



# The comparison of normalization procedures based on different classification systems



Yunrong Li, Javier Ruiz-Castillo\*

Departamento de Economía, Universidad Carlos III, Spain

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## ABSTRACT

In this paper, we develop a novel methodology within the IDCP measuring framework for comparing normalization procedures based on different classification systems of articles into scientific disciplines. Firstly, we discuss the properties of two rankings, based on a graphical and a numerical approach, for the comparison of any pair of normalization procedures using a single classification system for evaluation purposes. Secondly, when the normalization procedures are based on two different classification systems, we introduce two new rankings following the graphical and the numerical approaches. Each ranking is based on a double test that assesses the two normalization procedures in terms of the two classification systems on which they depend. Thirdly, we also compare the two normalization procedures using a third, independent classification system for evaluation purposes. In the empirical part of the paper we use: (i) a classification system consisting of 219 sub-fields identified with the Web of Science subject-categories; an aggregate classification system consisting of 19 broad fields, as well as a systematic and a random assignment of articles to sub-fields with the aim of maximizing or minimizing differences across sub-fields; (ii) four normalization procedures that use the field or sub-field mean citations of the above four classification systems as normalization factors; and (iii) a large dataset, indexed by Thomson Reuters, in which 4.4 million articles published in 1998–2003 with a five-year citation window are assigned to sub-fields using a fractional approach. The substantive results concerning the comparison of the four normalization procedures indicate that the methodology can be useful in practice.

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## 1. Introduction

Differences in publication and citation practices have been known for decades to create serious difficulties for the comparison of raw citation counts across different scientific disciplines. Since the early eighties various normalization proposals have been suggested (see the review by [Schubert & Braun, 1996](#)). Moreover, the normalization problem has recently attracted renewed interest.<sup>1</sup> Consequently, there is a need to develop methods for the comparison of the performance achieved by different normalization procedures in empirical situations.

*Abbreviation:* IDCP, citation inequality due to differences in citation practices.

\* Corresponding author. Tel.: +34 91 624 95 88.

E-mail address: [jrc@eco.uc3m.es](mailto:jrc@eco.uc3m.es) (J. Ruiz-Castillo).

<sup>1</sup> Among the target, or cited-side variety of normalization procedures, see [Glänzel \(2011\)](#), [Radicchi, Fortunato, and Castellano \(2008\)](#), [Radicchi and Castellano \(2012\)](#), [Crespo, Li, and Ruiz-Castillo \(2013\)](#) and [Crespo, Herranz, Li, and Ruiz-Castillo \(2013\)](#), as well as the review of the percentile rank approach by [Bornmann and Marx \(2013\)](#). Among the source, or citing-side variety, see inter alia [Zitt and Small \(2008\)](#), [Moed \(2010\)](#), [Leydesdorff and Opthof \(2010\)](#), and [Waltman and Van Eck \(2013a\)](#).

Lacking information on the citing side, we focus on normalization procedures of the target or cited-side variety, where each procedure is based on a priori given classification system of publications in the periodical literature into a set of scientific disciplines. The paper studies the evaluation of alternative normalization procedures in two scenarios. In the first one, there is only a single classification system for the implementation as well as the evaluation of two (or more) normalization procedures. In this case, all that is needed is a method for assessing the performance of the contesting normalization procedures using the single classification system for evaluation purposes. In the second scenario, there are two (or more) classification systems for the implementation and the evaluation of two (or more) normalization procedures. As far as we know, this is the first paper that presents a complete discussion of this case (see, however, the contributions by [Sirtes, 2012](#); [Waltman & Van Eck, 2013b](#), that will be discussed below).

Given a classification system, we evaluate the performance of normalization procedures using the measurement framework recently introduced in [Crespo, Li, et al. \(2013\)](#), where the number of citations received by an article is a function of two variables: the article's underlying scientific influence, and the discipline to which it belongs. Consequently, the citation inequality of the distribution consisting of all articles in all disciplines – the *all-sciences case* – is the result of two forces: differences in scientific influence within each homogeneous discipline, and differences in citation practices across the set of heterogeneous disciplines. Essentially, as we will see below, the effect of the latter on citation inequality is captured by an *IDCP* term – where *IDCP* stands for citation inequality attributable to differences in citation practices.

A key aspect of this framework is that it serves to evaluate any set of normalization procedures in terms of any given classification system as required in the first scenario. The evaluation can take a graphical, or a numerical form.<sup>2</sup> In this paper, we establish that the graphical approach does not provide a complete ranking, i.e. we show that there are situations in which a pair of normalization procedures is non-comparable according to the graphical criterion. We also establish that the rankings according to the two approaches are logically independent, that is, we show that there exists at least one pair of normalization procedures that are ordered differently by the two rankings.

The canonical example of the second scenario arises in the presence of a number of classification systems at different aggregate levels. Assume for simplicity that there are only two hierarchically nested classification systems into what we call *sub-fields* and *fields*, so that every sub-field at the lower aggregation level belongs to only one field at the higher aggregate level. The question we study in this paper is how to compare one normalization procedure based at the sub-field level with another based at the field level. The problem is that we only know how to assess alternative normalization procedures using a single classification system for evaluation purposes. Therefore, the performance of the first procedure evaluated at the sub-field level cannot be directly compared with the performance of the second procedure evaluated at the field level. Our solution to this problem is the introduction of a new ranking based on a double test that assesses both normalization procedures in terms of the two classification systems on which they depend. For a procedure to dominate the other according to the double test, it should perform better than the other under both classification systems.

This idea is applicable to the comparison of any two normalization procedures based on different classification systems independently of the method followed for their evaluation. However, it should be remembered that in our measuring framework the evaluation of normalization procedures could take a graphical and a numerical approach. Therefore, in our case we must introduce *two* new rankings, each of them relying on a double test that compares the two normalization procedures using for evaluation purposes the two classification systems on which they depend. For a procedure to strongly dominate another according to the graphical (or the numerical) approach it should exhibit a better graphical (or numerical) performance under both classification systems. We establish that the two rankings are logically independent; therefore, strict dominance according to one ranking does not necessarily imply dominance according to the second.

This strategy deserves two closely related comments. Firstly, satisfying either of the two dominance criteria is a strong requirement. Consequently, we expect that neither of the two new rankings is complete. Secondly, [Sirtes \(2012\)](#) first suggested that the assessment of two classification-system-based normalization procedures would be generally biased in favor of the normalization procedure based on the system used for evaluation purposes. [Waltman and Van Eck \(2013b\)](#) concur with this idea, and provide further arguments about the possibility of this bias. In a double test, the presence of a bias of this type would favor the first (and the second) procedure under comparison when the first (and the second) classification system is used for evaluation purposes. Therefore, the bias would increase the probability that the two procedures are non-comparable. In any case, we confirm that neither of the two rankings is complete.

