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The method of leader's overthrow in networks based on Shapley value

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A B S T R A C T

Quantitative methods for leaders' detection and overthrow are useful tools for decision-making in many real-life social networks. In the given research, we present algorithms that detect and overthrow the most influential node to the weaker leadership positions following the greedy method in terms of structural modifications. We employ the concept of Shapley value from the area of cooperative game theory to measure a node's leadership and to develop the leader's overthrow algorithms. Specifically, we introduce a quantitative approach to analyze prospective structural modifications in social networks to make the initially identified network leader less influential. The resulting mechanism is based on the symbiosis of game-theoretic and algorithmic concepts. It presents a useful tool for the technical analysis of the primary structural data in the initial steps of multifaceted quantitative network analysis where the raw data (i.e., linkages) is frequently the only knowledge about interrelations in social networks.

1. Introduction

The variety of game and graph theoretical approaches plays an important role in formalizing the leadership of agents in social networks based on the analysis of node centrality metrics. In terms of practical use, one of the first research applications of centrality measures was introduced in the 1940s at the Group Networks Laboratory, M.I.T. Later, Cohn & Marriott [9] applied different centrality metrics to analyze the diversity of Indian social life. Pitts [32] used centrality-based concepts for examination of communication paths in the context of urban development while Czepiel [12] applied centrality computation in the analysis of a technological innovation in the steel industry. The practical application of centrality metrics has grown fast in the last fifty years. For example, Moore, Eng & Daniel [29] used centrality scores for the estimation of aid coordination between the non-governmental organizations (NGOs) in Mozambique (i.e., NGOs involved in the flood operations). Estrada & Bodin [14] used network centrality metrics to manage landscape connectivity. Faris & Felmlee [15] explored gender segregation and cross-gender aggression based on centrality measures.

In the given research, we present a quantitative analysis of leadership in social networks employing the idea of network centrality. We use the term “node” as an equivalent to the terms “agent” and “player” since we do quantitative social network analysis (SNA) based on graph and game theoretical approaches.

Agents' leadership is one of the core ideas in SNA. Different evaluation methods exist. Degree [17], betweenness [3,16], and closeness [5,34] are the most widely known metrics that assess the structural centralities of nodes. The algorithmic measures of nodes' leadership are well presented in Kleinberg [26] and Page et al. [31]; where the notion of leadership is given based on the analysis of link structures. An interesting approach to characterize the role of nodes within networks is given by Scripps et al. [38]; where the community-based metric in the symbiosis with the degree-based measure is introduced in the context of nodes' roles classification. The problem of leadership analysis in networks is well described by Balkundi & Kilduff [4] and Hoppe & Reinelt [23].

In contrast, there is yet another problem of understanding how the network's structure should be efficiently modified in order to overthrow the current network leader to the weaker positions.

Generally, modern social networks are presented by large-scale structures with poorly formalized data flows due to their uncertain social nature [41]. Consequently, basic information about a network structure (i.e., presence/absence of links between nodes) is often only the well-formalized data about the social network. In this case, network structure presents basic knowledge about interrelations that can be employed for the decision-making related to a network's modifications.

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Consider large-scale terrorist networks that are often characterized by the lack of knowledge about specific information flows within existing interrelations. On the initial steps of network analysis, the structural factor plays a key role for understanding interrelations and detecting agents' leadership. In networks, such as criminal networks with hidden patterns, or money laundering networks, the overthrow of the detected leader may seriously damage the network structure or even bring about irreversible destruction [13,24,35].

In this paper, we consider the case when all nodes in the network participate in the process of the leader's overthrow. Specifically, the main goal of the given research is to analyze what sufficient set of links has to be established in a network (based on the greedy approach) in order to decrease the leader's influence on other nodes.

Agents establish links between each other for different socio-economic reasons depending on the nature of the social network. For example, in organizational networks [11], agents compete for leadership positions in informal organizational networks by creating new relations in order to improve their leadership positions in the formal organizational structures. Building leadership is a competitive process that is based on establishing new relations and overthrowing the existing leaders. Another example is related to criminal and terrorist networks. A sleeper agent might incite other agents to create new relations in order to improve their leadership position and, simultaneously, to weaken current network leaders [37].

Two agents in a network are motivated to create a link between each other only if both of them make some profit from it. Specifically, in our research we consider profit as an improvement of an agent's leadership. To measure the level of a node's leadership we employ the concept of Shapley value [39] from the area of cooperative games. Specifically, we use the Shapley value (SV) centrality metric developed by Aadithya et al. [1].

In the context of our quantitative network analysis, we determine the leader as a node that has the highest SV. This means that it has the highest SV-based leadership in the network where cooperation is presented by links between nodes. The leader's overthrow is a procedure of structural modifications that results in the leader's SV-decrease in terms of the distribution of a total surplus generated by the coalition of all nodes.

The quantitative analysis of agents' leadership in terms of SV distribution allows us to build potential scenarios for structural modifications in a network. As a result, we get primary information as part of the multi-faceted leadership analysis in networks. This primary information regarding the leadership distribution (in terms of SV) does not play a key role in the decision-making process for network modifications, but it reflects quantitative SV-based leadership distribution based on the structural factor.

In the given research, we show the advantages of the SV-based concept compared to the traditional centrality metrics and explain why SV is employed to measure leadership in networks. Based on the given SV-based game theoretic approach we develop two overthrow algorithms that establish sets of links to overthrow the initial leader with the highest SV to the weaker positions.

Next, we test the SV-based algorithms based on the trivial network topologies and on the real-world structures retrieved from the co-authorship networks of the Norwegian School of Economics (NHH) and BI Norwegian Business School [6].

The scientific finding of the given research is based on the idea of building a quantitative mechanism that detects prospective structural modifications in social networks based on the game theoretic approach. The advantage of the SV-based game theoretic approach (compared to classical well-known network metrics) is presented in the following section.

2. Shapley value as the network's centrality metric

Shapley value is one of the fundamental concepts of game theory [33]. The core idea of SV is the payoffs' distribution among players according to their personal contributions to the overall gain in a cooperative game. Since SV measures players' leadership based on their mutual cooperation, it is applicable in the domain of social network analysis. Specifically, it reflects the real-world players' interrelations since it counts mutual influence of players in all possible coalitions in networks.

