating the common space into interlocking yearly multi-dimensional spaces.

†I would like to thank Susan Cozzens, Bob Coward, and Henry Small at the Institute for Scientific Information for providing data and assisting in the analysis of the analgesia, plate tectonics, and Lesch-Nyhan data sets respectively. This material is based upon work supported by the National Science Foundation under Grant SRS-7912157.

The centroid method is based on the simple principle that linked items should be located near each other in a space. For citation data, the centroid method locates each article at the centroid of the articles it cites and at the centroid of the articles it references. So, if one knows the locations in space of all cited items, then each citing item is placed at the centroid of its reference list. Similarly, given the location of the citing articles, each cited article is placed at the centroid of the articles that cite it. Placing articles when neither cited nor citing articles are fixed in the space may be done using an iterative procedure. Starting with an arbitrary assignment of scale values to the citing articles, place each cited article at the arithmetic mean of the scale values of the articles referencing it. These are the new estimates of the cited-article scale values. Next, compute new estimates of the citing-article scale values by placing each citing article at the arithmetic mean of the scale values of the cited articles it references. This procedure is repeated many times until the assignment of scale values for both citing and cited articles is invariant from one iteration to the next (up to multiplication by an expansion factor). The actual computation procedure involves a somewhat more sophisticated method as illustrated using the following example. Consider the case where two articles contain references to three other articles with article A citing articles C and D while article B cites D and E (a matrix representation of this citation pattern appears in Table 1 where a one indicates that the row article cites the column article and a zero indicates the lack of a citation). This matrix is next expanded into the square matrix that appears in Table 2. A one in this matrix means that either the row article cites the column article or the row article is cited by the column article. Next, since each article is placed at the mean scale values of those articles to which it is linked, divide the elements in each row by the number of 1's in that row (see Table 3). The matrix is now ready to be processed using a standard eigenvalue–eigenvector routine as outlined in the appendix. One solution is:

$$A = -\,-2^{-3/2} \quad B = 2^{-3/2}$$

$$C = -\,-\tfrac{1}{2} \qquad D = 0 \qquad E = \tfrac{1}{2}$$

with expansion factor $= 2^{-1/2}$.

Table 1. A hypothetical citation matrix in which article A cites articles C and D and article B cites D and E

		cited articles		
		C	D	E
citing articles	A	1	1	0
	B	0	1	1

Table 2. Expanded form of citation matrix in Table 1. A one indicates that the row articles cites or is cited by the column article

	A	B	C	D	E
A	0	0	1	1	0
B	0	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	0
E	0	1	0	0	0

Table 3. Expanded citation matrix with each element divided by its row sum

	A	B	C	D	E
A	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
B	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
C	1	0	0	0	0
D	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
E	0	1	0	0	0

To see that this is a stable solution note that A = mean of the scale values for the papers it references divided by the expansion factor

$$A = (C + D)/2 \div 2^{-1/2} = (-\tfrac{1}{2} + 0)/2 \div 2^{-1/2} = -2^{-3/2}$$

similarly

$$B = (D + E)/2 \div 2^{-1/2} = (0 + \tfrac{1}{2})/2 \div 2^{-1/2} = 2.$$

C = mean of the scale values for the papers citing it divided by the expanion factor

$$C = A \div 2^{-1/2} = -2^{-3/2} \div 2^{-1/2} = -2^{-1} = -\frac{1}{2}$$

and similarly,

$$D = (A + B)/2 \div 2^{-1/2} = (-2^{-3/2} + 2^{-3/2}) \div 2^{-1/2} = 0$$

$$E = B \div 2^{-1/2} = 2^{-3/2} \div 2^{-1/2} = 2^{-1} = \tfrac{1}{2}.$$

This is the essence of the eigenvalue–eigenvector solution since all scale values are stable (up to multiplication by a contant) from one iterate reassignment of values to the next.

