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The self-similar science system [1]

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Abstract

A system with a self-similar property is scale-independent and statistically exhibits that property at all levels of observation. In addition, a power law describes the distribution of a scale-independent property. Many investigators have observed social activities and structures, particularly in the science system, that are best described by a power-law distribution. However, unlike classical physical power laws that are used in the design of complex technical systems, social power laws are not used to develop social policy. Using the science system as a model social system and peer-reviewed publications and citations to these papers as the data source we will demonstrate the existence of two power law distributions that are then used to predict the existence of two additional power laws. In fact, it will be shown that in four UK sectoral, six OECD national, a regional and the world science systems the Matthew effect can be described by a power-law relationship between publishing size (papers) and recognition (citations). The exponent of this power law is 1.27 ± 0.03, it is constant over time and relatively independent of system size and nationality. The policy implications of these robust self-similar social properties as well as the need to develop scale-independent policy are discussed. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

A system with a self-similar property exhibits a statistically similar characteristic when examined at the level of individual entities, collections of entities or the system as a whole (Mandelbrot, 1983). In other words the same general characteristic can be seen locally and globally and thus it is independent of the scale at which the observation is made. Furthermore, the scale-independent nature of a self-similar property is characterised by a power-law distribution. We will show that the Matthew effect in science (Merton, 1968; Merton, 1988) and other structural features of the science system are self-similar from the level of a sectoral, domestic, and regional science system through to the level of the world science system.

For over a hundred years, observers of society have noticed that some human activities and structures are self-similar and characterised by a power-law relationship generally defined by the following relationship

$$y = ax^n \quad (1)$$

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For example, Zipf (1949) noticed that the frequency of each word in a written communication decreased in a power-law relationship with respect to the relative frequency (rank) of the word in a text. Lotka (1926) found that the number of scientists publishing a given number of papers in a fixed period of time decreased in power-law relationship to the number of papers published. And Pareto (1987) saw that in some countries the number of people earning an income of a given amount decreased as a power of the income. Many investigators have empirically confirmed these and other power-law distributions enhancing their status as candidates for consideration as 'social laws' (Haitun, 1982a,b,c).

In two general ways, these potential 'social power laws' differ from classical physical power laws such as the gravitational force between two masses which varies as the inverse square of the distance between them. First, the observed exponent of a social power law is usually non-integer while that of a physical law is integer. [2] Second, the exponent of a social power law usually varies somewhat from one social system to the next. [3] On the other hand, the exponent of classical physical laws like the gravitational law is the same for all systems throughout the universe. These two differences hinder our ability to use social power laws to formulate social policy even though scientists and engineers can use physical power laws to construct complicated technical systems such as buildings, computers and space stations.

First, this paper will examine some general characteristics of a power-law distribution that is associated with a self-similar property. Next it will explore the science system as an example of a social system and discuss the Matthew effect in science. Then, using publication and citation data, we will show that the Matthew effect and other structures in a science system can be quantified with a power law establishing that the science system has self-similar properties. Finally the paper will conclude with a discussion of the policy implications of a self-similar social system.

1.1. Power laws

A power-law distribution is unique and has some interesting features. A power law can be represented graphically on a log–log scale as a straight line. In other words Eq. (1) is transformed into

$$\log(y) = b + n\log(x) \tag{2}$$

where $b = \log(a)$ and Eq. (2) has the general form of a linear relationship. The slope of the linear log–log line is simply the exponent of the power law and it is frequently called the Hausdorff–Besicovitch or fractal dimension (Stewart, 1989). Simply speaking an object with, for example, a Hausdorff–Besicovitch dimension of 1.28 has a power-law characteristic that resides in a dimension larger than 1.0 (such as a line) but lower than 2.0 (such as a plane). In other words, the characteristic occupies more than one dimension but less than two dimensions; it occupies a fractional dimension and it is said to have a *fractal dimension* of 1.28.

A power law is one of the common signatures of a nonlinear dynamical process, i.e., a chaotic process, which is at a point of self-organised (Bak, 1991) criticality or residing on the boundary between order and disorder. Such a system is often called a self-organised system (Gellman, 1994) because it exhibits structure not merely in response to inputs from the outside but also, indeed primarily, in response to its own internal processes (Krugman, 1996).

Finally, a power law is indicative of the existence of a scale-independent or self-similar (Mandelbrot, 1983) property. A self-similar property is, statistically speaking, a property that is similar at all levels of observation, i.e., from the individual to the whole. Also, a scale-independent distribution is characterised differently than a scale-dependant distribution. A scale-dependant distribution like the normal distribution is usually characterised by a mean and a variance (or standard deviation). While a scale-independent distribution like a power-law distribution may have a mean, it can be unstable over time and

[2] Or the reciprocal of an integer (e.g., square root).

[3] For example the exponent of the Lotka distribution will vary depending on the community of authors that is examined (e.g., the collection of journals used when calculating the exponent).

its variance is usually undefined or infinite [4] (Peters, 1991; Shlesinger et al., 1993). A power-law distribution is normally characterised by its exponent and the standard error of the exponent. The jaggedness of a coastline is an example of a self-similar property and it can be graphically illustrated by a geometric fractal. When observed up close or far away the jaggedness of a coastline can look similar. The power law that describes the distribution of the length of a coastline (Stewart, 1989) with respect to the observation resolution usually has a fractal dimension between 1.15 and 1.25. The fractal dimension of two coastlines may be similar but are rarely exactly the same.

A prerequisite for finding any law in a natural, physical or social system is having accurate, detailed observations about a system (Katz and Katz, 1999). Locating reliable data is difficult and acquiring such data can be costly. Some likely candidates as sources of social data are census, economic and publication data. However, census and much of the economic data are frequently derived from self-reporting questionnaire surveys and the accuracy of such data can be highly questionable. In addition, personal privacy laws and corporate confidentiality make the accurate collection of this type of social data even more problematic. Publication data uses information published in the public domain such as scientific papers and technical patents. These data are not noise free but even so, a carefully constructed index of publications can provide reasonably comprehensive data about formal and informal communications in a social system, in this case the communication of research findings among scientists through journal papers. Although the index data may contain some noise due to data entry and acquisition errors or spelling, address and referencing errors in the original publication, it is likely that at least refereed scientific papers will likely contain less noise than self-reported economic and census data. In addition, many observers of the scientific community such as Derek de Solla Price have confirmed the existence of power-law distributions in the publishing activity of scientists (De Solla Price, 1963).