For large-scale networks with lack of detailed information regarding internal processes, the structural factor becomes important for a quantitative assessment of the leadership potential of nodes. Often, structure is the only well explored factor. Therefore, it is important to have an efficient measure that evaluates node leadership based on the network structural characteristics.

To understand why the concept of Shapley value is employed for the network leadership analysis, it is necessary to understand when it is useful to employ it.

The SV-based centrality metric is not the unique or the only solution to estimate leadership. Considering conventional centrality metrics, e.g., those based on node degree, closeness, and betweenness, it should be specified that each of them reflects a node's leadership depending on the particular application. Depending on the context of the problem, an appropriate centrality metric should be employed. Consider an abstract transportation network, where the set of locations (i.e., nodes) is connected by roads (i.e., links). The degree-centrality reflects an immediate influential power showing how many locations are directly reachable from the current node, but it does not count the global network structure, because it takes into consideration only the neighboring nodes approachable in one hop (i.e., within one-link distance). The closeness centrality measures how fast it is possible to travel between different locations in terms of the overall network. It is based on the calculation of the inverse sum of the node's shortest distances to all other nodes. The betweenness centrality reflects the leadership of the node in terms of how often it is required to go through the location along the shortest route between two other locations. In many real-life networks there is a great proportion of nodes that do not appear on the shortest paths between any other two nodes [30]. This means that many nodes can get the betweenness value equal or close to zero. Since closeness and betweenness centralities take into account the overall network structure, they are more advanced measures compared to the degree centrality, but “prohibitively expensive to compute, and thus impractical for large networks” [25].

The efficiency of conventional centrality metrics depends on the application area. Nevertheless, they have a generic nature that is characterized by an “individualistic” measurement approach. This means that they “fail to recognize that in many network applications, it is not sufficient to merely understand the relative importance of nodes as *stand-alone* entities. Rather, the key requirement is to understand the importance of each node in terms of its utility when combined with other nodes” [1]. This means that conventional centrality metrics do not consider the interdependencies of nodes' failures. They reflect the resulting effect (i.e., after-effect) of multi-node failures in terms of a network's structure. It is important to specify a key point that “such an approach is inadequate because of *synergies* that may occur if the functioning of nodes is considered in groups” [28]. The SV-based centrality metric counts the mutual effect of all possible nodes' combinations measuring the leadership of nodes within a graph [19].

Aadithya [1] study found the following:

"The SV of each agent (node) in the game is interpreted as a centrality measure because it presents the average marginal contribution made by each node to every possible combination of the other nodes. This paradigm of SV-based network centrality thus confers a high degree of flexibility (which was completely lacking in traditional centrality metrics) to model real-world network phenomena." (p. 2)

The advantage of the SV-concept for the leadership measurement is based on the idea of quantitative analysis of interpersonal and intergroup cooperation in networks that originally comes from the domain of cooperative game theory. In contrast to conventional measurements, SV reflects internal collaborations occurring within all possible combinations of network agents, i.e., all possible groups of agents that can be formed within a network [20]. Consequently, the SV-based leadership measurement encapsulates the effect of all possible interrelations between agents (within groups) considering how each of them (being a part of different coalitions in the network) contributes to the leadership positions of other agents. More details about advantages of SV compared to classical metrics are well described in Michalak et al. [28].

The SV-concept is based on the analysis of cooperation games. Since coalition building is a natural feature of agents' behavior for leadership formation in social networks [7], SV has a practical importance for real-life applications. For example, consider the analysis of infectious diseases networks [28]. The main goal in this type of networks is to detect agents who have the highest potential to spread diseases and, subsequently, have to be monitored for prevention purposes. SV-based analysis considers the involvement of each agent in all possible coalitions in the network detecting who has the biggest influence on other agents and plays a key role in passing on diseases.

Another example is co-authorship networks [2,18], where research collaboration can be tracked based on bibliometrics. SV-based analysis helps to detect how agents form scientific coalitions and what their leadership positions are based on joint publications.

In the following section, we present the formalization of the SV concept adopted for network analysis by Aadithya et al. [1] and Michalak et al. [28]. The latter shows the transition from the classical interpretation of SV to the network-based interpretation accompanied by formal proof. Next, we introduce two Shapley-based algorithms that aim to detect an agent with the highest SV (i.e., network leader) based on Michalak et al. [28] approach and to find a set of links required to decrease the leader's influence on other nodes (measured by SV). First, we introduce an algorithm, where the decision-maker can specify the number of links to be established in the network in order to diminish the leader's influence. Second, we present an algorithm that detects a set of links in order to overthrow the initial leader to the weakest position.

In the given research, the process of nodes' linking to decrease SV of the initial leader is called an overthrow procedure. The importance of the leader's overthrow is based on the practical need to analyze the prospective network modifications required to change the leader. The straightforward way to get a new leader is based on the deletion of the agent from the network, for example, by dismissal of the person in the organizational network, or killing the leader in the terrorist/criminal network. In contrast, this research aims to show the way for the "evolutional" leader's overthrow based on the network modifications. In the real world this process can be presented by managerial decisions for structural transformations of the organizational network [11] or by marketing decisions to rearrange leadership positions in the customer networks [40].

3. Leader's overthrow algorithm

In the given chapter, we use the adapted version of SV for network analysis [1,28] and introduce two algorithms that employ the SV-concept.

Consider graph *G(V,E)* and $v_i \in V$. All nodes (i.e., neighbors), which are reachable from v_i in at most one hop within *G(V,E)* are denoted by $N_G(v_i)$. The degree of node v_i is defined by $deg_G(v_i)$. According to Aadithya et al. [1]; the SV definition for node v_i in *G(V,E)* is the following (see Eq. (1)):

$$SV(v_i) = \sum_{v_j \in \{v_i\} \cup N_G(v_i)} \frac{1}{1 + deg_G(v_j)} \tag{1}$$

Based on Eq. (1) Aadithya et al. [1] introduced the algorithm to measure nodes' leadership in a network (see Algorithm 1).

Algorithm 1: SV-COMPUTING *(G)*

FOR each $v \in V(G)$ do
- ShapleyValue $[v] = \frac{1}{1+deg_G(v)}$
- FOR each $u \in N_G(v)$ do
  - ShapleyValue $[v]$ += $\frac{1}{1+deg_G(u)}$
- END

END

return: L = List of SV-s for all nodes

Algorithm 1. Algorithm 1 returns the SVs for all nodes and reflects their leadership positions within the analyzed network. Based on the given mechanism we introduce two algorithms that identify sets of links to be established in order to overthrow the strongest node (i.e., leader).