APPLICATIONS OF THE CENTROID METHOD

Citation data

The simplest application of the centroid method separates the articles into two classes: cited articles and citing articles, and generates one multidimensional space for each. In Noma[2], the centroid method scaled 11 cited articles and 52 citing articles on analgesia. The 2 dimensional space reveals a clustering of the 63 articles into three scientific specialties. A two dimensional plot of the analgesia articles (Fig. 1) is similar to maps produced by multidimensional scaling or factor analysis of the 11 article co-citation matrix. However, the centroid method also permits

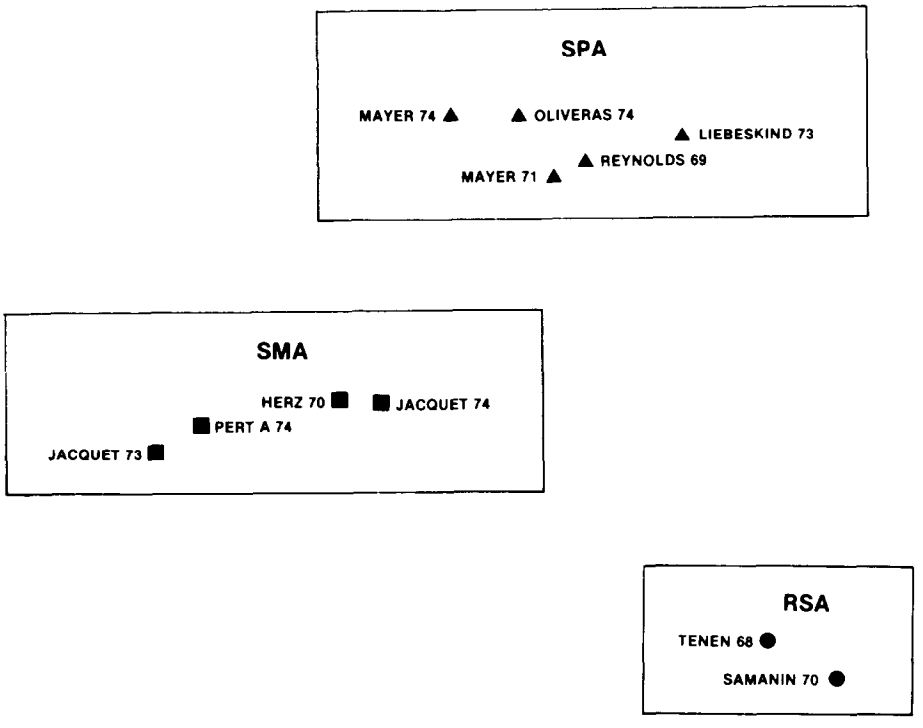

Fig. 1. Plot of two-dimensional centroid solution for cited analgesia papers.

one to plot all 63 cited and citing articles in the same graph as shown in Fig. 2. In this figure, the relative locations of the 3 clusters of citing articles is revealed as well as the locations of the citing article clusters. An alternative method for graphic presentation is to visualize the cited and citing articles as points lying two parallel planes in the space. In Fig. 3 the 63 analgesia papers are plotted on two planes with the cited articles in the lower plane and the citing articles are in the upper plane. Lines connecting all citing–cited article pairs complete the description of the citation network. To emphasize the pattern of citations among the three clusters all clustered articles may be collapsed into a point in either the cited or citing plane (Fig. 4). Lines between clusters indicate that at least 10% of the potential cluster to cluster citation links are present in the citation data.

The centroid procedure may also be used to reorder the rows and columns of original citation matrix. In this matrix the rows correspond to the citing articles and the columns correspond to the cited articles with a one in row 10 column 6 meaning that the 10th citing article referenced the 6th cited article. Lack of a citation is represented by a zero. Table 4 shows the citation matrix for the analgesia data set with the rows and columns ordered according to the first dimension of the centroid solution (for clarity blanks replace zeros in the table). The general pattern is a "broken parallelogram" in which the ones (citations) tend to concentrate around the main diagonal with zeros (lack of a citation) concentrating in the upper right and lower left corners. Citations deviating from the "broken parallelogram" may be of substantive interest and some of the block modeling[5] techniques may be used to model the citation pattern.

So far in this paper the centroid method has been applied only to a data set in which there exist two distinct classes of articles: cited and citing, with no links within either the cited set or the citing set. The general model of the centroid method, however, is not restricted to the placement of two disjoint sets of articles in a space. The method merely places each article at the centroid of the articles on its reference list and at the centroid of the articles that cite it. This means that an entire citation network may be scaled using this method. In such a network, an article may be a cited article with respect to some and a citing article with respect to others.