1.2. The science system

Science is a social activity that affects society. While there may be some debate about the magnitude of the scientific community's economic contribution to society there is no question that it is significant (Martin and Salter, 1996). In this paper, the science system will be used as an example of a social system. As mentioned earlier, there are some excellent sources of high-quality data about scientific publishing and citation activities in the science community and some power-law relationships have already been established. First let us explore some general characteristics of the science system and its publishing process.

Scientists strive to understand nature. They reside mainly within publicly funded institutions such as universities, hospitals, research council laboratories and government agencies. This can be confirmed by examining institutional participation in refereed UK research papers indexed in the 1994 Science Citation Index (Hicks and Katz, 1996). Of the more than 43,000 publications university, hospital and research council researchers participated in 65%, 27% and 10%, respectively, of these publications. On the other hand, although significant, the participation from the industrial and nonprofit sectors was only 8% and 2%, respectively. Moreover, more than 50% of these papers were produced in collaboration with public sector scientists.

Scientific research is costly. For example, in 1994 the UK government spent US$4 billion or 0.4% of GDP on public R&D. Funding to the science community was channeled through government agencies like the Higher Education Funding Council of England (HEFCE) and the Office of Science and Technology. Dynamic processes internal to the scientific

[4] Self-similar distributions can also be described by the Levy distribution which is given by $\log(f(t)) = i\delta t - \gamma|t|^{\alpha}(1 + i\beta(t/|t|)\tan(\alpha\pi/2))$ where δ is the location parameter of the mean, γ is a scale parameter, β is a measure of the skewness (ranges from -1 to $+1$) and α measures the peakedness of the distribution (ranges from 0 to 2) and the fatness of the tail. Also, α is equal to the fractal dimension of a times series (see Peters, 1991). If $\alpha = 2$ then the distribution describes a normal distribution with a stable mean and a well-defined variance. If $1 \le \alpha < 2$ then the distribution has a stable mean but the variance is undefined, or infinite. And if $0 < \alpha \le 1$ then the distribution has an unstable mean and an undefined variance (Peters, 1991).

community largely determine how much support is given to a research activity. However, the broad structure and relative size of general research disciplines such as medicine are strongly influenced by national priorities and public attitudes.

In general, *anonymous peer review* mechanisms provide some quality control (Pasternack, 1966) of the evaluation and public reporting processes in the scientific community. For example, project funding and journal publications are usually evaluated anonymously by a small group of international peers who are knowledgeable about the research area. The origins of peer review can be traced to the minutes of Royal Society from March 1, 1664 to 1665 when the council ordered that publications in *Philosophical Transactions* be reviewed first by some of its members (Porter, 1964). Although peer review has its inefficiencies, Merton (1973) claims that practicing scientists see it as crucial to the development of science.

Scientific investigation can be rewarding and generate professional recognition. This recognition can come in several forms. It can come through the peer review process exemplified by receiving a research grant from a funding agency or having a scientific paper accepted for publication in a refereed journal. The latter is recognition that the reported findings appear to make a contribution to the existing knowledge base and the journal editor and referees feel that the paper should be placed in the public domain for wider scrutiny. If the published research impacts on another researcher, either positively or negatively, then scientific etiquette requires that this fact should be recognised through a citation to the published work in subsequent publications. A significant contribution, that is, one that impacts on a large portion of the scientific community, may be recognised by more prestigious rewards such as an honorary degree or a Nobel prize. What is the nature of the relationship between scientific output and recognition?

1.3. The Matthew effect in science

Recognition appears to accumulate with increasing presence in the science system. This effect was named the ‘Matthew effect’ in science by Robert Merton in a 1968 *Science* paper which he updated 20 years later in an *ISIS* article (Merton, 1968, 1988). This classical observation in the sociology of science is based on the general observation that those with a large presence in a community gain more recognition compared to those with little presence or as the old adage says, *the rich get richer while the poor get poorer*. In other words, as scientists and scientific institutions participate in the science system they gain an accumulative advantage that brings them increasing rewards. Each successive increment of advantage widens the gap between the have and have-nots in science (as well as in many other social domains). By way of evidence Merton pointed to the skewed publication and citation distributions for scientists as well as the skewed distribution of resources and productivity for institutions. In fact, Merton says

Intellectual property in the scientific domain that takes the form of recognition by peers is sustained, then, by a code of common law. This provides socially patterned incentives, apart from the intrinsic interest in inquiry, for attempting to do good scientific work and for giving it over to the common wealth of science in the form of an open contribution available to all who would make use of it, just as common law exacts the correlative obligation on the part of the users to provide the reward of peer recognition by reference to that contribution. (ISIS, p. 622)

Merton used a collection of observations, most of them power-law distributions, and speculated that as the output of scientists and institutions increases the recognition they accumulate increases in a disproportionate manner.

This paper will examine the Matthew effect in science in greater detail and attempt to quantify a relationship between publication size and recognition at various levels of aggregation in the science community. Size will be measured using numbers of peer-reviewed scientific papers and recognition [5] measured using the number of citations received by these papers. There is some evidence to support the

[5] As previously mentioned, recognition comes in many forms (awards, prizes, position, etc.). From this point on we will only explore the recognition that is acquired by citation to peer-reviewed publications.

notion that this relationship is a power law. Naranan (1970) reported a power-law distribution for citations to articles in the Genetics Citation Index (1961). Both Naranan and Allison (1980) have suggested that perhaps recognition as well as publication size has a power-law distribution throughout the scientific community.

This paper will demonstrate using accurate data and counting techniques that it is possible to quantify a Matthew effect in science. It will be shown that from the unintentional collective action of scientists in institutions publishing peer reviewed papers and then recognising these papers in the references of subsequent publications

- there emerges a power-law relationship between the amount of recognition (citations) received by members of a scientific community and their publishing size; and
- the exponent of the power law, 1.27 ± 0.03, is fairly constant with time and relatively independent of the nationality and size of a science system.

In addition to the Matthew effect, this paper will also demonstrate that there is a power-law relationship between publishing size (papers) and rank order of the publishing size of communities in the science system. It will be shown that for the twenty-five largest publishing communities in a science system

- there emerges a power-law relationship between the size of a scientific community (papers) and its size rank in the community; and
- the exponent of the power law, -0.44 ± 0.01, is relatively independent of the nationality and size of the science system; and
- the rank of a specific community in a science system can be quite unpredictable.

Finally, using the power-law relationships between (1) recognition and size, and (2) size and size rank power-law relationships between (a) recognition and recognition rank and (b) impact (citations per paper) and impact rank will be predicted and measured.

2. Data source and methodology

This section will explore the data source and the methodology that was used in this research.