Algorithm 2. Algorithm 2 iteratively detects links to be established in order to overthrow the initial leader in terms of SV allocation within a network. Following the greedy approach, the algorithm establishes k links. The practical importance of the k-parameter is that it gives a flexibility for the decision-maker to choose how many links should be created.

Algorithm 3. Algorithm 3 iteratively detects links to be established in order to overthrow the initial leader to the weakest position in terms of SV allocation within a network. The number of created edges can be much bigger than the existing number of edges in the network. Establishing that many links is unlikely in the real-life scenarios, but the purpose of the algorithm is to show the ultimate set of links required to overthrow the node to its lowest leadership position. Algorithm 3 detects the set of links for the ultimate overthrow, which is applicable in the analysis of how it is hard to decrease an agent's leadership (in terms of required number of links to be created).

Algorithm 2: *K*-OVERTHROW-COMPUTING *(G, k)*

```
1   L = SV-COMPUTING (G)
2   Target = node with MAX(L)
3   SV_sub = SV(Target)
4   Link = Ø
5   E_add = Ø
6   n = 0
7   WHILE n < k:
8   |     FOR each v ∈ V(G):
9   |     |     FOR each u ∉ N_G(v):
10  |     |     |     Create trial link (v,u)
11  |     |     |     L = SV-COMPUTING (G)
12  |     |     |     IF SV_sub > SV(Target):
13  |     |     |     |     THEN: Link = (v,u)
14  |     |     |     |           SV_sub = SV(Target)
15  |     |     |     |     ELSE: Delete trial link (v,u) from G(V,E)
16  |     |     |     END
17  |     |     END
18  |     END
19  |     append Link → E AND append Link → E_add // Include Link to E and E_add
20  |     Link = Ø
21  |     n = n + 1
22  END
23  return: E_add
```

Line 2:

MAX*(L)* detects the maximal Shapley Value (SV) in the list L. *Target* is the initially detected leader that has to be overthrown. Its value is constant in the algorithm.

Line 3—5:

SV_{sub} is a temporary variable used to compare the leader's SVs before and after the trial link was created.

Link is a temporary variable that contains the link approved on the current iteration.

E_{add} is a set of approved links.

Lines 6—7:

Counter n is initially equal to zero. It is used to control the number of established links.

The loop continues while the number of established links (i.e., n) is not equal to the allowed number of links (i.e., k). In each iteration of the WHILE loop, the algorithm approves the link that gives the maximal decrease of SV(*Target*)-value. We consider k as a constraint for the number of links to be established. To reflect the real-life cases, the value of k cannot be greater than the existing number of edges in the initial network G: $1 \leq k \leq |E|$.

Algorithm 3: MAX-OVERTHROW-COMPUTING *(G)*

```
1   L = SV-COMPUTING (G)
2   Target = node with MAX(L)
3   SV_sub = SV(Target)
4   Link = Ø
5   E_add = Ø
6   WHILE  SV(Target) ≠ MIN(L) OR
7   |     [SV(Target) = MIN(L) AND [SV(Target) = SV(j) AND j ≠ Target]
8   |     AND G is NOT complete]:
9   |     FOR each v ∈ V(G):
10  |     |     FOR each u ∉ N_G(v):
11  |     |     |     Create trial link (v,u)
12  |     |     |     L = SV-COMPUTING (G)
13  |     |     |     IF SV_sub > SV(Target):
14  |     |     |     |     THEN: Link = (v,u)
15  |     |     |     |           SV_sub = SV(Target)
16  |     |     |     |     ELSE: Delete trial link (v,u) from G(V,E)
17  |     |     |     END
18  |     |     END
19  |     END
20  |     append Link → E AND append Link → E_add // Include Link to E and E_add
21  |     Link = Ø
22  END
23  return: E_add
```

Line 2:

MAX*(L)* detects the maximal Shapley Value (SV) in the list *L*. *Target* is the initially detected leader that has to be overthrown. Its value is constant in the algorithm.

Lines 6–8:

WHILE SV*(Target)* ≠ ***MIN****(L)* **OR**
[SV(*Target*) = ***MIN***(*L*) **AND** [SV(*Target*) = SV(*j*) **AND** *j* ≠ *Target*] **AND** *G* is **NOT** complete]

In each iteration of the WHILE loop, the algorithm approves the link that gives the maximal decrease of SV(*Target*)-value. The compound WHILE loop checks two main conditions:

1. SV(*Target*) ≠ ***MIN***(*L*)

MIN(*L*) detects the minimal Shapley Value (SV) in the list *L*.

The loop continues while SV of the initially detected leader is not the minimal one.

2. [SV(*Target*) = ***MIN***(*L*) **AND** [SV(*Target*) = SV(*j*) **AND** *j* ≠ *Target*] **AND** *G* is **NOT** complete]

This condition is required to control cases, when the Target-node approaches the lowest SV, but there exist other node(s) with the same SV: SV(*Target*) = SV(*j*) **AND** *j* ≠ *Target*. In other words, it is required to check if *Target* has a potential to get a lower Shapley Value. It is possible only if the updated *G*-graph is not complete (*G* is **NOT** complete).

It is important to notice that the given overthrow algorithms are applicable to connected graphs.

In the following section, we show how they work on the trivial topologies. Next, we test them on the real-life networks retrieved from Belik & Jörnsten [6].

4. Testing on the trivial topologies

Any large-scale network consists of the trivial topologies with different characteristics [21]:

- "point-to-point", or "line"
- "star"
- "ring"
- mixed, i.e., topologies that are based on the previous three types

Trivial topologies form a basement for large-scale networks. In the given section, we present the detailed explanation of the overthrow mechanism based on the given structures. Since the number of links in the tested trivial topologies is small (i.e., in the range between two and eight), we show how the SV-based leader's overthrow procedure works running Algorithm 3. Our main goal in this section is to explain the computational SV-based mechanism step-by-step.

Later (in Section 5) we give the detailed results running both algorithms (i.e., Algorithm 2 and Algorithm 3) on the real-life networks.