In order to avoid the bias, [Waltman and Van Eck \(2013b\)](#) compare source and target normalization procedures using an independent classification system for evaluation purposes. On our part, we believe that this is a recommendation worth pursuing. Thus, in the second scenario we suggest the comparison of any pair of classification-system-based normalization procedures using two strategies: the double tests that only involve the two classification systems on which the normalization procedures are based and, by analogy with [Waltman and Van Eck's \(2013b\)](#) procedure, the evaluation in terms of a third, independent classification system. Therefore, to illustrate this methodology in empirical situations we need to specify a

<sup>2</sup> Both forms have been previously used in two instances: (i) to compare the performance of different normalization procedures based on the same classification system ([Crespo, Li, et al., 2013](#); [Crespo, Herranz, et al., 2013](#); [Li, Castellano, Radicchi, & Ruiz-Castillo, 2013](#)), and (ii) to compare two types of normalization procedures, namely, those target procedures in which the disciplines' mean citations in different classification systems are used as normalization factors, and a variety of source normalization procedures independent of any classification system ([Waltman & Van Eck, 2013b](#)).

number of elements, namely: (i) a minimum of three classification systems; (ii) a minimum of two classification-system-based normalization procedures, and (iii) a rich enough dataset.

- (i) We use two types of classification systems. Firstly, we use two nested classification systems at the field and the sub-field aggregation levels. Secondly, following Zitt, Ramana-Rahari, and Bassecouard (2005) we focus on a pair of classification systems that organize the available data in two polar ways: assigning papers to sub-fields in a systematic manner, so as to make the differences between sub-fields as large as possible; or assigning papers to sub-fields in a random manner, so as to make the differences between sub-fields as small as possible. We refer to them as the *systematic* and *random assignments* at the sub-field level.
- (ii) As far as normalization procedures are concerned, recall that, since the inception of Scientometrics as a field of study, differences in citation practices across scientific disciplines in the all-sciences case are usually taken into account by choosing the world mean citation rates in each discipline as normalization factors (see inter alia Braun, Glänzel, & Schubert, 1985; Moed & van Raan, 1988; Moed, Burger, Frankfort, & van Raan, 1985; Moed, De Bruin, & van Leeuwen, 1995; Schubert & Braun, 1986; Schubert & Braun, 1996; Schubert, Glänzel, & Braun, 1983, 1987; Vinkler, 1986, 2003). More recently, other contributions support this traditional procedure on different grounds (Crespo, Li, et al., 2013; Crespo, Herranz, et al., 2013; Li et al., 2013; Radicchi & Castellano, 2012; Radicchi et al., 2008). Consequently, in this paper we choose this type of normalization procedure for every one of the four classification systems introduced in the previous paragraph.<sup>3</sup>
- (iii) Using a dataset of approximately 4.4 million articles published in all scientific disciplines in 1998–2003 with a five-year citation window, we identify the sub-fields at the lowest aggregation level permitted by our data with the 219 Web of Science subject categories distinguished by Thomson Reuters. As is well known, a practical problem is that documents in Thomson Reuters databases are assigned to sub-fields via the journal in which they have been published. Many journals are assigned to a single sub-field, but many others are assigned to two, three, or more sub-fields. There are two alternatives to deal with this problem: a fractional strategy, according to which each publication is fractioned into as many equal pieces as necessary, with each piece assigned to a corresponding sub-field; and a multiplicative strategy in which each paper is wholly counted as many times as necessary in the several sub-fields to which it is assigned. Fortunately, Crespo, Herranz, et al. (2013) establishes that the effect on citation inequality of differences in citation practices at the sub-field level according to the two strategies is very similar, so that it suffices to work with one of the two alternatives. In this paper we focus on the fractional strategy.

A second well known difficulty is that there is no generally agreed-upon Map of Science that allows us to climb from the sub-field up to other aggregate levels (see inter alia Boyack, Klavans, & Börner, 2005; Leydesdorff, 2004, 2006; Leydesdorff & Rafols, 2009; Small, 1999; Waltman & Van Eck, 2012 as well as the references they contain). Among the many alternatives, in this paper we consider an intermediate level consisting of 19 broad fields taken from Albarrán, Crespo, Ortuño, and Ruiz-Castillo (2011), a contribution that borrows from the schemes recommended by Tijssen and van Leeuwen (2003) and Glänzel and Schubert (2003) with the aim of maximizing the possibility that a power law represents the upper tail of the citation distributions involved. It is not claimed that this scheme provides an accurate representation of the structure of science. It is rather a convenient simplification for the discussion of aggregation issues in this paper.

The remaining part of this paper is organized into five sections. Section 2 summarizes the measurement framework introduced in Crespo, Li, et al. (2013), and presents the estimates of the effect on citation inequality of differences in citation practices across the fields and sub-fields included in our four classification systems. Section 3 is devoted to the comparison of normalization procedures using a single classification system for evaluation purposes within the graphical and the numerical approach. Section 4 introduces the two double tests for the comparison of any pair of normalization procedures using two classification systems for evaluation purposes. Section 5 discusses the empirical results on the comparison of our four normalization procedures using both the two double tests, as well as the strategy recommended by Waltman and Van Eck (2013b). Finally, Section 6 offers some concluding remarks.

## 2. The effect on citation inequality of differences in citation practices across scientific disciplines

### 2.1. A measurement framework

Every classification system  $K$  consists of  $D$  disciplines, indexed by  $d = 1, \dots, D$ . Let  $\mathbf{C}$  be the initial citation distribution consisting of the citations received by  $M$  articles. For simplicity, we assume that every article in  $\mathbf{C}$  is assigned to a single discipline. Let  $M_d$  be the number of articles in discipline  $d$ , so that  $\sum_d M_d = M$ . Denote each discipline citation distribution by  $\mathbf{c}_d = \{c_{di}\}$  with  $i = 1, \dots, M_d$ , where  $c_{di}$  is the number of citations received by article  $i$  in discipline  $d$ . The original citation distribution is simply the union of all the discipline distributions, that is,  $\mathbf{C} = \cup_d \mathbf{c}_d$ .

<sup>3</sup> As indicated in the concluding section, the methods developed in this paper can be equally used for the comparison of other types of normalization procedures.

Of course, the problem is that every discipline is characterized by idiosyncratic publication and citation practices. Consequently, the direct comparison of raw citation counts between articles belonging to different disciplines is plagued with difficulties. In this paper, we measure the effect of differences in citation practices using a simple model introduced in Crespo, Li, et al. (2013). As we will presently see, given a classification system  $K$ , in the implementation of this model using an additively decomposable inequality index the citation inequality attributed to differences in citation practices across disciplines is captured by a between-group inequality term – denoted  $IDCP(K)$  – in a certain partition by discipline and citation quantile.

Partition each citation distribution  $\mathbf{c}_d$  into  $\Pi$  quantiles,  $\mathbf{c}_d^\pi$ , of size  $M_d/\Pi$ , indexed by  $\pi = 1, \dots, \Pi$ . Thus,  $\mathbf{c}_d = (\mathbf{c}_d^1, \dots, \mathbf{c}_d^\pi, \dots, \mathbf{c}_d^\Pi)$ . Let  $\mu_d^\pi$  be the average citation of quantile  $\mathbf{c}_d^\pi$ , and let  $\boldsymbol{\mu}_d^\pi$  be the vector where each article in quantile  $\mathbf{c}_d^\pi$  is assigned the average citation  $\mu_d^\pi$ . For every  $\pi$ , consider the distribution  $\mathbf{m}^\pi = (\mu_1^\pi, \dots, \mu_d^\pi, \dots, \mu_D^\pi)$  of size  $M/\Pi$ . Under the assumptions of the model in Crespo, Li, et al. (2013), for each  $\pi$  the citation inequality of  $\mathbf{m}^\pi$ ,

$$I(\mathbf{m}^\pi) = I(\mu_1^\pi, \dots, \mu_d^\pi, \dots, \mu_D^\pi), \tag{1}$$

is attributed to differences in citation practices across disciplines at that quantile. Therefore, any weighted average of the expressions  $I(\mathbf{m}^\pi)$  over all quantiles constitutes a good measure of the phenomenon we are interested in.