2.1. Science Citation Index

The Science Citation Index (SCI) produced by the Philadelphia-based company, Institute for Scientific Information, is a unique scientific publication index database and was used as the primary data source. Unlike some index databases, the SCI provides reasonably comprehensive coverage of the significant contributions to most science areas and more details about the publications in the journals it indexes. The SCI indexes all publications published in about 3500 of the world's leading scientific and technical journals. For each item in a journal the SCI records standard bibliographic information such as journal name, title, volume, page, etc. It also records all the names and institutional addresses for the authors involved in each publication, the type of publication (article, note, review, conference proceeding, biography, etc.) and the references to other publications.

The publication type is useful for identifying peer-reviewed papers. Publication types *article*, *note* and *review* are almost always refereed while *conference proceeding* publications are sometimes refereed. Other publication types such as *biographies* are rarely refereed. ISI uses the references to compute the number of times a publication is cited by other publications indexed in ISI database. As previously discussed, citations are used as a measure of recognition in this study.

How do we know if the SCI covers a large percentage of significant contributions in science? Intriguingly, the journal coverage in the SCI is based on Bradford's law of scattering (Bradford, 1950). Bradford did not have a mathematical equation to support his law and it took a while for information scientists to formulate one (Naranan, 1970; Garfield, 1971). In general, the Bradford distribution of scientific articles in journals for a given subject can be expressed in the functional form

$$J(p) \propto p^{-\gamma}$$

where $J(p)$ is the number of journals containing p articles on a subject (Wang and Wang, 1998).

Eugene Garfield, the founder of ISI, suggested that a small number of journals could account for a large percentage of the significant contributions to all of science. In other words, what Bradford postulated for a single discipline, Garfield postulated for science as a whole. A recent citation analysis (Garfield, 1996) has shown that as few as 150 journals account for one half of the most frequently cited papers and one quarter of all published scientific papers in the SCI. In other words, Garfield speculates that an index database need only cover a small percentage of the scientific literature in order to capture most of the significant scientific contributions.

In fact, underlying Garfield's observation are two power laws. Using his data (Garfield, 1996) for the top 50 most productive and highly cited journals [6] in 1989 and 1994 we find a power-law relationship between the number of journal articles, j_a, and rank by number of papers, r_a, given by

$$j_a = k(t)\, r_a^{-0.42 \pm 0.01} \tag{3}$$

and between the number of journal citations, j_c, and rank by number of citations, r_c, given by

$$j_c = k'(t)\, r_c^{-0.60 \pm 0.01} \tag{4}$$

In both instances the exponent apparently remains constant over time. The intercepts increase with time indicating an increase in the total number of articles and citations. Also, the rank of some journals changes with time. For example, the *Journal of Applied Physics* moved from 7th rank to 3rd rank when ranked by number of articles. And *Science* moved from 5th to 4th rank and the *Journal of Chemical Physics* moved from 6th to 9th rank when ranked by number of citations. A constant exponent can coexist with dynamically changing ranks. This will be discussed more fully later.

Recall our objective is to establish a relationship between recognition and publishing size of a community, but how do we define a community using publication data? Simply, a community is defined as a group of individuals who have something in common. Therefore, a group of scientists who publish in the same group of journals defines a community. The size of the community can be measured using numbers of refereed publications as the indicator.

2.2. Methodology

Let us explore this idea more closely using ISI's 1981–1996 National Science Indicators (NSI) on diskette. This data is available from ISI at a cost of about US$5000. The NSI information for the world and various countries consists of

- the number of refereed publications (article, note, review and conference proceedings),
- in 102 Current Contents research fields, and
- citations to these publications
- for items indexed in ISI's science, social science and arts and humanities citation index databases.

For example, according to the NSI data the largest publishing community of researchers in the world science system is composed of authors publishing in journals that ISI collectively call *applied physics, condensed matter and material sciences*. Between 1981 and 1996 this community published 559,880 refereed papers and received 4,094,172 citations to these papers in the same time period. The Current Contents classification scheme is based on journals that are uniquely assigned to one research area. Now, using NSI data for the world system let us explore the relationship between recognition and publishing size for 102 communities defined by using the Current Contents research fields.

Fig. 1 is a scattergram plot showing the relationship between the number of refereed papers published in the world science system between 1981 and 1996 in each of 102 Current Contents fields and the number of citations received by these papers in the same time period. In other words, the recognition (citations) received by papers published by different research communities (Current Contents fields) in the world are plotted against the publishing size of each community. It is apparent that as publishing size increases recognition increases but exactly what is the relationship? A regression analysis showed that a power law ($r^2 = 0.75$) might be a better fit to the data than a linear ($r^2 = 0.70$), polynomial [7]

[6] These data are available on the internet from the following URLs: http://www.the-scientist.library.upenn.edu/yr1996/sept/research_a_960902.html and http://www.the-scientist.library.upenn.edu/yr1996/sept/research_b_960902.html.

[7] 2nd to 6th order polynomials were tested.