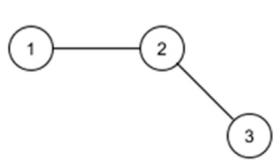

Fig. 1. "Point-to-point" network topology in the initial state.

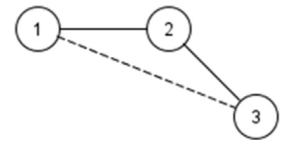

Fig. 2. Modified "Point-to-point" network topology.

Table 1
Results for the "point-to-point" topology.

Initial		Overthrow			Final	
Node	Shapley value	Link	SV*(Target)*	Decrease	Node	Shapley value
1	0.83	(1,3)	1.00	0.33	1	1.00
2	1.33				2	1.00
3	0.83				3	1.00

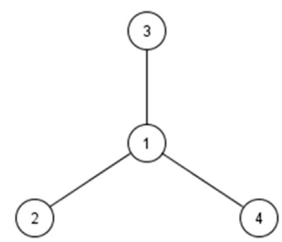

Fig. 3. "Star" network topology in the initial state.

Table 2
Results for the "Star" topology.

Initial		Overthrow			Final	
Node	Shapley value	Link	SV*(Target)*	Decrease	Node	Shapley value
1	1.75	(2,3)	1.42	0.33	1	1.00
2	0.75	(2,4)	1.17	0.25	2	1.00
3	0.75	(3,4)	1.00	0.17	3	1.00

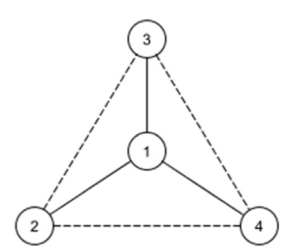

Fig. 4. Modified "Star" network topology.

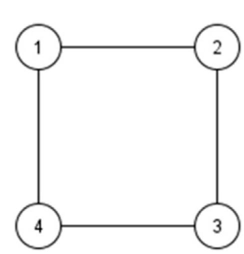

Fig. 5. The initial state of the "Ring" network topology with even number of nodes.

Table 3
Results for the "Ring" topology with even number of nodes.

Initial		Overthrow			Final	
Node	Shapley value	Link	SV(*Target*)	Decrease	Node	Shapley value
1	1.00	(2,4)	0.83	0.17	1	0.83
2	1.00				2	1.17
3	1.00				3	0.83
4	1.00				4	1.17

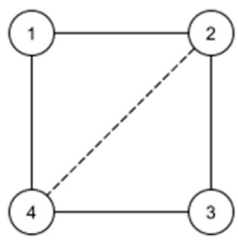

Fig. 6. Modified "Ring" network topology with even number of nodes.

4.1. "Point-to-point" topology

The "Point-to-point" topology is presented in Fig. 1.

Initially, Algorithm 3 calculates the SVs for the given topology. It detects that node 2 is the leader (i.e., it has the highest SV). Next, the link (1,3) is established in order to decrease the level of leadership for node 3. Since we get the complete graph, the algorithm stops, and we get SV(1) = SV(2) = SV(3) = 1. The results for all algorithm's steps are presented in Fig. 2 and in Table 1.

4.2. "Star" topology

The "Star" topology is characterized by the existence of central hub that is presented by node 1 in Fig. 3. Following Algorithm 3, we get the results presented in Table 2.

Node 1 was detected by the algorithm as the leader. Algorithm 3 creates three links in order to overthrow node 1 to the weakest

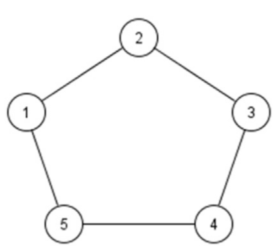

Fig. 7. The initial state of the "Ring" network topology with odd number of nodes.

Table 4
Results for the "Ring" topology with odd number of nodes.

Initial		Overthrow			Final	
Node	Shapley value	Link	SV(*Target*)	Decrease	Node	Shapley value
1	1.00	(2,5)	0.83	0.17	1	0.83
2	1.00				2	1.17
3	1.00				3	0.92
4	1.00				4	0.92
5	1.00				5	1.17

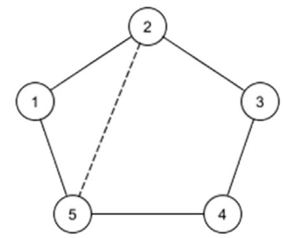

Fig. 8. Modified "Ring" network topology with odd number of nodes.

position (i.e., SV(1) = 1). It stops on the iteration when the graph becomes complete and no more links can be established. The resulting modified "Star" topology is presented in Fig. 4.

4.3. "Ring" topology

The "Ring" topology is characterized by sequential connections of odd or even numbers of nodes forming the cycle. First, we consider the structure with an even number of nodes (see Fig. 5).

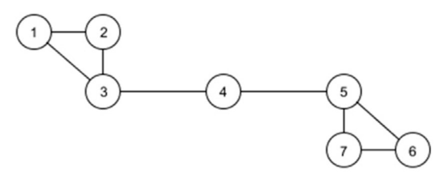

Fig. 9. Mixed network topology in the initial state.

Table 5
Results for the mixed topology.

Initial		Overthrow			Final	
Node	Shapley value	Link	SV(*Target*)	Decrease	Node	Shapley value
1	0.92	(1,4)	1.08	0.17	1	1.24
2	0.92	(2,4)	0.95	0.13	2	1.04
3	1.25	(1,5)	0.90	0.05	3	0.7
4	0.83	(2,5)	0.85	0.05	4	1.24
5	1.25	(1,6)	0.82	0.03	5	0.99
6	0.92	(2,6)	0.78	0.03	6	0.99
7	0.92	(4,6)	0.75	0.03	7	0.82
		(4,7)	0.73	0.02		
		(1,7)	0.70	0.02		

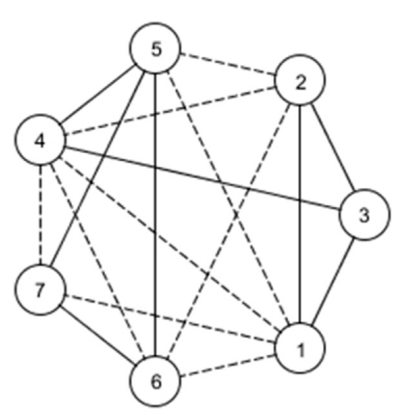

Fig. 10. Modified mixed topology.