Once the centroid method has located articles in a space, all articles published in a given year may be displayed as points in a multi-dimensional space aligned with other yearly spaces. If

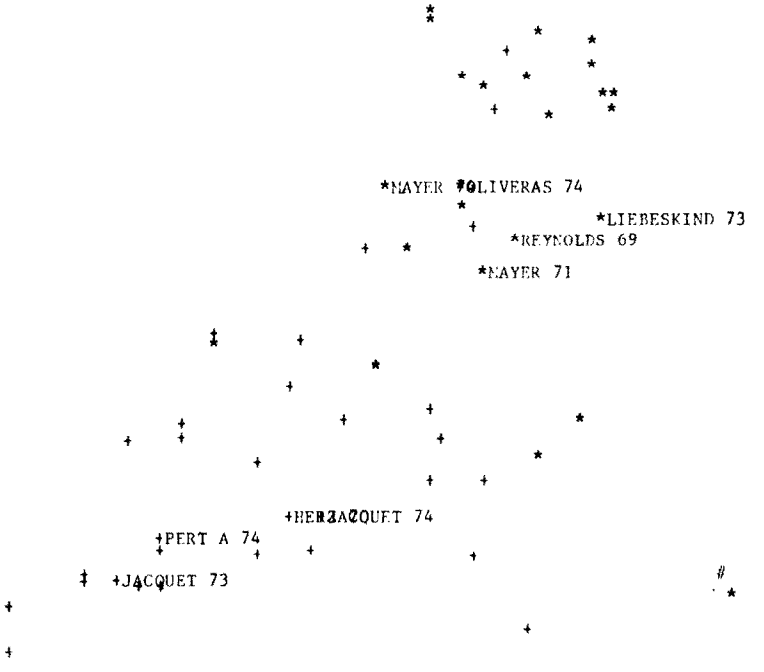

Fig. 2. Two-dimensional plot of centroid solution showing the locations of both the citing and cited analgesia articles. The symbols classify the papers according to research area: sites of morphine action = solid square, stimulation produced analgesia = solid triangle, role of serotonin in analgesia = solid circle, unassigned = open square.

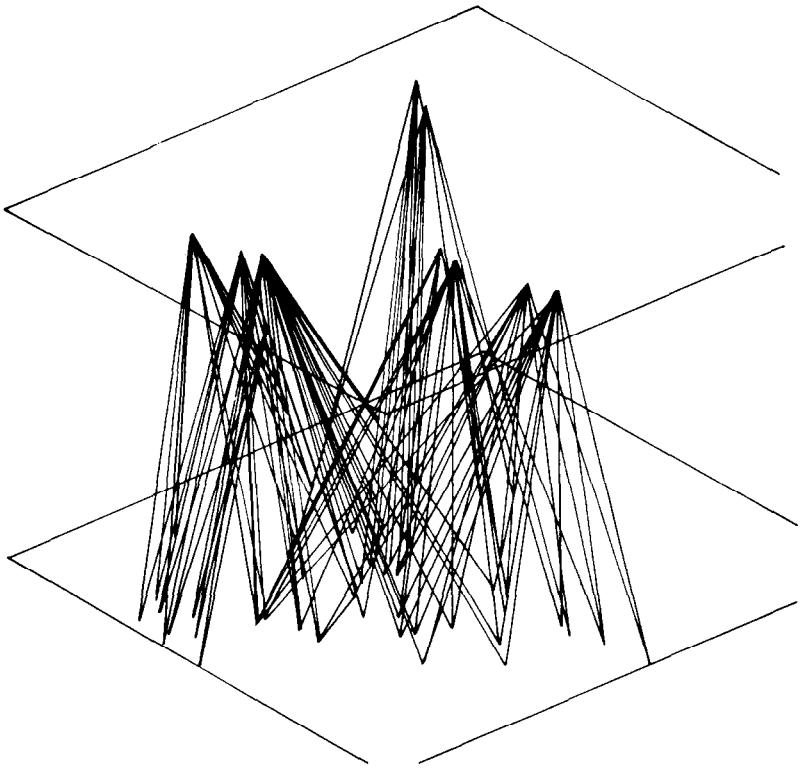

Fig. 3. Two two-dimensional plots of centroid solution for analgesia articles with each line corresponding to a citation from a citing article on the lower plane to a cited article on the upper plane.