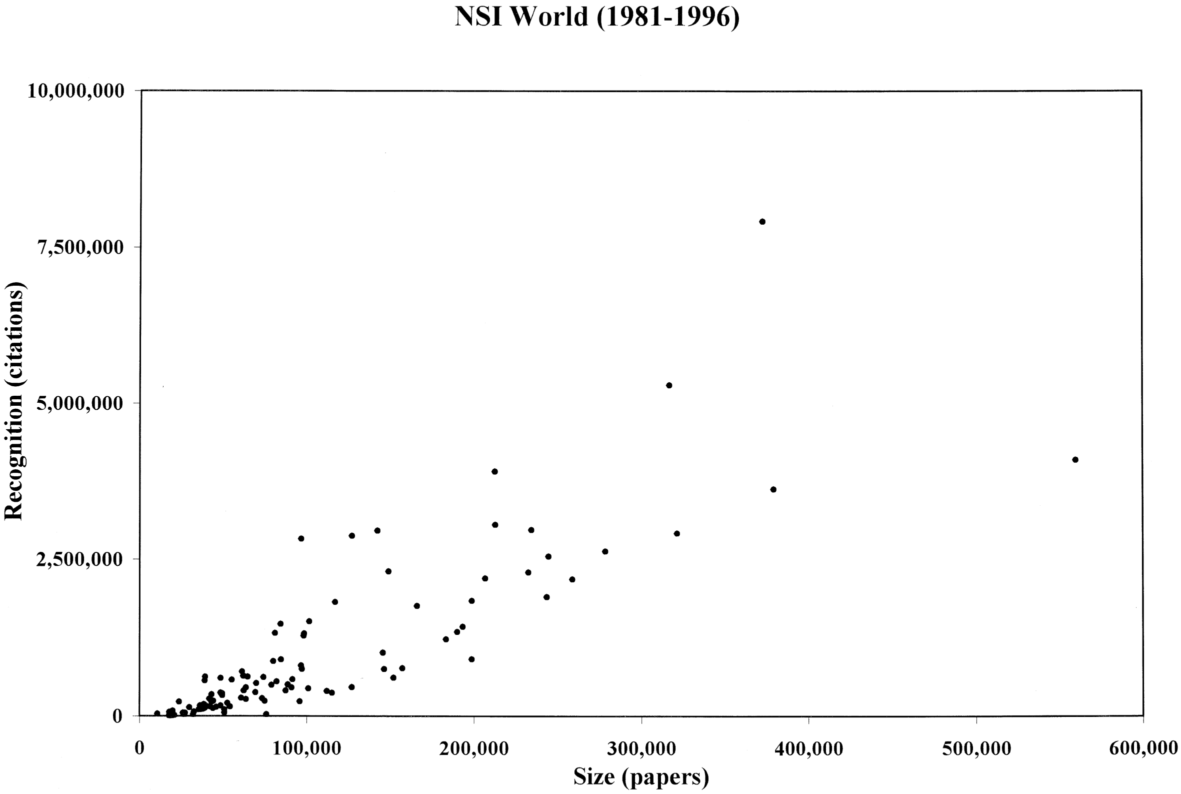

Fig. 1. Recognition vs. publishing size for the world science system (data source: ISI 1981–1996 National Science Indicators).

($r^2 = 0.71$) or exponential fit ($r^2 = 0.51$). However, the statistical difference between the various fits is not significant enough to come to a definitive conclusion.

Perhaps by using different counting techniques than the ones used by ISI to produce the NSI data some noise can be eliminated, the accuracy and resolution increased and the statistical significance of one of the regression fits improved. For example, in order to confine our observation to the science system those publications indexed in the social science and arts and humanities databases should not be included. In addition, conference proceedings could be removed from the list of peer-reviewed publication types since many of these publications are not refereed. Although this change will make relatively little difference it will confine the observation to mostly peer-reviewed publications.

ISI has a detailed journal classification scheme that it uses to classify SCI publications. There are over 150 categories in this scheme but unlike Current Contents [8] a journal can be classified into one or more categories. By allowing journals to overlap categories we get a more accurate reflection of the interdisciplinary nature of the research communities. Also, we nearly double the number of scientific communities for which we can get a publishing size measure from about 80 to over 150. More importantly, while the communities derived from the Current Contents categories in the NSI range in size from about 10,000 papers to 600,000 papers we shall see that the detailed classification gives us communities that range in size from about 500 to 300,000 papers. In other words the resolution of the publish-

[8] Within Current Contents editions, there is no overlap of journals within categories; between Current Contents editions, there may be overlapping journal assignments.

ing size measurement is increased by more than two orders of magnitude.

A reduction in noise can be achieved by changing the way citations are counted. Citations are usually counted using either a fixed or a variable citation window. The NSI data are counted using a variable citation window. In other words, a paper published in 1981 accumulates citations over the whole time interval (1981–1996) while a paper published in 1996 only accumulates citations in the publication year. Narin (1976) has shown that on average the annual number of citations to a paper increases and peaks in the 3rd or 4th year after publication. Thereafter the rate declines and about 8 years after publication 80% of the total number of citations have been received. The rate of decline differs slightly with scientific field. Thus, by using a variable window the NSI citation counts are composed of a mixture of increasing and decreasing citation rates. By using a fixed citation window we can ensure the rate at which citations accumulate has less variation than by using a viable citation window. Furthermore, by counting citations on or before the citation peak by using a 3-year fixed citation window (i.e., publication year and two subsequent years), thus minimising the effect of the variable rate of decline, a clearer relationship might be found between recognition and size.

Publication and citation counts produced using these techniques are not available from a standard ISI product like the NSI. However, as part of a NERC [9] evaluation exercise special data was purchased from ISI. The data set consisted of 1981–1994 publication and citation counts in the sciences for the US, UK, France, Germany [10], Canada, Australia, Europe [11] and the world. Unlike the NSI data only article, note and review publication types were counted and the papers were classified into 152 science fields using the 1994 version of the SCI journal categorisation scheme. Citations were counted using a fixed 3-year citation window.

The data purchased from ISI was supplemented with UK specific data. Over the past 5 years SPRU has developed a database of SCI papers that involved at least one UK author. By using the corporate addresses indexed in the SCI, each UK institution has been assigned to one of approximately 6500 standard corporate names and each corporation has been assigned to one of four institutional sectors: education (i.e., universities), medical (primarily hospitals and medical clinics), industry and other (including research council and government laboratories).

In general, the findings in this research are based on a data set that consists of

- the annual number of refereed publications (article, note or review) indexed in the SCI between 1981 and 1992
- for each of 152 communities (SCI journal categories)
- in each of four UK institutional sectors (education, medical, industry and other), six nations (UK, US, France, Germany, Canada and Australia), one region (the EU) and the world
- and the annual number of citations to these papers counted using a 3-year citation window.

For simplicity this data set will be called the BEST (Bibliometric Exploration of Science and Technology) data.

3. Results

3.1. Size and recognition

We shall explore the Matthew effect in science using the BEST data. Fig. 2 is a scattergram plotted on a log–log scale of publishing size and recognition for 152 communities in the world science system. Size was measured using refereed papers published between 1981 and 1992 and recognition was measured by counting citations to those papers between 1981 and 1994 calculated using a 3-year citation window. The straight line on the graph was derived using a regression analysis of a power-law function through the data points. The power-law relationship between recognition, c, and publishing size, p, is given by

$$c = k_a p^{1.27 \pm 0.03} \quad (5)$$

where the intercept, $k_a = 0.15 \pm 0.04$, and the coefficient of determination, r^2, is 0.92. Other functions

[9] Natural Environment Research Council.
[10] East and West Germany.
[11] Fifteen member nations.