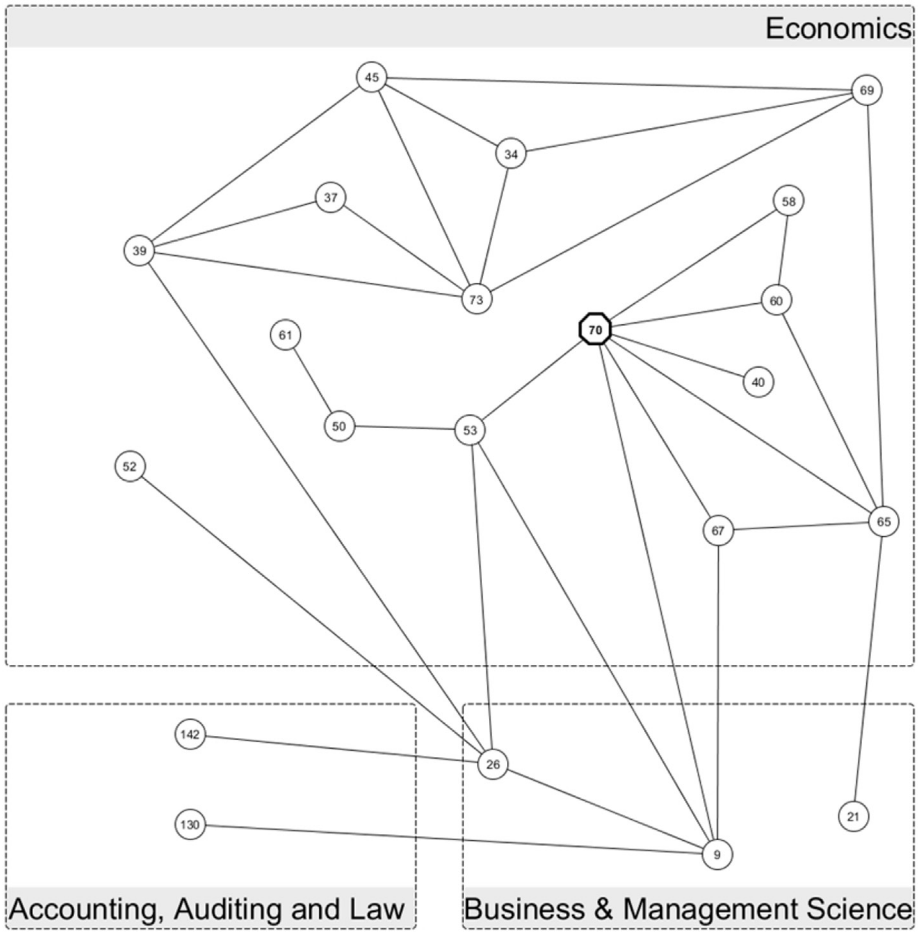

Fig. 11. The NHH largest component.

Running Algorithm 3, we get the results represented in Table 3.

According to Table 3, initially all nodes have equal SVs. The algorithm chooses node 1 as the *Target*. By establishing link (2,4) SV(1) decreases by 0.17, and the resulting SV(1) becomes equal to 0.83. Link (1,3) is not created by Algorithm 3, because it increases SV(1) back to the initial value that is equal to one. For the given "Ring" network topology with an even number of nodes the SV*(Target)* is decreased to its minimum value of 0.83. The resulting network is presented in Fig. 6.

Next, we test the "Ring" structure with an odd number of nodes (see Fig. 7). Applying Algorithm 3 to the graph presented in Fig. 7, we get the results represented in Table 4.

According to Table 4, link (2,5) is sufficient to overthrow node 1 to the weakest position in the network. Specifically, $\Delta SV(1) = -0{,}17$. The resulting graph is presented in Fig. 8.

4.4. Mixed topology

We analyze the symmetric mixed topology that includes "Point-to-point", "Star" and "Ring" based sub-graphs. The given network is presented in Fig. 9.

According to the results in Table 5, node 3 is the leader (the initial $SV(3) = 1.25$). Following Algorithm 3, nine links are created to overthrow node 3 to the weakest position with $SV = 0.7$. The resulting network is presented in Fig. 10.

5. Testing on the real-life networks

In the given section, we illustrate Algorithm 2 and Algorithm 3 based on two real-life networks. The first network is the largest connected component of the NHH interdepartmental co-

Table 6
Initial results for the NHH largest component.

Node	Shapley value	Node	Shapley value	Node	Shapley value	Node	Shapley value
9	1.41	40	0.62	58	0.71	69	0.98
21	0.67	45	1.02	60	0.87	70	1.99
26	1.73	50	1.03	61	0.83	73	1.35
34	0.82	52	0.67	65	1.49	130	0.67
37	0.7	53	0.99	67	0.71	142	0.67
39	1.07						

Table 7
Established links in the NHH network following Algorithm 2.

k	Link	*Target*			k	Link	*Target*		
		SV	Decrease	Position			SV	Decrease	Position
1	(40,58)	1.742	0.248	1	17	(21,67)	0.911	0.014	11
2	(40,60)	1.608	0.133	2	18	(21,40)	0.897	0.014	12
3	(40,67)	1.508	0.1	2	19	(21,58)	0.883	0.014	13
4	(53,58)	1.425	0.083	2	20	(21,60)	0.869	0.014	14
5	(58,67)	1.358	0.067	2	21	(26,58)	0.858	0.011	14
6	(40,65)	1.301	0.057	4	22	(26,40)	0.847	0.011	14
7	(53,60)	1.244	0.057	6	23	(26,60)	0.836	0.011	14
8	(60,67)	1.196	0.048	6	24	(26,67)	0.825	0.011	15
9	(9,58)	1.149	0.048	6	25	(9,21)	0.816	0.009	16
10	(9,40)	1.107	0.042	6	26	(21,53)	0.807	0.009	17
11	(40,53)	1.071	0.036	7	27	(34,40)	0.798	0.009	17
12	(58,65)	1.036	0.036	7	28	(26,65)	0.789	0.009	17
13	(9,60)	1.004	0.032	8	29	(34,58)	0.78	0.009	17
14	(53,67)	0.972	0.032	8	30	(34,60)	0.77	0.009	17
15	(9,65)	0.947	0.025	9	31	(34,67)	0.761	0.009	17
16	(53,65)	0.925	0.022	9	32	(37,58)	0.754	0.008	17

authorship network and the second one is the largest component of the BI interdepartmental co-authorship network. The detailed analysis of the NHH and BI networks is presented in Belik & Jörnsten [6].