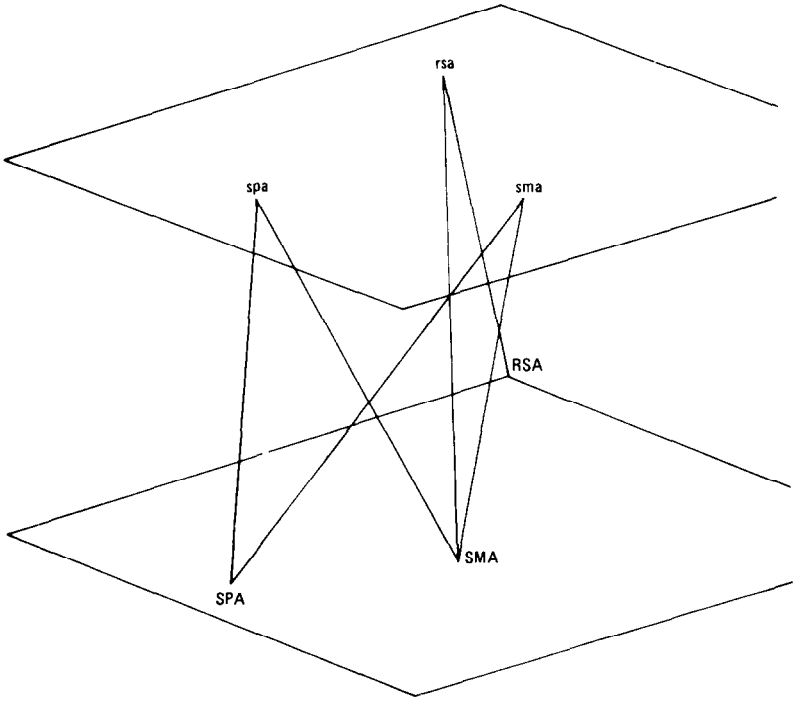

Fig. 4. Two two-dimensional plots of the centroid solution for the analgesia papers with the papers condensed according to their classifications as stimulation produced analgesia (spa), sites of morphine action (sma), or role of seritonin on analgesia (rsa). The condensed citing articles appear on the lower plane and the cited articles appear on the upper plane.

Table 4. Analgesia citation matrix with rows (citing articles) and columns (cited articles) ordered by centroid scale values

	JACQUET 73	PERT A 74	HERZ 70	JACQUET 74	MAYER 74	OLIVERAS 74	MAYER 71	REYNOLDS 69	LIEBESKIND 73	TENEN 68	SAMANIN 70
PERT CB (A)	1	1									
CHANG	1	1									
PERT CB (C)	1	1									
PERT CB (B)	1	1	1								
YAKSH (A)	1	1	1								
ELSOBKY	1	1			1						
LASCHKA		1	1								
WEI	1		1	1							
LEYBIN	1		1	1							
PERT A	1	1					1				
YAKSH (C)	1	1	1			1					
GORDON		1			1						
CHIA	1	1			1	1					
LOH			1	1							
DOSTROVS.		1	1	1		1					
HARRIS		1	1	1	1		1				
SHERMAN	1	1		1	1		1	1			
YAKISH	1	1	1			1				1	
CRISWELL	1		1	1			1		1		
FREDERIC (B)	1				1		1	1			
BRADLEY			1				1				
BELLUZZI			1				1				
AKIL			1	1	1	1	1	1			
MELZACK					1	1					
POMERANANZ					1	1					
CHAN		1	1	1	1	1	1		1	1	
LEBARS	1	1	1	1	1	1	1		1	1	1
MAYER	1	1	1	1	1	1	1	1	1	1	1
FUNG					1	1	1				
BOWSHER		1				1	1	1	1		
DEY		1	1	1	1		1	1		1	1
JURNA			1			1			1		
JACQUET	1		1	1	1	1		1	1	1	1
GIESLER					1	1	1	1			
FREDERIC. (A)					1		1	1			
YOKOTA						1	1				
HAIGLER		1		1			1	1		1	1
BEALL					1	1		1	1		
WANG			1				1	1		1	
MCCREERY					1				1		
RICHARDS							1	1			
ALBEFESS			1				1	1	1		1
HUANJI						1			1		
SASA						1			1		
SOPER							1	1	1		
ADAMS							1		1		
SOVAK							1		1		
SAMANIN						1			1	1	1
RANDIC							1		1	1	1
WARWICK										1	1
STEFANIN										1	1
TULUNAY										1	1