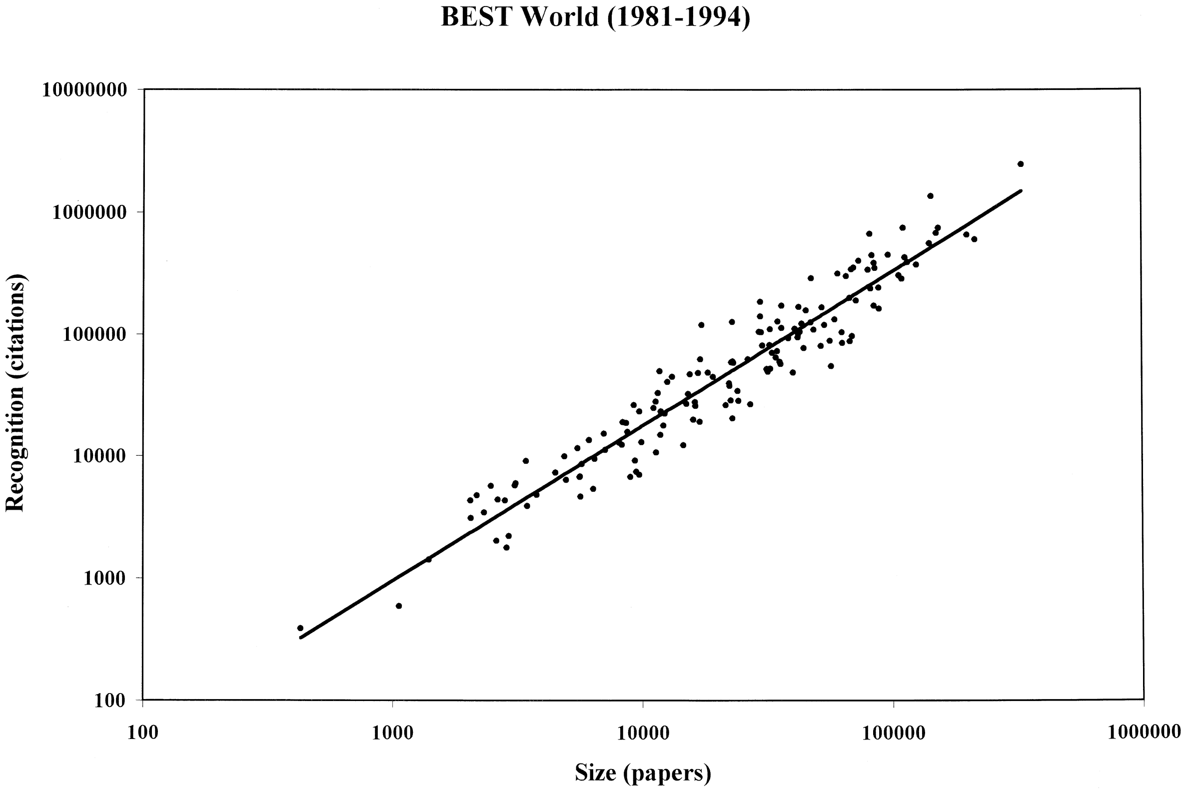

Fig. 2. Recognition vs. size for world science system (data source: purchased from ISI).

(e.g., linear, exponential, etc.) were tested but none of them exhibited as high r^2 values as the power law.

Table 1
Annual exponent for power-law relationship between size and recognition for the world science system

Year	Exponent	r^2
1981	1.25 ± 0.02	0.92
1982	1.26 ± 0.03	0.92
1983	1.26 ± 0.03	0.92
1984	1.24 ± 0.03	0.91
1985	1.26 ± 0.03	0.92
1986	1.26 ± 0.03	0.91
1987	1.27 ± 0.03	0.91
1988	1.27 ± 0.03	0.91
1989	1.27 ± 0.03	0.91
1990	1.28 ± 0.03	0.91
1991	1.24 ± 0.03	0.91
1992	1.26 ± 0.03	0.91

In order to determine the stability of this relationship with time the exponent of the power law was calculated on an annual basis for the world data. The results displayed in Table 1 illustrate that the exponent remained remarkably constant over the time interval.

Finally, using the BEST data the exponent of the power-law relationship was computed for an economic region (EU), six OECD countries (US, UK, France, Germany, Canada and Australia) and four national sectors (UK education, UK medical, UK industry, UK other). The results are given in Table 2. [12]

It can be seen that the exponents are similar for all science systems suggesting that a similar dynami-

[12] The intercepts are not given because they simply reflect the number of papers produced by the most productive of the 152 communities in each country.

Table 2
Exponents for power-law relationship between size and recognition for an economic region, six OECD countries and four national sectors

Country	Exponent	r^2
World	1.27 ± 0.03	0.92
EU	1.25 ± 0.02	0.94
US	1.34 ± 0.04	0.89
UK	1.27 ± 0.03	0.93
France	1.22 ± 0.02	0.95
Germany	1.23 ± 0.03	0.93
Canada	1.25 ± 0.03	0.90
Australia	1.20 ± 0.03	0.91
UK		
Education	1.23 ± 0.03	0.93
Medical	1.23 ± 0.03	0.95
Industry	1.16 ± 0.05	0.81
Other	1.28 ± 0.03	0.90

cal process is at work in each system. All the exponents are within one standard deviation of the world exponent except for Australia and UK industry, which are within two standard deviations. The average of the exponents for the UK, France and Germany equals the EU exponent. The exponent for the US science system is larger than the exponent for the world system while the exponent for the EU is lower. This might reflect the influence of the size of the domestic science base [13] and the English language bias in the SCI journal coverage.

In summary, an empirical measure of the Matthew effect in science based on scientific papers indexed in the SCI and citations to these papers suggests that there is a power-law relationship between size and recognition with an average exponent of 1.27 ± 0.03. This self-similar relationship is stable with time and as expected it is reasonably independent of the size and the nationality of a science system.

3.2. Size and size rank

In addition to the power law discussed in the introduction of the paper, de Solla Price [14] also noted that there was a power-law relationship between the number of papers published by a scientist and the publishing size rank of the scientist in a community (De Solla Price, 1963). Let us see if a similar organisational structure exists in the science system by examining the relationship between the publishing size of a community and its publishing size rank in a science system.

Fig. 3 is a scattergram plot on a log–log scale of the size and size rank in the world system. The inset graph shows the relationship between the publishing size of each of the 152 communities in the world science system and their publishing size rank. It can be seen that this graph is composed of a linear region ranging over about the first 40 ranks and a curved region covering the remaining ranks. The main graph is an expanded graph showing only the power-law relationship between size and size rank for only the top 25 ranked communities. These communities account for more than 50% of the total number of papers indexed in the SCI between 1981 and 1994. It was found that for these top 25 communities the relationship between publishing size, p, and size rank, r, is given by

$$p = k_b r^{-0.44 \pm 0.01} \qquad (6)$$

where the intercept $k_b = 311{,}065$ and $r^2 = 0.99$.