5.1. NHH network

The network structure of the NHH largest component is presented in Fig. 11.

First, we test Algorithm 2 in order to detect and overthrow a leader applying different *k*-values. Since the NHH largest component has 32 links connecting 21 nodes, we run the algorithm for all values of *k* in the range [1, 32].

First, the algorithm calculates the initial SVs (see Table 6).

Node 70 is detected as the *Target*-node with SV = 1.99. Next, the algorithm establishes *k*-links allowed to build in order to overthrow node 70. Table 7 shows the list of consequently established links. For each link, we provide the following details:

1. Current SV of the *Target*-node for the latest established link.
2. The difference between SVs of the *Target*-node before and after the link was established (i.e., "Decrease").
3. The current position (i.e., SV-based rank) of the *Target*-node within the network.

For example, "Position = 3" means that the node is the third-most influential (out of 21 nodes) in terms of SV-based analysis.

Each value in the "Link" column shows the latest link established for the current *k*. For example, for *k* = 3 three links were established. The first two links (i.e., (40,58) and (40,60)) are reflected in the previous rows, and the latest link (i.e., (40,67)) is presented in the row *k* = 3.

It is important to notice that each approved link guarantees the SV-decrease of the initial leader (i.e., *Target*-node), but it is not necessary that each approved link gives a decrease in terms of its "Position" value. In fact, each approved link makes the *Target*-node weaker, but it also affects the rearrangement of SVs for all other nodes in the network. Therefore, it is a very common situation when more than one link has to be established in order to decrease the "Position" value of the *Target*-node.

Next, we apply Algorithm 3 to the NHH largest component in order to detect and overthrow the current leader to its weakest position.

First, the algorithm calculates the initial SVs. The results are presented in Table 6.

Node 70 is detected as the *Target*-node with SV = 1.99. Next, the algorithm establishes the set of links in order to overthrow node 70 to the weakest position. The list of consequently established links is presented in Appendix A. For each link we provide the details about the current SV*(Target)* and the difference between SVs of the *Target*-node before and after the link was established.

According to Appendix A, sixty-seven links were created to overthrow node 70 from the position of the leader to the SV-based weakest position in the network. The resulting SVs for all nodes are presented in Table 8.

5.2. BI network

The network structure of the BI largest component is presented in Fig. 12.

Applying Algorithm 2, we get the initial SVs on the first step (see Table 9). Since the BI largest component has 38 links connecting 28 nodes, we run the algorithm for all values of *k* in the range [1, 38].

According to Table 9, node 242 is detected as the most

Table 8
Resulting SVs for the NHH largest component based on Algorithm 3.

Node	Shapley value	Node	Shapley value	Node	Shapley value	Node	Shapley value
9	1.87	40	1.37	58	1.37	69	0.6
21	0.56	45	1.06	60	1.37	70	0.56
26	1.23	50	1.04	61	0.61	73	0.72
34	0.97	52	0.59	65	1.44	130	0.56
37	0.78	53	1.37	67	1.37	142	0.59
39	0.96						

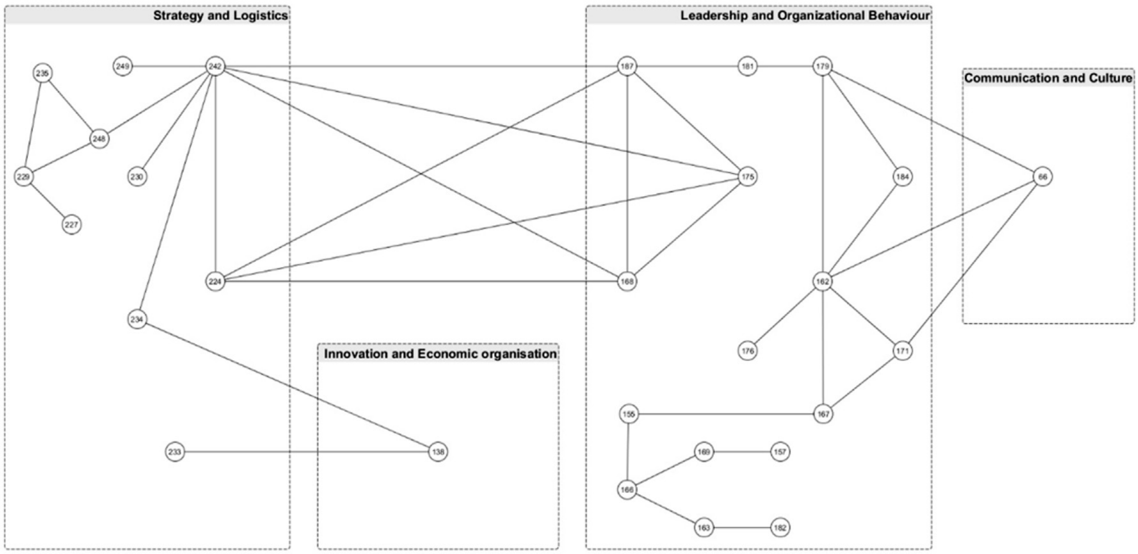

Fig. 12. The BI largest component.

Table 9
Initial results for the BI largest component.

Node	Shapley value	Node	Shapley value	Node	Shapley value	Node	Shapley value
66	0.84	167	0.98	181	0.7	230	0.61
138	1.17	168	0.88	182	0.83	233	0.83
155	0.83	169	1.08	184	0.68	234	0.78
157	0.83	171	0.89	187	1.21	235	0.83
162	1.93	175	0.88	224	0.88	242	2.46
163	1.08	176	0.64	227	0.75	248	0.94
166	1.25	179	1.26	229	1.33	249	0.61

influential (SV(242) = 2.46). Next, the algorithm establishes *k*-links allowed to build in order to overthrow node 242. Table 10 shows the list of consequently established links.

Next, we apply Algorithm 3 to the BI largest component in order to detect and overthrow the leader to its SV-based weakest position.