For reasons explained in Crespo, Li, et al. (2013), it is convenient to work with a certain member of the so-called Generalized Entropy family of citation inequality indices, the index  $I_1$ . This index has the important property that, for any partition of the population into sub-groups, the overall citation inequality is additively decomposable into a within- and a between-group term, where the within-group term is a weighted sum of the sub-group citation inequality terms with weights that add up to one. Using this index, it can be shown that the citation inequality of the initial distribution,  $I_1(\mathbf{C})$ , can be expressed as the sum of three terms, one of which is the  $IDCP$  term.<sup>4</sup> For classification system  $K$ , this term is defined as follows:

$$IDCP(K) = \sum_{\pi} v^\pi I_1(\mathbf{m}^\pi) = \sum_{\pi} v^\pi I_1(\mu_1^\pi, \dots, \mu_d^\pi, \dots, \mu_D^\pi), \tag{2}$$

where  $v^\pi = \sum_d v_d^\pi$ , and  $v_d^\pi$  is the share in the total citations of discipline  $d$  of the citations received by articles in quantile  $\mathbf{c}_d^\pi$ . Thus,  $v^\pi$  is the share of total citations in the all-sciences case received by articles in quantile  $\mathbf{c}_d^\pi$  for all disciplines, and  $\sum_{\pi} v^\pi = 1$ . Therefore, Eq. (2) indicates that  $IDCP(K)$  is a weighted average of the key expressions in (1), with weights  $v^\pi$  that add up to one.<sup>5</sup> It should be noted that, due to the skewness of science, in practical applications the weights  $v^\pi$  tend to increase dramatically with  $\pi$ .

## 2.2. Classification systems

As indicated in Section 1, we work with four classification systems.

1. System  $A$  consists of 219 sub-fields, indexed by  $s = 1, \dots, 219$ , identified with the corresponding Web of Science categories distinguished by Thomson Reuters. Let  $N_s$  be the number of articles in sub-field  $s$  in system  $A$ , so that  $\sum_s N_s = M$ , and let us denote sub-field  $s$  citation distribution by  $\mathbf{c}_s = \{c_{si}\}$  with  $i = 1, \dots, N_s$ .
2. Consider the following systematic assignment of publications into the 219 sub-fields of system  $A$ . Start by ordering all articles in the dataset from the least to the most cited, assigning the most highly cited articles to the smallest sub-field in  $A$ . We proceed in this fashion until the largest sub-field is assigned the least cited articles.<sup>6</sup> The classification system based on this systematic assignment is denoted by  $S$ . Sub-field  $s$  citation distribution in system  $S$  is denoted by  $\mathbf{c}_s^S = \{c_{si}^S\}$  with  $i = 1, \dots, N_s$ , and  $s = 1, \dots, 219$ .
3. The third classification system corresponds to the random assignment of publications into the 219 sub-fields of system  $A$ . We start by randomly selecting one sub-field in  $A$ , whose size is denoted by  $N_1$ . Then, we randomly draw  $N_1$  articles from the dataset, leaving the rest for the next step. We proceed in this way, using each time the articles remaining after the previous random draws, until the last sub-field is assigned the articles left at the next-to-last step. The classification system based on this random assignment is denoted by  $R$ . Sub-field  $s$  citation distribution in system  $R$  is denoted by  $\mathbf{c}_s^R = \{c_{si}^R\}$  with  $i = 1, \dots, N_s$ , and  $s = 1, \dots, 219$ .
4. System  $B$  consists of 19 fields, indexed by  $f = 1, \dots, 19$ , obtained by aggregation of the 219 sub-fields in system  $A$  according to the rule suggested in Albarrán et al. (2011). Let us denote by  $N_f = \sum_{s \in f} N_s$  the number of articles in field  $f$  in system  $B$ . For any  $f$ , the field citation distribution  $\mathbf{c}_f$  is the union of sub-fields in that field, that is,  $\mathbf{c}_f = \cup_{s \in f} \mathbf{c}_s = \{c_{fg}\}$  with  $g = 1, \dots, N_f$ .

<sup>4</sup> As far as the two remaining terms in the decomposition are concerned, one refers to the citation inequality that takes place within the  $\mathbf{c}_d^\pi$  quantiles, while the other measures the citation inequality in the distribution where each article in any discipline is assigned the mean citation of the quantile to which it belongs. For high  $\Pi$ , the first term is expected to be small, while the second – capturing the skewness of science in the all-sciences case – is expected to be large. For details, see Crespo, Li, et al. (2013).

<sup>5</sup> Naturally, both the weighting system  $v^\pi$  and the distributions  $\mathbf{m}^\pi$  depend on the classification system we use for evaluation purposes. However, for simplicity we do not express this dependency by writing  $v^\pi(K)$  and  $\mathbf{m}^\pi(K)$  in each case. Unless otherwise indicated, writing  $IDCP(K)$  is enough for our purposes.

<sup>6</sup> When we proceed in the opposite direction, that is, beginning with the assignment of the most highly cited articles to the largest sub-field, we end up with a very large number of sub-fields consisting entirely of uncited articles.

**Table 1**

The effect on citation inequality of differences in citation practices at the sub-field and field levels when the raw data is organized according to four classification systems.

Classification systems	IDCP term (1)	Overall citation inequality (2)	(1)/(2) in % (3)
<b>S</b>	0.8642	0.8644	99.98
<b>A</b>	0.1552	0.8644	17.95
<b>B</b>	0.1079	0.8644	12.48
<b>R</b>	0.0024	0.8644	0.28

System *S* = systematic assignment at the sub-field level. System *A* = articles classified into the 219 sub-fields corresponding to the Web of Science subject-categories (fractional approach). System *B* = articles classified into 19 broad fields according to the aggregation scheme developed in Albarrán et al. (2011). System *R* = random assignment at the sub-field level.

Note that the systematic and random assignments provide a test to verify whether the IDCP method works well in practice. Under the systematic (random) classification system differences between sub-fields would be at a maximum (minimum). Therefore, we expect IDCP(*S*) to be well above IDCP(*A*) and IDCP(*B*), while IDCP(*R*) is expected to be well below IDCP(*A*) and IDCP(*B*). Table 1 includes the values for the IDCP term for the four classification systems when the number of quantiles *II* is equal to 1000 – a choice maintained in the sequel.