Table 3 gives the exponent of the power-law relationship among the 25 largest publishing communities and their rank for the EU, six OECD countries (US, UK, France, Germany, Canada and Australia) and four national sectors (UK education, UK medical, UK industry, UK other). The size rank in each system was determined by ranking each of the 152 communities in a given system by the number of published papers.

It can be seen from Table 3 that the exponents for the EU, US, UK, France, Canada and Australia are within one standard deviation of the world exponent while Germany's exponent is within three standard deviations. Also, the largest institutional sector in the UK, the education sector, is within one standard

[13] Researchers tend to cite papers published by domestic authors more frequently than foreign researchers compared to the share of world publications published by the citing nation [see Table 5-55 in the National Science Board, Science and Engineering Indicators, 1998; p. A-325]. Since the US has the largest science system it would have the largest number of domestic citations which could yield a larger exponent.

[14] See Fig. 14 (De Solla Price, 1963).

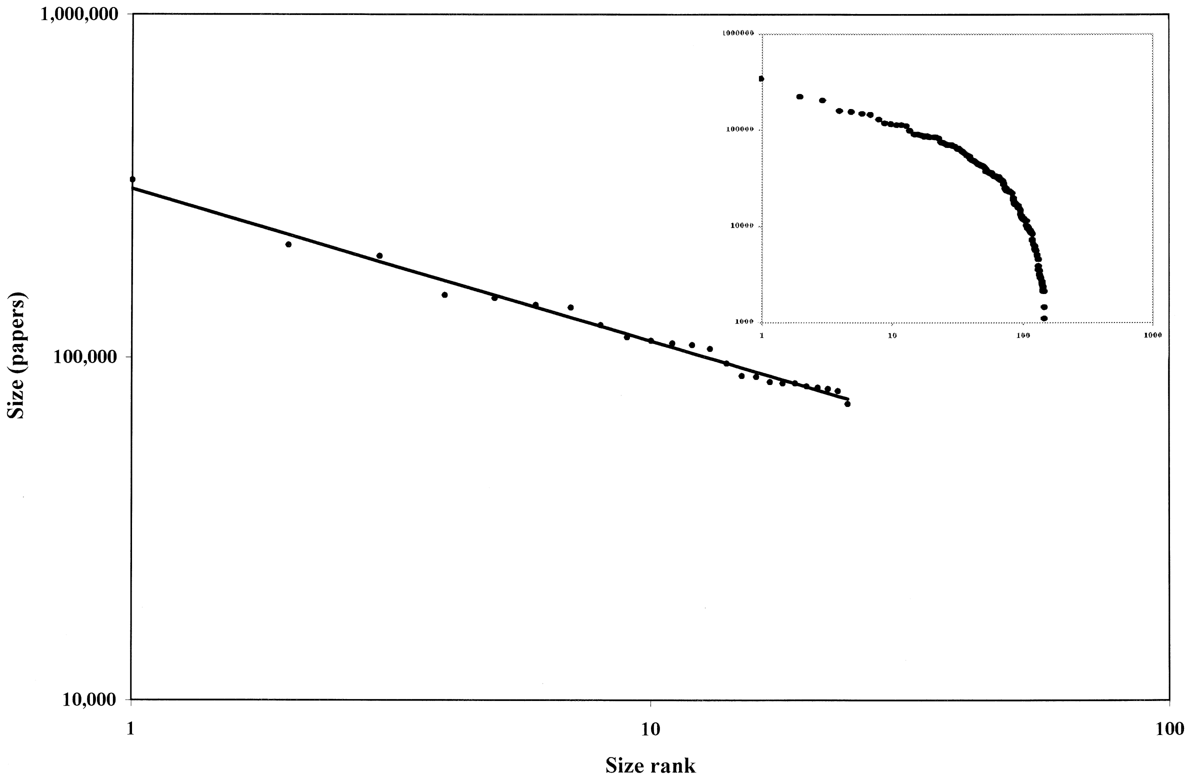

Fig. 3. Publishing size vs. rank of publishing size (data source: purchased from ISI).

deviation of the world exponent while the smaller sectors are within three standard deviations.

Although the exponents for the power-law relationship between publishing size and publishing size rank are similar for most of the science systems there is diversity in the rank of various communities in the different systems. Table 4 lists the rank in each science system of the twenty-five largest publishing communities in the world system.

Biochemistry and molecular biology is the largest publishing community in the world, US, UK, France, Canada, Australia systems as well as in UK education and UK other sectors. It ranks 2nd in Germany and 5th and 3rd respectively in UK medical and industry. On the other hand, *chemistry*, which ranks 2nd worldwide is 3rd in the EU, 6th in the US and Australia, 9th in the UK and France, 1st in Germany, 5th in Canada, UK education and UK industry and 51st in UK medical and 49th in UK other. As one moves down the list the disparity between the world rank and various national and sectoral ranks increases. Thus, even though there is similarity in the exponents of the power-law relationship there is diversity in the size rank of a given community

Table 3
Exponents for a power-law relationship between size and size rank for an economic region, six OECD countries and four national sectors

Country	Exponent	r^2
World	-0.44 ± 0.01	0.99
EU	-0.44 ± 0.01	0.99
US	-0.43 ± 0.02	0.96
UK	-0.44 ± 0.02	0.96
France	-0.46 ± 0.02	0.95
Germany	-0.51 ± 0.02	0.97
Canada	-0.46 ± 0.01	0.98
Australia	-0.41 ± 0.02	0.95
UK		
Education	-0.44 ± 0.02	0.94
Medical	-0.54 ± 0.03	0.92
Industry	-0.56 ± 0.01	0.98
Other	-0.51 ± 0.02	0.98

Table 4
Size rank of the top 25 communities in the world science system in Europe, six national and four sectoral science systems