First, the algorithm calculates the initial SVs. The results are presented in Table 9.

Node 242 is detected as the *Target*-node with SV = 2.46. Next, the algorithm establishes the set of links in order to overthrow

Table 10
Established links in the BI network following Algorithm 2.

k	Link	*Target*			k	Link	*Target*		
		SV	Decrease	Position			SV	Decrease	Position
1	(230,249)	2.128	0.332	1	20	(175,248)	1.003	0.025	10
2	(230,234)	1.961	0.167	1	21	(248,249)	0.980	0.023	10
3	(234,249)	1.828	0.133	2	22	(187,234)	0.958	0.022	11
4	(230,248)	1.728	0.100	2	23	(66,224)	0.946	0.011	12
5	(168,249)	1.644	0.083	2	24	(66,168)	0.935	0.011	14
6	(224,230)	1.578	0.067	2	25	(66,175)	0.924	0.011	16
7	(234,248)	1.511	0.067	2	26	(66,230)	0.913	0.011	17
8	(175,249)	1.444	0.067	2	27	(66,249)	0.902	0.011	18
9	(168,234)	1.397	0.048	2	28	(66,187)	0.893	0.009	18
10	(175,230)	1.349	0.048	2	29	(66,234)	0.884	0.009	18
11	(187,248)	1.302	0.048	2	30	(138,224)	0.875	0.009	18
12	(224,249)	1.254	0.048	4	31	(138,168)	0.866	0.009	18
13	(168,230)	1.218	0.036	6	32	(138,175)	0.857	0.009	18
14	(175,234)	1.183	0.036	7	33	(138,230)	0.847	0.009	18
15	(187,249)	1.147	0.036	8	34	(138,249)	0.838	0.009	18
16	(224,248)	1.111	0.036	8	35	(155,234)	0.831	0.008	21
17	(187,230)	1.083	0.028	8	36	(66,248)	0.823	0.008	21
18	(168,248)	1.056	0.028	10	37	(138,187)	0.816	0.008	21
19	(224,234)	1.028	0.028	10	38	(155,168)	0.808	0.008	21

Table 11
Resulting SVs for the BI largest component based on Algorithm 3.

Node	Shapley value	Node	Shapley value	Node	Shapley value	Node	Shapley value
66	0.9	167	0.79	181	0.59	230	1.46
138	1.03	168	1.46	182	0.59	233	0.6
155	0.68	169	0.66	184	0.6	234	1.34
157	0.63	171	0.57	187	1.67	235	0.64
162	1.82	175	1.46	224	1.46	242	0.54
163	1.1	176	0.57	227	0.75	248	1.82
166	0.79	179	1.02	229	1.14	249	1.34

node 242 to the weakest position. The list of consequently established links is presented in Appendix B. Ninety-six links were created to overthrow node 242 from the leader's position to the SV-based weakest position in the network. The resulting SVs for all nodes in the network are presented in Table 11.

6. Conclusion

An important factor in the analysis of leadership formation is to use a suitable measure. For this purpose, we employed the concept of Shapley value in the interpretation of Aadithya et al. [1]. The adapted Shapley value for the analysis of leadership in networks is based on the theory of cooperative games. Its advantages compared to traditional metrics are described in section 2 and deeply analyzed in Aadithya et al. [1] and Michalak et al. [28].

In the given research, we developed two algorithms (based on [1] and [28] that detect the network's most influential nodes and overthrow them to the weaker positions). Specifically, Algorithm 2 establishes *k* links allowed by the decision-maker and Algorithm 3 establishes the set of links to get the leader to the weakest leadership position. Both algorithms are based on the greedy approach. Checking all new possible links on each iteration, algorithms approve the link that gives the maximum decrease of the leader's SV in terms of the overall surplus distribution generated by the coalition of all agents in the network.

Initially, we showed how our approaches work based on the trivial network topologies. Next, we tested them based on two real-life networks. Specifically, we applied algorithms to the NHH and BI largest connected components.

The practical importance of the presented algorithms is based on their applicability for the analysis of real-life networks. As described in section 2, SV-based overthrow mechanisms help to develop managerial scenarios for the “evolutional” leader's overthrow based on the network modifications. In practice, this process can be presented by managerial decisions for structural transformations of organizational networks [11] or by marketing decisions to rearrange leadership positions in the customer networks [40]. The mechanisms introduced can also be employed for the leadership analysis in criminal networks with hidden patterns [27] or money laundering networks [36], where it is important to detect prospective organization modifications based on the analysis of initial structural (quantitative) characteristics. In this kind of networks, the overthrow of the detected leader may cause serious damage to the functioning of the network.

It is important to notice that in real-life networks, the presented algorithms are not the unique solutions, but they are useful methods to detect and to plan the prospective modifications. They present a useful tool for the technical analysis of the primary structural data in the initial steps of multifaceted quantitative network analysis [10] where the raw data (i.e., linkages) is frequently the only knowledge about the network interrelations.

The results of the research point to an interesting direction for future work.

Modern SNA requires the framework development for the technical (quantitative) analysis [8,22,42]. Since many social networks are characterized by complex and large-scale structures, the need for efficient quantitative methods of SNA is growing fast and, respectively, it requires new interdisciplinary mechanisms to be developed. In the given context, future work implies integration of the developed overthrow algorithms to the modern framework of multi-factor technical SNA and testing on the real-life large-scale networks. Another direction for future work is based on the idea of developing symbiotic models for leadership analysis combining the introduced SV-based approach with other centrality metrics, e.g., those based on node betweenness, closeness, etc. This will give new opportunities to extend multi-factor models of SNA and open new opportunities for the comprehensive understanding of networks' internal mechanisms.