Three comments are in order. Firstly, since the 4.4 million articles in the dataset are simply differently organized in the four cases and our citation inequality index is invariant to data permutations, overall citation inequality is the same for all classification systems. This is the value 0.8644 that appears in column 2 in Table 1. Secondly, it should be noted that the results in row *A* are taken from Crespo, Herranz, et al. (2013). They indicate that the IDCP(*A*) term represents, approximately, 18% of overall citation inequality. As expected, the IDCP term represents a smaller proportion of overall citation inequality when articles are classified into broad fields: IDCP(*B*)/I<sub>1</sub>(*C*) = 12.5%. Thirdly, it can be concluded that the IDCP method responds very well to the two polar cases in the sense that IDCP(*S*) represents practically 100% of overall citation inequality, while IDCP(*R*) is barely above zero (see column 3 in Table 1).

### 3. The evaluation of normalization procedures in terms of a given classification system

#### 3.1. Normalization procedures

Let us denote by  $\mu_s^A$ ,  $\mu_s^S$ , and  $\mu_s^R$  the average citation of sub-field *s* in systems *A*, *S*, and *R*, respectively. The procedures that use such means as normalization factors at the sub-field level are denoted by *NA*, *NS*, and *NR*, respectively. The normalization of every article *i* in sub-field *s* within its respective system proceeds as follows:

$$\begin{aligned} c_{si}^{A*} &= c_{si} / \mu_s^A \text{ in system } A; \\ c_{si}^{S*} &= c_{si} / \mu_s^S \text{ in system } S; \\ c_{si}^{R*} &= c_{si} / \mu_s^R \text{ in system } R. \end{aligned}$$

Similarly, denote by  $\mu_f^B$  the mean citation of field *f* in system *B*. Of course, for any *f* we have  $\mu_f^B = \sum_{s \in f} (N_s / N_f) \mu_s^A$ . The normalization of every article *g* in field *f* proceeds as follows:

$$c_{fg}^{B*} = c_{fg} / \mu_f^B \text{ in system } B.$$

Of course, for any *f* we have  $\mathbf{c}_f^* = \{c_{fg}^*\} = \cup_{s \in f} \mathbf{c}_s^* = \cup_{s \in f} \{c_{si}^*\}$ . After normalization by procedure *NK*, for *K* = *S*, *A*, *B*, and *R*, the IDCP terms and the citation distributions in the all-sciences case are denoted by IDCP<sup>NK</sup>(*K*) and  $\mathbf{C}^{K*}$ .

We are also interested in evaluating every procedure using other classification systems that are different from the one on which it is based. Let us start by evaluating procedure *NA* in terms of system *S*. Note that, for every article *i* in sub-field *s* in system *S*, there exists some article *j* in some sub-field *r* in system *A* such that  $c_{si}^S = c_{rj}^A$ . Therefore, for the evaluation of *NA* in terms of *S* the normalization of each article *i* in sub-field *s* in system *S* proceeds as follows:

$$c_{si}^{AS*} = c_{rj} / \mu_r^A.$$

In this case, the IDCP term after normalization is denoted by IDCP<sup>NA</sup>(*S*). Next, to evaluate *NA* in terms of system *B*, note that, for every article *g* in field *f* in system *B*, there exists some article *k* in some sub-field *t* in system *A* such that  $c_{fg}^B = c_{tk}^A$ . Therefore, for the evaluation of *NA* in terms of *B* the normalization of each article *g* in sub-field *f* in system *B* proceeds as follows:

$$c_{fg}^{AB*} = c_{tk} / \mu_t^A.$$

In this case, the IDCP term after normalization is denoted by IDCP<sup>NA</sup>(*B*). We leave to the reader how to evaluate any other procedure in terms of a system that is different from the one on which the procedure is based.

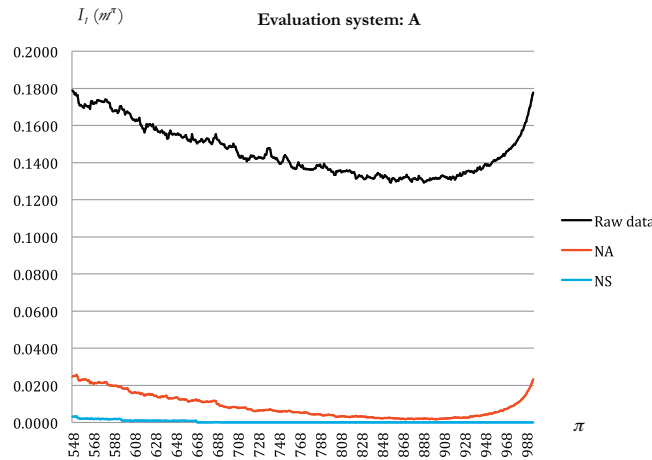


Fig. 1. The comparison of normalization procedures NS and NA using classification system A for evaluation purposes (quantile interval [548, 995]).

In general, given any normalization procedure  $NK$  and a classification system  $G$  – not necessarily equal to  $K$  – the  $IDCP$  after normalization is written  $IDCP^{NK}(G)$ . Similarly, the citation distribution in the all-sciences case after normalization by  $NK$  evaluated in terms of system  $G \neq K$  is denoted by  $\mathbf{C}^{KG^*}$ . Note that, given any procedure  $NK$ ,  $\mathbf{C}^{KG^*}$  for  $G \neq K$  is a mere permutation of distribution  $\mathbf{C}^{K^*}$ . Hence, given  $NK$ , total citation inequality of the normalized citation distribution in the all-sciences case is independent of the system used for evaluation purposes, i.e. for any  $K$ ,  $I_1(\mathbf{C}^{KG^*}) = I_1(\mathbf{C}^{K^*})$  for all  $G \neq K$ . For later reference, the values for  $IDCP^{NK}(G)$  and  $I_1(\mathbf{C}^{K^*})$  are presented in Appendix.

To facilitate the description of the graphical and numerical evaluation approaches, it is essential to realize that, for any  $K$  and  $G$ , the term  $IDCP^{NK}(G)$  is a weighted average of expressions capturing the citation inequality of the normalized citation distributions according to procedure  $NK$  attributable to differences in citation practices across disciplines in system  $G$ . Formally, we have:

$$IDCP^{NK}(G) = \sum_{\pi} v_{*}^{\pi}(G) I_1^{NK}[\mathbf{m}_{*}^{\pi}(G)], \tag{3}$$

where the asterisk in  $v_{*}^{\pi}(G)$  and  $\mathbf{m}_{*}^{\pi}(G)$  denotes that we are referring to a normalized distribution within classification system  $G$ , while the superscript in  $I_1^{NK}[\cdot]$  indicates that the normalization takes place according to  $NK$ . Using this notation, we can proceed to discuss the graphical and numerical evaluation approaches.

### 3.2. The graphical method

It should be remembered that, given the skewness of science, for any normalization procedure and any classification system, the weights  $v_{*}^{\pi}(\cdot)$  in Eq. (3) tend to increase dramatically with  $\pi$ . Therefore, it is convenient to make the evaluation before the weighting system is applied, namely, in terms of the expressions  $I_1^{NK}[\mathbf{m}_{*}^{\pi}(G)]$  that, for any  $\pi$ , capture the effect of differences in citation practices in system  $G$  after normalization by procedure  $NK$ . Thus, given any pair of procedures  $NK$  and  $NL$  and a single classification system  $G$  – not necessarily distinct from  $K$  or  $L$  – for evaluation purposes, we proceed by comparing  $I_1^{NK}[\mathbf{m}_{*}^{\pi}(G)]$  and  $I_1^{NL}[\mathbf{m}_{*}^{\pi}(G)]$  at any  $\pi$ .