ISI category	World	EU	US	UK	France	Germany	Canada	Australia	UK			
									Education	Medical	Industry	Other
Biochemistry and molecular biology	1	1	1	1	1	2	1	1	1	5	3	1
Chemistry	2	3	6	9	9	1	5	6	5	51	5	49
Pharmacology and pharmacy	3	2	3	3	3	3	4	7	2	3	1	21
Neurosciences	4	5	2	4	7	9	2	8	6	4	29	8
Physics	5	6	10	10	2	4	9	23	4	80	30	24
Multidisciplinary sciences	6	17	4	8	6	28	27	18	11	36	21	5
Medicine, general and internal	7	4	8	2	13	12	17	2	12	1	23	16
Chemistry, organic	8	7	20	5	4	7	24	27	3	69	4	64
Physics, applied	9	24	13	33	12	19	43	45	18	95	6	42
Physics, condensed matter	10	10	24	28	5	5	31	63	16	116	19	43
Immunology	11	11	5	7	10	21	10	9	22	10	20	7
Plant sciences	12	9	15	6	18	10	3	3	9	82	36	2
Chemistry, physical	13	8	27	12	8	8	22	26	8	70	7	54
Oncology	14	13	7	16	19	24	26	30	35	7	46	14
Engineering, electrical and electronic	15	30	12	15	36	37	13	39	14	77	2	47
Chemistry, analytical	16	19	32	38	22	27	37	29	32	48	10	35
Microbiology	17	14	21	11	24	15	14	22	15	21	17	12
Surgery	18	23	9	13	47	33	19	15	51	2	108	84
Physiology	19	26	11	26	28	35	6	13	20	33	79	31
Biophysics	20	16	18	29	11	17	25	32	21	35	41	25
Chemistry, inorganic and nuclear	21	12	47	14	15	6	44	24	7	79	42	67
Cytology and histology	22	15	17	23	14	14	23	21	27	24	57	13
Cardiovascular system	23	18	16	25	17	26	40	40	58	6	43	58
Genetics and heredity	24	22	22	18	23	30	18	16	26	23	26	6
Radiology and nuclear medicine	25	28	14	27	48	16	28	59	64	8	49	40

across the science systems. The publishing size rank of a community in a given system appears to be more dependent on national priorities, historical precedence, funding, research facilities and available skills than on global influences. In a given system the precise size rank of a scientific community, especially those in the lower ranks, will likely be unpredictable from one time period to the next.

In summary, a power-law relationship exists between the publishing size of a scientific community and its size rank with an average exponent of the power law, -0.44 ± 0.01. The exponent is fairly independent of nationality and size of the science system. This relationship suggests that there is a self-similar structure in the rate of decrease of publishing size among size-ranked communities in each science system even though the size rank of a given community in a particular science system may be quite unpredictable.

3.3. Recognition and recognition rank

In Sections 3.1 and 3.2, two power laws were identified (Eqs. (5) and (6)). Using these relationships an additional power law can be predicted and measured using the BEST data. From Eqs. (5) and (6), the relationship between recognition, c, and rank

Table 5
Exponents for a power-law relationship between recognition and recognition rank for an economic region, six OECD countries and four national sectors

Country	Exponent	r^2
World	-0.61 ± 0.02	0.97
EU	-0.57 ± 0.03	0.96
US	-0.70 ± 0.03	0.96
UK	-0.68 ± 0.01	0.99
France	-0.66 ± 0.02	0.98
Germany	-0.65 ± 0.02	0.97
Canada	-0.56 ± 0.02	0.96
Australia	-0.55 ± 0.02	0.98
UK		
Education	-0.59 ± 0.03	0.96
Medical	-0.65 ± 0.03	0.96
Industry	-0.75 ± 0.03	0.97
Other	-0.88 ± 0.02	0.98

by citations, r, for the 25 highest ranked communities can be solved and is given by

$$c = k_d r^{-0.56 \pm 0.03} \quad (7)$$

The values for the exponents measured using the BEST data are given in Table 5.

The predicted world exponent for the relationship between recognition and recognition rank was -0.56 ± 0.03, which is within the standard error limit of the measured value of -0.61 ± 0.02. The measured exponents for the EU, Germany and UK education are within one standard deviation of the world measured exponent while the US, France, Canada and Australian exponents are within two standard deviations. It is interesting to observe that the exponents for the six OECD nations appear to increase with the scientific size of the nation. This might suggest a propensity for the recognition of communities in a science system to decrease more rapidly with rank in larger nations than smaller nations. Finally, just as we showed that there was diversity in the publishing size rank of communities in the individual science system (Table 4) there is also diversity in the citation rank in each system for the twenty-five most highly cited communities in the world system. [15]

[15] The data are not given here as the specific ranks are of no importance but the fact that the ranks vary from system to system is important.

3.4. Impact and impact rank

A power-law relationship between impact (citations per paper), i, and impact rank, r, for the top 25 impact communities can be predicted by manipulating Eqs. (6) and (7) and is given by

$$i = k_e r^{-0.12 \pm 0.05} \quad (8)$$

Indeed this power law was found in the BEST data but there was substantial discrepancy between the predicted and measured exponents (see Table 6). These discrepancies are due to the fact that Eqs. (6) and (7) predict the best average fit power-law relationship between (1) recognition and size and (2) size and size rank. The exponent of the power-law relationship between impact and impact rank is predicted by taking the ratio between Eqs. (7) and (6). One can hardly expect the ratio of these two averages to predict the average best fit exponent between impact and rank accurately. However, it is important to note that even though the exponents cannot be predicted the power-law relationship was predicted and can be measured. Furthermore, the exponents in most of the systems have reasonably similar values.

4. Policy implications of self-similar social characteristics

Let us explore some of the implications of self-similar or scale-independent social activity for policy

Table 6
Exponents for a power-law relationship between impact and impact rank for an economic region, six OECD countries and four national sectors

Country	Exponent	r^2
World	-0.28 ± 0.01	0.99
EU	-0.28 ± 0.01	0.97
US	-0.33 ± 0.01	0.97
UK	-0.32 ± 0.01	0.98
France	-0.25 ± 0.01	0.94
Germany	-0.33 ± 0.01	0.98
Canada	-0.23 ± 0.02	0.85
Australia	-0.26 ± 0.01	0.96
UK		
Education	-0.23 ± 0.01	0.96
Medical	-0.39 ± 0.03	0.87
Industry	-0.33 ± 0.02	0.94
Other	-0.42 ± 0.01	0.99

makers and analysts. An important fact to remember is that while a power-law distribution that characterises a self-similar property may have a mean but its variance is usually undefined or infinite. In other words unlike a normal distribution that has a mean and a variance (or standard deviation) at a specific scale, a self-similar property may be exhibited over many orders of magnitude where the notion of a mean is meaningless and may not have a real-world interpretation. A social policy directed at a self-similar social characteristic but based on an average is surely derived from an indicator that misrepresents the actual nature of the system. For example, if Pareto's law of income distribution holds in a nation then a wealth creation policy striving to increase family income but based on the notion of an average household income could be inaccurate and probably doomed to fail.

Similarly, the comparison of impact (citations per paper) among national science systems can produce misleading results. In this paper it has been demonstrated that the amount of recognition received by a community increases as a power of the publishing size of the community. Mathematically it follows that the impact of a community also increases in a power-law relationship to the size of the community. Since the size of national science systems varies then we would expect their impact also to vary.