yearly two dimensional planes are ordered from early years to later years, the graphic display is a natural elaboration of the citing-cited parallel planes (Fig. 3). This model of multiple planes, each with articles published within a given 12 month period, is similar to Price's[6] onion skin model. It is also similar to Garfield's graphical presentation of citation data in a "historical map" or "historiograph"[4], in which articles are separated by year of publication and by dissimilar citation patterns. Figure 5 shows the evolution of the citation network for papers dealing with purine metabolism and Lesch–Nyhan syndrome (for a complete discussion of the data collection and analysis procedure see Small[7]). The vertical axis represents the year of publication for each of the papers, while the dispersion of points on parallel planes was determined by a centroid scaling of the citation matrix. Each line indicates a citation. The papers were selected by picking a starting paper (Kornberg, node 1) describing some early work on the isolation and purification of the enzyme, later known as HGPRT. This paper is important since the metabolism of HGPRT is defective in Lesch-Nyhan patients. To this single-element, first generation set are added a second generation of all papers citing Kornberg (node 1) that are themselves cited at least 35 times. To the first and second generation of papers are added all papers referencing one or more of the generation one and two papers and receiving at least 80 citations themselves. The centroid method then scaled these 41 papers with their accompanying citation links to generate one multidimensional representation for each year from 1955 to 1979. Plotting the first two dimensions of each yearly configuration plus the year-of-publication dimension (Fig. 6) reveals four main branches of research stemming from the original HGPRT enzyme paper by Kornberg. First, an unrelated branch of blood presentation (nodes 13, 27, and 40) has only one link with the Kornberg paper. The first dimension scale values for papers in this branch clearly separate them from all other papers. The second dimension reveals a branch for the regulation of enzyme activity (nodes 22, 23, 35, 38, and 41). In addition to these two peripheral branches, there are two main branches corresponding to the biochemistry of purine metabolism with one branch concentrating on the HGPRT enzyme and the other concentrating on the closely related PPRP enzyme system. Interestingly, even though only one paper (Seegmiller, node 25) deals specifically with Lesch–Nyhan, it is correctly aligned with the four