Appendix A. Algorithm 3 applied to the NHH largest component

#	Link	SV(*Target*)	Decrease	#	Link	SV(*Target*)	Decrease
1	(40,58)	1.742	0.248	35	(34,65)	0.731	0.008
2	(40,60)	1.608	0.133	36	(37,40)	0.723	0.008
3	(40,67)	1.508	0.100	37	(37,60)	0.716	0.008
4	(53,58)	1.425	0.083	38	(37,67)	0.708	0.008
5	(58,67)	1.358	0.067	39	(39,60)	0.702	0.006
6	(40,65)	1.301	0.057	40	(37,65)	0.696	0.006
7	(53,60)	1.244	0.057	41	(39,67)	0.689	0.006
8	(60,67)	1.196	0.048	42	(9,37)	0.683	0.006
9	(9,58)	1.149	0.048	43	(37,53)	0.676	0.006
10	(9,40)	1.107	0.042	44	(39,40)	0.670	0.006
11	(40,53)	1.071	0.036	45	(39,58)	0.663	0.006
12	(58,65)	1.036	0.036	46	(9,39)	0.658	0.005
13	(9,60)	1.004	0.032	47	(39,53)	0.652	0.005
14	(53,67)	0.972	0.032	48	(39,65)	0.647	0.005
15	(9,65)	0.947	0.025	49	(40,45)	0.641	0.005
16	(53,65)	0.925	0.022	50	(45,58)	0.636	0.005
17	(21,67)	0.911	0.014	51	(45,60)	0.630	0.005
18	(21,40)	0.897	0.014	52	(45,67)	0.625	0.005
19	(21,58)	0.883	0.014	53	(9,45)	0.620	0.005
20	(21,60)	0.869	0.014	54	(40,50)	0.615	0.005
21	(26,58)	0.858	0.011	55	(45,53)	0.611	0.005
22	(26,40)	0.847	0.011	56	(45,65)	0.606	0.005
23	(26,60)	0.836	0.011	57	(50,58)	0.601	0.005
24	(26,67)	0.825	0.011	58	(50,60)	0.596	0.005
25	(9,21)	0.816	0.009	59	(50,67)	0.592	0.005
26	(21,53)	0.807	0.009	60	(9,50)	0.587	0.004
27	(34,40)	0.798	0.009	61	(40,52)	0.583	0.004
28	(26,65)	0.789	0.009	62	(50,65)	0.579	0.004
29	(34,58)	0.780	0.009	63	(52,53)	0.575	0.004
30	(34,60)	0.770	0.009	64	(52,58)	0.571	0.004
31	(34,67)	0.761	0.009	65	(52,60)	0.567	0.004
32	(37,58)	0.754	0.008	66	(52,67)	0.563	0.004
33	(9,34)	0.746	0.008	67	(9,52)	0.559	0.004
34	(34,53)	0.739	0.008				

Appendix B. Algorithm 3 applied to the BI largest component

#	Link	SV(*Target*)	Decrease	#	Link	SV(*Target*)	Decrease
1	(230,249)	2.128	0.332	42	(155,249)	0.778	0.008
2	(230,234)	1.961	0.167	43	(157,168)	0.771	0.006
3	(234,249)	1.828	0.133	44	(155,187)	0.765	0.006
4	(230,248)	1.728	0.100	45	(157,175)	0.759	0.006
5	(168,249)	1.644	0.083	46	(138,248)	0.752	0.006
6	(224,230)	1.578	0.067	47	(157,224)	0.746	0.006
7	(234,248)	1.511	0.067	48	(157,230)	0.739	0.006
8	(175,249)	1.444	0.067	49	(157,234)	0.733	0.006
9	(168,234)	1.397	0.048	50	(157,249)	0.726	0.006
10	(175,230)	1.349	0.048	51	(155,248)	0.721	0.005
11	(187,248)	1.302	0.048	52	(157,187)	0.716	0.005
12	(224,249)	1.254	0.048	53	(162,168)	0.710	0.005
13	(168,230)	1.218	0.036	54	(162,175)	0.705	0.005
14	(175,234)	1.183	0.036	55	(162,224)	0.699	0.005
15	(187,249)	1.147	0.036	56	(162,230)	0.694	0.005
16	(224,248)	1.111	0.036	57	(162,234)	0.688	0.005
17	(187,230)	1.083	0.028	58	(162,249)	0.683	0.005
18	(168,248)	1.056	0.028	59	(157,248)	0.678	0.005
19	(224,234)	1.028	0.028	60	(162,187)	0.673	0.005
20	(175,248)	1.003	0.025	61	(163,168)	0.668	0.005
21	(248,249)	0.980	0.023	62	(163,175)	0.663	0.005
22	(187,234)	0.958	0.022	63	(163,224)	0.659	0.005
23	(66,224)	0.946	0.011	64	(163,230)	0.654	0.005
24	(66,168)	0.935	0.011	65	(163,234)	0.649	0.005
25	(66,175)	0.924	0.011	66	(163,249)	0.644	0.005
26	(66,230)	0.913	0.011	67	(162,248)	0.640	0.004
27	(66,249)	0.902	0.011	68	(163,187)	0.636	0.004
28	(66,187)	0.893	0.009	69	(166,168)	0.632	0.004
29	(66,234)	0.884	0.009	70	(166,175)	0.628	0.004
30	(138,224)	0.875	0.009	71	(166,224)	0.624	0.004
31	(138,168)	0.866	0.009	72	(166,230)	0.619	0.004
32	(138,175)	0.857	0.009	73	(166,234)	0.615	0.004
33	(138,230)	0.847	0.009	74	(166,249)	0.611	0.004
34	(138,249)	0.838	0.009	75	(166,187)	0.607	0.004
35	(155,234)	0.831	0.008	76	(163,248)	0.604	0.004
36	(66,248)	0.823	0.008	77	(167,168)	0.600	0.004
37	(138,187)	0.816	0.008	78	(167,175)	0.596	0.004
38	(155,168)	0.808	0.008	79	(167,224)	0.593	0.004
39	(155,175)	0.801	0.008	80	(167,230)	0.589	0.004
40	(155,224)	0.793	0.008	81	(167,234)	0.585	0.004
41	(155,230)	0.785	0.008	82	(167,249)	0.582	0.004
#	Link	SV(*Target*)	Decrease	#	Link	SV(*Target*)	Decrease
83	(166,248)	0.578	0.003	90	(169,249)	0.556	0.003
84	(167,187)	0.575	0.003	91	(167,248)	0.553	0.003
85	(168,169)	0.572	0.003	92	(169,187)	0.550	0.003
86	(169,175)	0.569	0.003	93	(171,224)	0.547	0.003
87	(169,224)	0.565	0.003	94	(168,171)	0.544	0.003
88	(169,230)	0.562	0.003	95	(171,175)	0.541	0.003
89	(169,234)	0.559	0.003	96	(171,230)	0.538	0.003

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