Instead of comparing the average impact among nations in science one might be tempted to compare the impact of one community, for example the *biochemistry and molecular biology* community, in different national science systems. However, this approach is also likely to be in error since if the Matthew effect is truly scale-independent it should hold from the level of the individual or perhaps the group to the level of the world. In other words, if one were to explore the relationship between publishing size and recognition for subcommunities within a given community, for example, compare institutional publishing size and recognition within the *biochemistry and molecular biology* community, again one would expect to find a power-law relationship for the Matthew effect.

Power-law analysis does provide us with a new way of comparing communities of differing size within the same or different systems. For example, assume the exponent of the Matthew effect has been empirically determined for a range of communities within a science system. In order to compare two communities of differing size one can use this exponent to compute the amount of recognition each community should expect to receive. By comparing the expected recognition for a given size community with the measured recognition one can directly compare their relative performance. A similar method could be used to compare two communities in different science systems if the exponent for the Matthew effect has been measured in each system.

Frequently, the Matthew effect in science manifests itself in other ways such as the manner in which funds are allocated to higher educational institutions. Sometimes smaller institutions complain that the larger institutions receive funds that are disproportionately large compared to their size. Policy makers tempted to try to rectify this situation should be careful not to use a scale-dependent policy (i.e., a policy focused on a limited range of institutional sizes) since it would likely have a limited or counterproductive effect when applied to scale-independent property. On the other hand, they might consider designing a scale-independent policy that, for example, strives to reduce the slope of the power-law distribution. If the slope could be reduced to one then the Matthew effect could be transformed from a nonlinear to a linear effect. This would produce a more equal effect because rewards would be proportional to size. Although this might seem to be a more equitable effect in some political and social ideologies it might be contrary to the naturally emerging properties of an internal process such as the peer-review process.

In a democracy, it might be difficult to design a policy to change the slope of a Matthew effect or other self-similar structures such as the one between size and size rank in the science system. In fact it would probably require nothing short of draconian measures to overcome the internal dynamics and momentum in the system. For example, there is evidence to show that the relationship between national funding and the publishing size of a community (Hart and Sommerfeld, 1998) is linear. Using this linear relationship and Eq. (6) we can calculate how the funding would have to be redistributed in order to interchange two communities separated by one size rank. For example, in order to interchange the 1st and 2nd ranked communities the funds for the 1st rank community would have to be decreased by 26.3% while the funds for the 2nd ranked community would have to be increased by 35.7%. In order to interchange the 2nd and 3rd rank the funds for the 2nd rank would have to be decreased by 16.3% while the funds for the 3rd rank community would have to be increased by 19.5%. Needless to say the political and social problems generated by trying to make funding changes of this size would be enormous. However, because the funding redistribution changes in a power law manner smaller amounts of funds have to be redistributed in order to interchange lower ranked communities. For example in order to interchange the rank of the 24th and 25th ranked communities the funds for the 24th ranked community would only have to be decreased by 1.8% and the 25th ranked community increased by 1.8%.

From this perspective, we can see that a self-similar system is quite robust. It is capable of tolerating fairly large economic shocks without experiencing a change in the dimensionality (slope) of its self-similar property or a change in its internal structure, especially among the highest ranked entities in the power-law distribution. For a self-similar social system perhaps the most a policy maker can hope to develop is policies that encourage communities to meet or exceed the expected performance that dynamically emerges from within the system.

5. Discussion

As mentioned in Section 1, numerous observers have identified the power-law nature of some social activities. However, few if any observers have clearly identified a power-law relationship underpinning a well-known social phenomenon such as the Matthew effect or a set of power laws that collectively portray social processes such as those found in the publishing and citing activities of the science community. In fact the author does not know of any research that has empirically demonstrated that a social process such as the Matthew effect is self-similar and thus scale-independent over a wide range of community sizes (i.e., communities ranging in publishing size from UK industry that published about 34,000 papers to approximately 4.6 million papers published in the world system). Before proceeding with the discussion let us summarise the findings of this study.

Using the science system as an example social system and data about peer-reviewed scientific publications and citations to these publications an attempt was made to measure the Matthew effect in science by examining the relationship between size and recognition. Publishing size was measured using refereed papers (articles, notes and reviews) indexed in the *Science Citation Index* between 1981 and 1992 for 152 science communities in various science systems—the world, an economic region (EU), six OECD nations (US, UK, France, Germany, Canada and Australia) and four national sectors (UK education, UK medical, UK industry and UK other). Recognition was measured using a 3-year citation window to count citations to these papers between 1981 and 1994. In addition to the Matthew effect the publishing size structure in science was explored using the relationship between size and publishing size rank.

A power-law relationship was found between (1) recognition and size and (2) size and size rank. In each case the exponents for these power laws were similar in each science system and relatively independent of size and nationality. Using these two relationships the exponent for a power-law relationship between recognition and recognition rank was predicted, measured and determined to be correct within measurement error. Finally, a power law was predicted between impact and impact rank and found in the BEST data.

In summary, these finding suggest that the Matthew effect in science (measured using recognition and size) and the size structure of the science system (measured using size and size rank are

- scale-independent processes that are
- relatively independent of size and nationality,
- indicative of self-organising characteristics that may exist from the level of a national sector science system to a national system and finally to the world science system, and
- can be used to predict the existence of other scale-independent relationships such as the power-law relationship between (1) recognition and recognition rank and (2) impact and impact rank

It may be possible to delineate the Matthew effect even more precisely by eliminating more noise in the data. Also it might be possible to identify more power-law relationships in the science system. In this study citations were counted using a 3-year citation window. However, to reduce cost and computational complexity the citation counts based on a 3-year citation window were computed using aggregate annual data. Thus a paper published in the later part of a calendar year, for example December, will actually only have citation counts for 2 years plus a month. By refining the counting techniques to use weekly, monthly or even quarterly aggregation the citation counts would be more accurately determined and this should increase the precision for measuring the Matthew effect in science. Also, by using coauthored papers as an indicator of collaboration it may be possible to establish power-law relationships between size, recognition and various types of collaboration (e.g., foreign and domestic). However, in order to do this research and confirm that the findings in this paper hold for other nations and at other scales, investigators would need access to the complete SCI database. Such a social science research project would require international cooperation and funding on par with a major natural science research project.

Finally, self-similar systems are robust. They are disturbed very little by fairly large external shocks. If we want to manage self-similar social systems, such as the science system, in a more effective way we will have to understand their nature more fully. In fact, we will have to develop new research methods and evaluation techniques designed specifically for self-similar systems if we wish to formulate more effective policies.

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