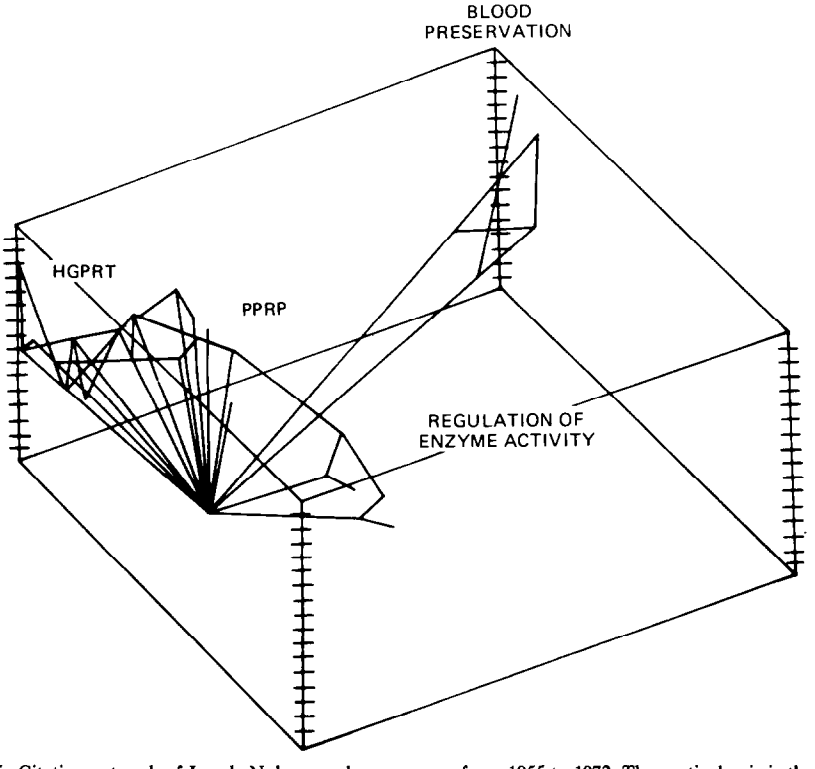

Fig. 5. Citation network of Lesch–Nyhan syndrome papers from 1955 to 1972. The vertical axis is the year of publication separating horizontal planes whose dimensions are the first two dimensions of the centroid solution. Nodes are papers and lines are citations between papers. See text for further description of the method for data collection and analysis.

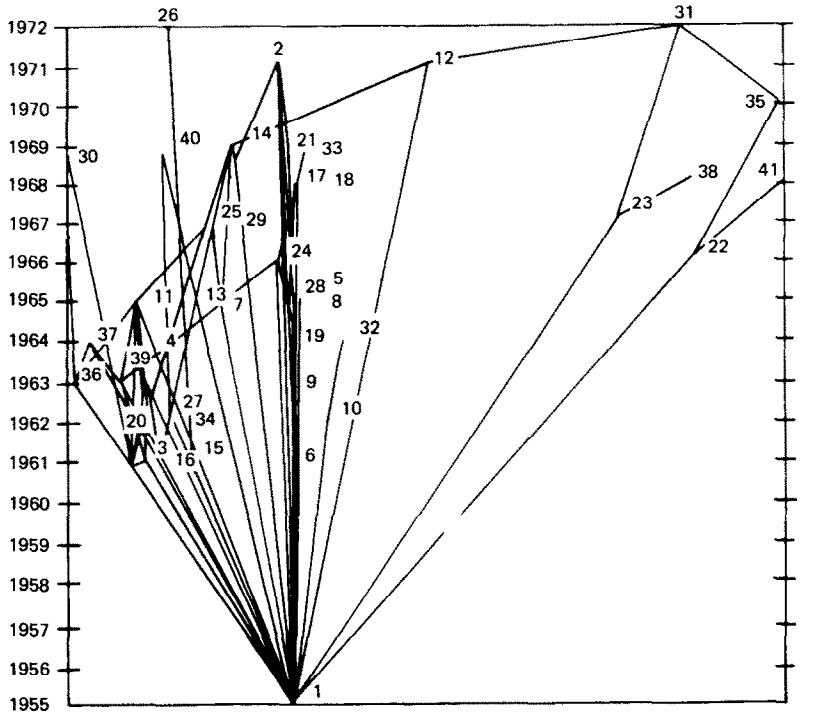

Fig. 6. Citation network of Lesch-Nyhan syndrome papers from 1955 to 1972. The vertical axis indicates the year of publication and the horizontal axis is the second dimension of the centroid solution. The numbers at the nodes are keyed to the papers. See text for further description.

HGPRT enzyme papers (nodes 1, 14, and 30). Also, examining only a part of the graphical output (Fig. 7) reveals a pentagonal pattern of papers all dealing with tumor resistance to anti-cancer drugs (4 of the 5 papers in this pattern originated at the Kettering Meyer laboratory).

Cluster data

The centroid method is not restricted to the scaling of citation matrices. It may, in fact, be applied to any data set in which each item is to be placed at the centroid of the items to which it is linked. The important consideration is that the lack of a link is not considered the negation of a link. Overlapping clusters of scientific articles may be viewed as one such data set. In this case, each cluster is an item to be placed in a space with linked clusters having one or more articles in common. To illustrate the scaling of clusters, groups of highly co-cited articles compiled from the *Science Citation Index* (*SCI*) were analyzed. Each year the Institute for Scientific Information extracts all highly cited scientific articles from the *SCI* and groups articles that are frequently cited together (see Garfield[8]) for a full description of the co-citation clustering procedure). In 1979 for instance, 10265 articles were grouped into 2336 article clusters. Next, articles that cluster in the current year are matched against articles that clustered in the previous year. Each article that clustered in both years is a link between the current year and previous year clusters it belongs to. Links between clusters are an indication of year to year continuity of a scientific specialty and permit a scaling of cluster strings across time. The number of articles shared by a pair of clusters may be used to construct a cluster to cluster linkage matrix similar to a citation matrix. The centroid scaling then operates on the matrix to place each cluster at the centroid of the clusters it is linked to. Figure 8 is the centroid solution for a small subset of the clusters extracted from the 1970 to 1979 *SCI*. Many of the Fig. 8 clusters had been previously identified with the geology of plate tectonics and the centroid scaling method organizes the clusters into a large central core of closely related tectonics clusters from which two branches sprout around 1977. These two branches consist of clusters containing with geochemistry articles, but articles in one branch are cited by articles researching the characteristics of the mantle while the other branch is cited by geochemistry articles.