We say that  $NK$  is uniformly better than  $NL$  under system  $G$  if  $I_1^{NK}[\mathbf{m}_{*}^{\pi}(G)] < I_1^{NL}[\mathbf{m}_{*}^{\pi}(G)]$  for all  $\pi$ , that is, if the curve  $I_1^{NK}[\cdot]$  as a function of  $\pi$  is uniformly below the curve  $I_1^{NL}[\cdot]$ . In this case, we write  $\{NK \succ_1(G) NL\}$ , while in the opposite case we write  $\{NL \succ_1(G) NK\}$ . However, the avoidance of the weighting issue comes at a cost: when the two curves exhibit one (or more) intersections, then we must conclude that two procedures are non-comparable according to this criterion, in which case we write  $\{NK \text{Non} \succ_1(G) NL\}$ .

Consider Example 1 where  $K=S$ ,  $L=A$ , and  $G=A$ , illustrated in Fig. 1 (since expressions  $I_1^{NS}[\cdot]$  and  $I_1^{NA}[\cdot]$  are very high for many quantiles in the lower tail of citation distributions and in the last quantiles in the upper tail, for clarity Fig. 1 only includes quantiles  $\pi$  in the interval [548, 995]). Three points should be noted. Firstly, the citation inequality due to differences in citation practices at any  $\pi$  in the raw data organized according to system  $A$  is measured by the curve  $I_1[\mathbf{m}^{\pi}(A)]$  in black in Fig. 1. Secondly, normalization gives rise to a clear decrease of the curves  $I_1^{NK}[\mathbf{m}_{*}^{\pi}(A)]$  for both  $NK=NS, NA$  below  $I_1[\mathbf{m}^{\pi}(A)]$ . Thirdly, in Fig. 1 the curve  $I_1^{NS}[\mathbf{m}_{*}^{\pi}(A)]$  is always below  $I_1^{NA}[\mathbf{m}_{*}^{\pi}(A)]$ , indicating that  $\{NS \succ_1(A) NA\}$  according to the graphical approach.<sup>7</sup>

<sup>7</sup> Of course, one could apply formal dominance methods to compare the two curves. However, we do not find it essential in the sequel, where a simple graphical approach will be applied. It should be noted that, in all cases whenever one normalization procedure dominates another one in the subset of quantiles shown in a figure, the dominance takes place uniformly over the entire domain.

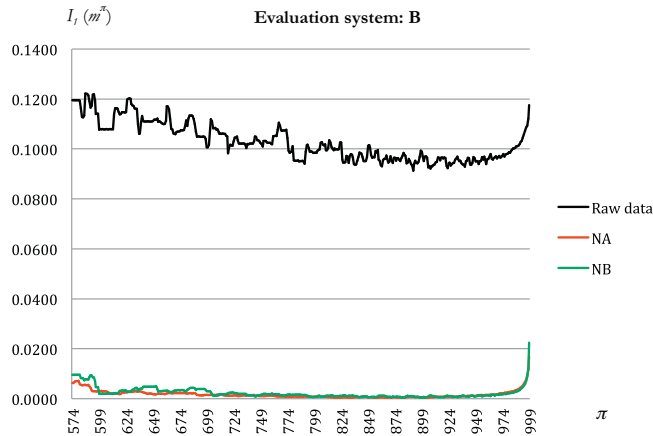


Fig. 2. The comparison of normalization procedures NA and NB using classification system B for evaluation purposes (quantile interval [532, 999]).

Table 2  
The impact of normalization under the four classification systems.

Normalization procedures	Change in the value of the IDCP term after normalization by the different procedures, in %			
	Classification system used for evaluation purposes:			
	S (1)	A (2)	B (3)	R (4)
NS	74.9	33.5	26.0	−304.7
NA	26.4	83.2	88.7	−216.3
NB	19.4	60.1	87.8	−158.6
NR	0.2	−8.1	−5.9	11.0

Next, consider Example 2 where  $K=A$ ,  $L=B$ , and  $G=B$ , illustrated in Fig. 2 (for the interval  $\pi \in [574, 1000]$ ).<sup>8</sup> This is an interesting case, where we expect the procedure constructed at the lowest aggregate level, NA, to perform better than NB. However, this needs to be confirmed in practice. As a matter of fact, Fig. 2 illustrates that, relative to the situation with the raw data, both procedures perform well but, because they repeatedly intersect, they are non-comparable. Thus, we conclude that  $\{NANon>_1(B)NB\}$ , which shows that, in general, the ranking  $>_1(\cdot)$  is not complete.

3.3. The numerical method

An alternative way to compare any pair of procedures NK and NL under any given classification system G, is by comparing the corresponding IDCP terms after normalization, that is, by comparing  $IDCP^{NK}(G)$  and  $IDCP^{NL}(G)$ . We find more useful expressing the result as the percentage that the differences  $[IDCP^{NK}(G) - IDCP(G)]$  and  $[IDCP^{NL}(G) - IDCP(G)]$  represent relative to the initial situation,  $IDCP(G)$ . Thus, given any pair of procedures NK and NL and a single system G for evaluation purposes, we say that NK is numerically better than NL under system G if the following condition is satisfied:

$$\frac{[IDCP(G) - IDCP^{NK}(G)]}{IDCP(G)} > \frac{[IDCP(G) - IDCP^{NL}(G)]}{IDCP(G)}. \tag{4}$$

In this case, we write  $\{NK>_{II}(G)NL\}$ , while if inequality (4) goes in the opposite direction, then we write  $\{NL>_{II}(G)NK\}$ . Note that the ranking of normalization procedures in the numerical approach is always complete, that is, for any pair of normalization procedures to be evaluated in terms of any classification system, we can always say whether one procedure is numerically better than the other.

To see how numerical comparisons work in practice we need to know the consequences of applying the different normalization procedures under all classification systems. Using the values of  $IDCP^{NK}(G)$  in Appendix, Table 2 presents the change in the IDCP term before and after each of the normalization operations. Consider, for example, the case in which normalization procedure NA is applied to the data organized according to system S. The consequences are captured by  $IDCP^{NA}(S)$  in row NA and column 1 in Appendix. In turn, recall that  $IDCP(S)=0.8642$  (see column 1 in row S in Table 1). Taking into account

<sup>8</sup> Note that the units in which magnitudes are measured along the vertical axis in every figure are quite different. This precludes the direct, visual comparability between them.

criterion (4), we are interested in the percentage change in the *IDCP* term before and after applying *NA* in *S*, that is, in the expression

$$\frac{100[IDCP(S) - IDCP^{NA}(S)]}{IDCP(S)} = \frac{100(0.8642 - 0.6385)}{0.8642} = 26.1.$$

The value of this expression appears in row *NA* and column 1 in Table 2, indicating that the effect of differences in citation practices across sub-fields in system *S* has been reduced by 26.1% as a consequence of normalization by *NA*. This compares, for example, with the reduction of 19.4% caused by normalization with *NB* using again system *S* for evaluation purposes (see row *NB* and column 1 in Table 2). On the other hand, the figures in columns 2, 3, and 4 in row *NA* are the values in expression

$$100[IDCP(K) - \frac{IDCP^{NA}(K)}{IDCP(K)}],$$

when the evaluation system is  $K=A, B,$  and  $R$  rather than  $S$ .

Once the meaning of each entry in Table 2 has been clarified, we are ready to compare normalization procedures in some paradigmatic examples using the numerical approach. Consider again Example 1 where  $K=S, L=A,$  and  $G=A$ . Since  $IDCP^{NS}(A)=0.1032$  and  $IDCP^{NA}(A)=0.0260$  (see column 3 in Appendix), *NA* exhibits a better numerical performance than *NS* under system *A*, that is,  $\{NA \succ_{II}(A)NS\}$  (see column 2 in Table 2). This shows that the rankings  $\succ_I$  and  $\succ_{II}$  are independent because we have simultaneously  $\{NS \succ_I(A)NA\}$  and  $\{NA \succ_{II}(A)NS\}$ .

On the other hand, consider again Example 2 where procedures *NA* and *NB* are compared in terms of system *B*. It is observed in Table 2 that *NA* performs numerically better than *NB*, so that  $\{NA \succ_{II}(B)NB\}$ . This shows that the two approaches are complementary and can be profitably used together: although the two procedures are non-comparable according to the graphical approach, *NA* is seen to perform numerically better than *NB* when system *B* is used for evaluation purposes in both cases.

#### 4. The evaluation of normalization procedures using different classification systems

##### 4.1. The first double test

Consider the comparison of any two procedures *NK* and *NL*. The problem, of course, is that they cannot be compared in terms of their own classification system. In other words, the terms  $IDCP^{NK}(K)$  and  $IDCP^{NL}(L)$  are not directly comparable because the classification systems *K* and *L* are different. Economists will note that this problem is akin to the lack of comparability of a country's Gross National Product (GNP) in two different time periods in nominal terms, say  $GNP_1$  and  $GNP_2$ . The reason is that  $GNP_1$  is the value of production in period 1 at prices of that period, while  $GNP_2$  is the value of production in period 2 at prices of that period.<sup>9</sup> For a meaningful comparison, GNP in the two periods must be expressed at common prices; that is, comparisons must be made only in *real* terms. However, we face what is known as an index number problem: which prices should be used in the comparison? For best results, production in both periods should be expressed at prices of period 1 and at prices of period 2. If  $GNP_2$  at prices of period 1 is greater than  $GNP_1$ , and  $GNP_2$  is greater than  $GNP_1$  at prices of period 2, then we say that GNP in real terms has unambiguously increased at both periods' prices. If both inequalities go in the opposite direction, then we say that GNP in real terms has unambiguously decreased. Otherwise, namely, if one inequality favors one period and the other inequality favors the other, then we say that GNP in both periods is non-comparable in real terms.

In our context, any pair of procedures, *NK* and *NL*, should be evaluated using both systems *K* and *L*. In the graphical approach, the extension gives rise to five possibilities.

- (i) We say that *NK* strongly dominates *NL* in the graphical sense if the first procedure performs uniformly better than the second using both systems for evaluation purposes, namely, if  $\{NK \succ_I(K)NL\}$  and  $\{NK \succ_I(L)NL\}$ . In this case, we write  $\{NKD_I(K, L)NL\}$ .
- (ii) If the opposite of (i) is the case, then we write  $\{NLD_I(K, L)NK\}$ .
- (iii) If *NK* performs uniformly better than *NL* according to one of the systems, but the two procedures are uniformly non-comparable according to the second system, namely if, for example,  $\{NK \succ_I(K)NL\}$  and  $\{NK \text{Non} \succ_I(L)NL\}$ , then we say that *NK* weakly dominates *NL* in the graphical sense and write  $\{NKWD_I(K, L)NL\}$ .
- (iv) If the opposite of (iii) is the case, then we write  $\{NLWD_I(K, L)NK\}$ .
- (v) Otherwise, that is, if one procedure performs uniformly better than another under one classification system and the opposite is the case under the other system, or both procedures are non-comparable under both classification systems, then the two procedures are non-comparable according to this double test and we write  $\{NK \text{Non} \text{--} D_I(K, L)NL\}$ .

<sup>9</sup> Of course, the same situation arises when we compare the GNP of two different countries in nominal terms. In this case production in the two countries is evaluated at their respective price systems.



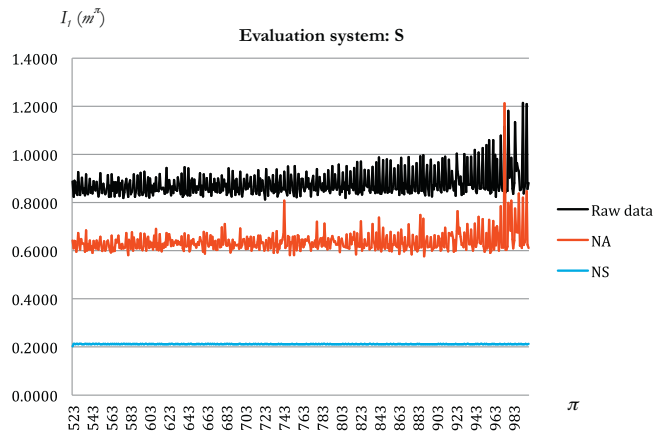


Fig. 3. The comparison of normalization procedures NS and NA using classification system S for evaluation purposes (quantile interval [523, 999]).

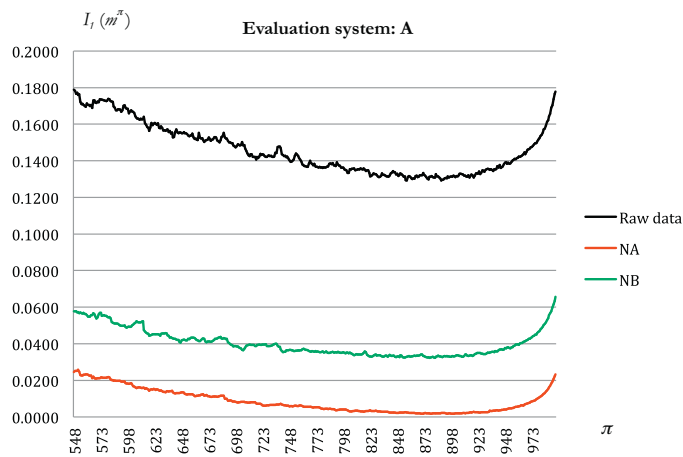


Fig. 4. The comparison of normalization procedures NA and NB using classification system A for evaluation purposes (quantile interval [548, 995]).

Consider Example 1 where  $K=S$ ,  $L=A$ , and  $G=A$ , already illustrated in Fig. 1, where  $\{NS>_1(A)NA\}$ . To this we must add the case  $K=S$ ,  $L=A$ , and  $G=S$ , illustrated in Fig. 3 (for the interval [523, 999]). It is observed that  $\{NS>_1(S)NA\}$ , so that NS strongly dominates NA in the graphical sense, i.e.  $\{NSD_1(A, S)NA\}$ . As a matter of fact, the same result is obtained when NS is compared with the two remaining procedures, that is,  $\{NSD_1(L, S)NL\}$  for  $L=B, R$  (for reasons of space, these results are available on request).