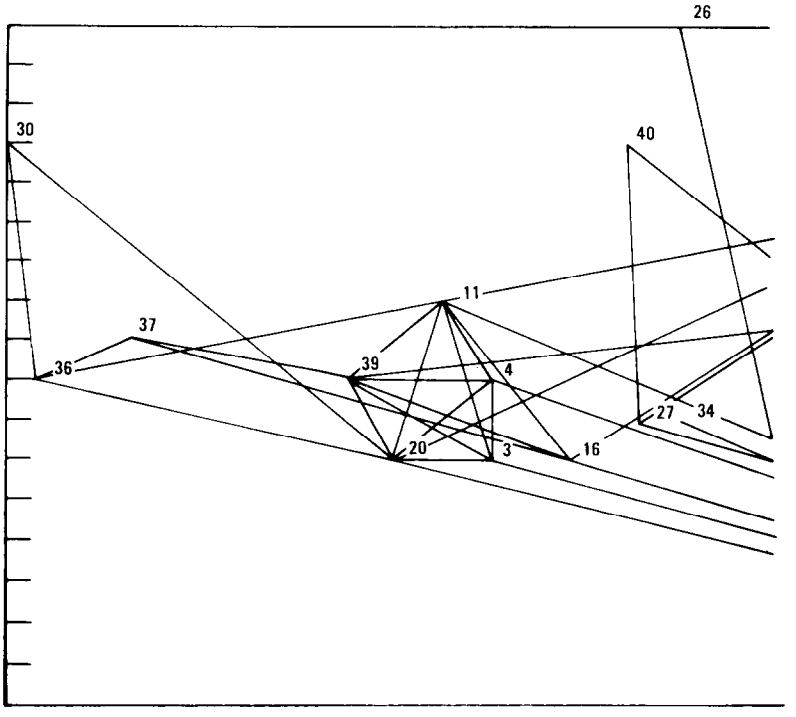

Fig. 7. Magnified view of dimension 2 of the citation network of the Lesch–Nyhan syndrome papers shown in Fig. 6.