Next, consider Example 2 where  $K=A$ ,  $L=B$ , and  $G=B$ , already illustrated in Fig. 2, where  $\{NANon>_1(B)NB\}$ . To this we must add the case  $K=A$ ,  $L=B$ , and  $G=A$ , illustrated in Fig. 4 (for the interval [548, 995]). It is observed that  $\{NA>_1(A)NB\}$ , so that NA weakly dominates NB in the graphical sense, i.e.  $\{NAWD_1(B, A)NB\}$ .

Finally, Figs. 5 and 6 (for the intervals [548, 995] and [604, 997]) illustrate the comparison of procedures NA and NR using A and R for evaluation purposes.<sup>10</sup> It is observed that  $\{NA>_1(A)NR\}$  while  $\{NR>_1(R)NA\}$ , so that  $\{NANon-D_1(A, R)NR\}$ . The same result (available on request) is obtained for the comparison between NB and NR, namely,  $\{NBNon-D_1(B, R)NR\}$ . The lack of comparability between these two pairs of normalization procedures shows that the first double test does not generate a complete ranking.

#### 4.2. The second double test

In the numerical approach we proceed as follows. There are three possible cases.

<sup>10</sup> It should be noted that the curve  $I_1[m^\pi(A)]$  as a function of  $\pi$  for the raw data in system A is practically unaffected by normalization according to NR. Thus, this curve is not included in Fig. 5 because it practically coincides with  $I_1^{NR}[m^\pi(A)]$ .

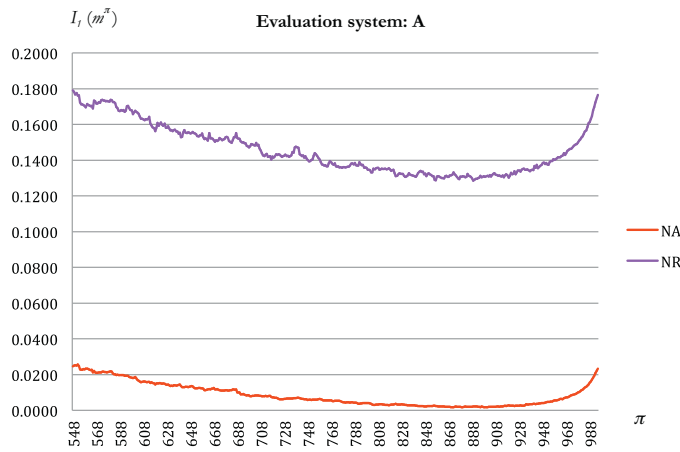


Fig. 5. The comparison of normalization procedures NA and NR using classification system A for evaluation purposes (quantile interval [548, 995]).

- (i) We say that  $NK$  dominates  $NL$  in the numerical sense if the first procedure performs numerically better than the second using both systems for evaluation purposes, namely, if  $\{NK >_{II}(K)NL\}$  and  $\{NK >_{II}(L)NL\}$ . In this case, we write  $\{NKD_{II}(K, L)NL\}$ .
- (ii) If the opposite of (i) is the case, then we write  $\{NLD_{II}(K, L)NK\}$ .
- (iii) Otherwise, that is, if one procedure performs numerically better than another under one classification system and the opposite is the case under the other system, then the two procedures are non-comparable and we write  $\{NKNon-D_{II}(K, L)NL\}$ .

The results in Table 2 allow us to illustrate the following two key cases. Firstly, the comparison of procedures NA and NB indicates that  $\{NA >_{II}(B)NB\}$  and  $\{NA >_{II}(A)NB\}$ , so that NA dominates NB in the numerical sense according to the second double test, i.e.  $\{NAD_{II}(B, A)NB\}$ . Secondly, for any of the remaining five pairs of normalization procedures, for example for NS and NA, it is observed that  $\{NS >_{II}(S)NA\}$  and  $\{NA >_{II}(A)NS\}$ , so that  $\{NANon-D_{II}(S, A)NS\}$ . Of course, this lack of comparability in five of the six possible cases establishes that the second double test does not generate a complete ranking.

Finally, recall that  $\{NSD_I(A, S)NA\}$  while  $\{NANon-D_{II}(A, S)NS\}$ . This shows that the two double tests are independent: strong dominance of NS over NA in the graphical sense does not imply dominance in the same direction in the numerical sense. Similarly, in spite of the fact that  $\{NAD_{II}(B, A)NB\}$  we have that  $\{NAWD_I(B, A)NB\}$ , that is, dominance in the numerical sense is compatible with just weak dominance in the graphical sense.

### 5. Discussion

In this section, we compare the six pairs of normalization procedures making precise in each case how far we can go with the two double tests, and which are the additional insights arising from the use of a third classification system – an idea that, as indicated in Section 1, we take from Waltman and Van Eck (2013b).

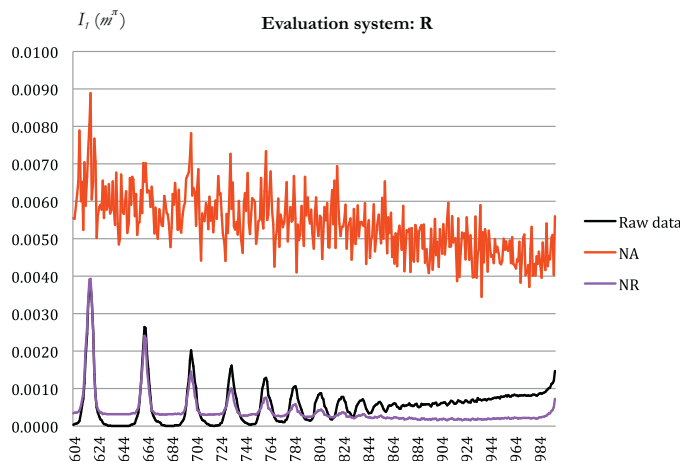


Fig. 6. The comparison of normalization procedures NA and NR using classification system R for evaluation purposes (quantile interval [604, 997]).

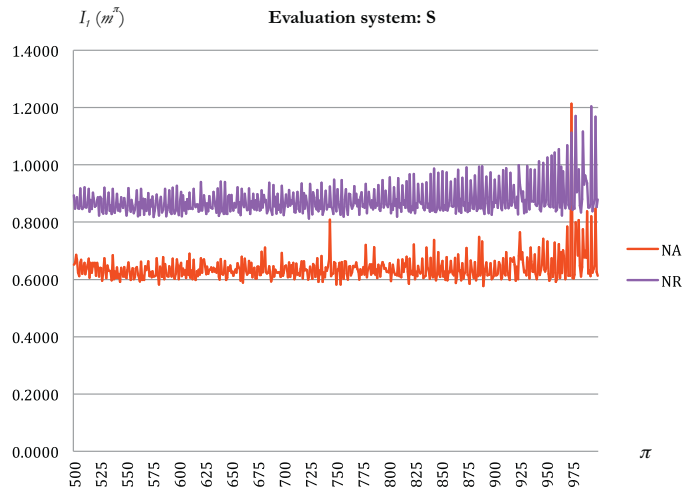


Fig. 7. The comparison of normalization procedures NA and NR using classification system S for evaluation purposes (quantile interval [500, 999]).