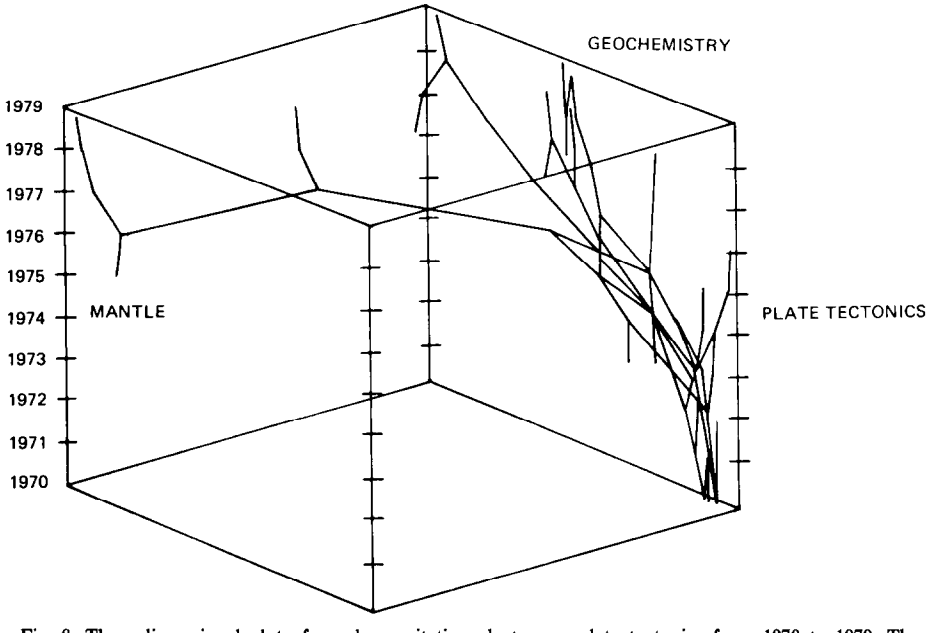

Fig. 8. Three-dimensional plot of yearly co-citation clusters as plate tectonics from 1970 to 1979. The vertical dimension is the year of the cluster and the two dimensions for each horizontal plane are the first two dimensions of the centroid solution. Each link indicates at least one article appears in both clusters. Links are only tabulated between clusters in consecutive years.

DISCUSSION

As we have seen, the centroid scaling method offers a useful way to untangle citation networks for a graphic display of the citation history. The method also signals a return to the original assumption underlying citation analysis: an article is substantively similar to articles on its reference list. The centroid method also may be viewed as a simultaneous analysis of co-citation and bibliographic coupling (see Noma[2]) so that both citing and cited articles are

concurrently scaled. Even though the centroid method successfully locates articles in a space, the problem of an effective method for presentation remains. The centroid alleviates some of these problems due to its tendency to split articles into widely separated branches, but eventually the number of citation links overwhelms the resolution of the graphic display. One possible solution to this problem may be plot cited article and draw a circle around it representing its capture radius for obtaining citations. Highly cited articles with references from diverse parts of the space would have large radii, while moderately cited articles receiving references only from extremely specialized papers will have small radii. The overlap of the radii may be interpreted in the same way as a co-citation measure (see Small[9]). Use of full color graphic displays may also prove useful in the search for structure inherent in the citation network. The centroid method, in all cases, simplifies the task of graphic presentation by offering a systematic procedure for placing items in a space using all the available citation data.

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APPENDIX

The centroid method assigns scale values to items locating each item at the centrod of the items to which it is linked. Computationally, start with a matrix, A, containing the linkage strengths between all items pairs. For citation data, $Aij = 1$ means that items i and j are linked with a strength of one (i.e. i cites j or i is referenced by j) and $Aij = 0$ indicates the lack of a link (i.e. lack of a citation or reference). Note that since i cites j only if j is referenced by i and vice versa, the A matrix is always symmetric ($Aij = Aji$) for citation data. Item i is located at the centroid of the items to which it is linked, so one may compute the location of item i by scanning the ith-row of matrix A and summing the coordinates of all items linked to A. Dividing the sum by the number of linked items yields the location of item i (up to multiplication by an expansion factor, λ, which will be discussed shortly):

$$\lambda Xi = D^{-1}ii(\Sigma AijXj) \quad \text{for all } i \text{ where } Dii = \Sigma Aij$$
$$j=1 \qquad\qquad j=1 \qquad\qquad \text{(A1)}$$
$$Dij = 0 \quad i \neq j.$$

Since all items must simultaneously satisfy equality (A1), this equality may be written in matrix form:

$$\lambda X = D^{-1}AX. \qquad \text{(A2)}$$

Fortunately, procedures for derriving the X's that satisfy eqn (A2) are readily available on many computer systems; the centroid scaling has been reduced to solving for the eigenvalues and eigenvectors of the $D^{-1}A$ matrix. The details of the computational procedures for generating solutions are discussed in Levine (1978) and Noma (1981) and will not be discussed here. Instead, emphasis is placed on the interpretation of resultant output.

The λ expansion factors (eigenvalues) are of interest when interpreting the solution, and determining the dimensionality of the representation of the journals in space. Each set of X's that satisfy equalities (A1) or (A2) have an expansion factor. Solutions with factor equal one partition items into one or more categories with no links between items across different partitions. There is at least one solution with expansion = 1. This solution places all items at the same point in the space and is ignored. Solutions with the next larger expansion factors ($\lambda \neq 1$) are the most representative of the linkage pattern between items and are used to plot points in the space. The first dimension of the centroid solution is the eigenvector with the largest non-unitary expansion factor. These scales are used in this paper. Solutions with smaller expansion factors are less representative of the linkage structure and may be ignored when plotting the items and finally, solutions with zero or negative expansion factors are ignored.