1. As we saw in Table 1, practically all citation inequality under system S is attributable to differences in citation practices. The other side of this coin is that, when highly cited sub-fields within S are normalized by high mean citations, differences in citation practices at every  $\pi$  are drastically reduced. As a matter of fact, the curve  $I_1^{NS}[\mathbf{m}^{\pi}(K)]$  as a function of  $\pi$  is always below  $I_1^{NL}[\mathbf{m}^{\pi}(K)]$  for all  $NL \neq NS$ , and all  $K=S, A, B$ , and  $R$  (see, for example, Figs. 1 and 3 for  $NL=NA$  and  $K=A, S$ ). In other words, procedure NS achieves the best possible results according to the first double test under the graphical approach:  $\{NSD_1(\mathbf{S}, L)NL\}$  for all  $L \neq S$ . On the other hand, NS performs also numerically better than the other procedures under system S itself. However, once the weighting system is taken into account and the evaluation is made in terms of any system  $L \neq S$ , we observe that the terms  $IDCP^{NS}(L)$  are very high (see column 1 in Appendix). In particular, this implies that NS is non-comparable with NA and NB according to the second double test, i.e.  $\{NLNon-D_{II}(\mathbf{S}, L)NS\}$  for  $L=A, B$ .

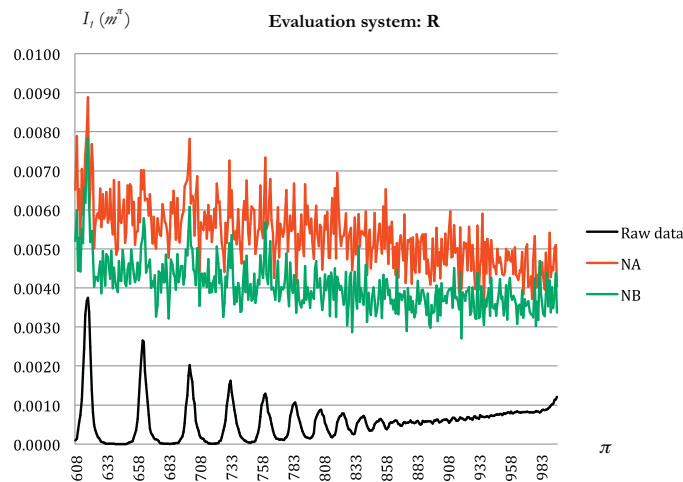
Thus, taking together the results of the two double tests, it would appear that the polar procedure NS exhibits a somewhat better performance than the two regular procedures NA and NB. In this rather worrisome situation, the evaluation of NS versus NA and NB in terms of an independent classification system becomes very relevant indeed. For the comparison with NA, for example, it is observed that  $\{NA>_{II}(\mathbf{R})NS\}$ . Exactly the same result is obtained for the comparison with NB (see column 4 in Table 2). Therefore, the weakness of NS relative to the regular procedures only manifests itself when the numerical evaluation uses an independent classification system.<sup>11</sup>

2. When articles are randomly assigned to sub-fields in system R, almost none of the citation inequality is attributable to differences in citation practices across sub-fields because the vast majority of citation inequality takes place within sub-fields. At the same time, since sub-field mean citations in R are very similar to each other, normalization by NR has practically no consequences when articles are organized according to the other systems. The implication is that the curve  $I_1[\mathbf{m}^{\pi}(K)]$  as a function of  $\pi$  for the raw data organized according to any system  $K \neq R$  is practically unaffected by normalization according to NR (see, for example, Fig. 5 for the case  $K=A$ ). Similarly, in the numerical approach normalization according to NR introduces almost no correction or even increases the  $IDCP^{NR}(K)$  term when  $K \neq R$  (see the last row in Table 2). Thus, in particular,  $\{NK>_I(\mathbf{K})NR\}$  and  $\{NK>_{II}(\mathbf{K})NR\}$  when  $K=A, B$ . However, the minimal impact of NR works in its favor when the evaluation is done using system R itself. Thus,  $\{NR>_I(\mathbf{R})NK\}$  for  $K=A, B$  in the graphical approach (see Fig. 6 for the case  $K=A$ ). Similarly,  $\{NR>_{II}(\mathbf{R})NK\}$  for all  $K=A, B$  in the numerical approach (see column 4 in Table 2). This leads to the conclusion that NR is non-comparable with NA and NB according to both double tests – a rather undesirable situation.

Interestingly enough, the weakness of NR relative to the regular procedures reveals itself when the evaluation is done in terms of an independent classification system. In the graphical approach, this is illustrated in Fig. 7 (for the interval [500, 999]) where NA and NR are compared using S for evaluation purposes (the case NB versus NR under S is available on request). Similarly, in the numerical approach we have  $\{NA>_{II}(\mathbf{S})NR\}$  and  $\{NB>_{II}(\mathbf{S})NR\}$  (see column 1 in Table 2).<sup>12</sup>

3. Next, we wish to compare the two polar normalization procedures NS and NR. For reasons explained in point 1 above, it is found that  $\{NSD_1(\mathbf{S}, \mathbf{R})NR\}$  under the graphical approach. However, NS is numerically non-comparable with NR according to the second double test, i.e.  $\{NSNon-D_{II}(\mathbf{S}, \mathbf{R})NR\}$ . This would lead us to indicate that procedure NS performs somewhat better than NR taking into account the two double tests. Again, the evaluation in terms of an independent classification system throws a definite light into this case. It is observed that  $\{NS>_{II}(\mathbf{G})NR\}$  for  $G=A$  or  $B$  (see columns 2

<sup>11</sup> Note that  $\{NA>_{II}(\mathbf{B})NS\}$  and  $\{NB>_{II}(\mathbf{A})NS\}$  (see columns 3 and 2 in Table 2). However, it could be argued that systems A and B are not entirely independent.  
<sup>12</sup> Note that  $\{NA>_{II}(\mathbf{B})NR\}$  and  $\{NB>_{II}(\mathbf{A})NR\}$  (see columns 3 and 2 in Table 2). However, as in note 9, it could be argued that systems A and B are not entirely independent.



**Fig. 8.** The comparison of normalization procedures *NA* and *NB* using classification system *R* for evaluation purposes (quantile interval [608, 996]).

and 3 in Table 2). Since a similar result is obtained in the graphical approach (results upon request), we conclude that the overall performance of *NS* is better than that of *NR*.

4. We now come to the more important comparison, namely, that of *NA* versus *NB*. Figs. 1 and 4 served to establish that  $\{NA_{Non-D_1}(B, A)NB\}$ , while the results in columns 2 and 3 in Table 2 indicate that  $\{NA_{D_{II}}(B, A)NB\}$ . Thus, in the nested case normalization at the lowest aggregate level has clear advantages according to both double tests. The possibility that using system *B* for evaluation purposes bias the results in favor of *NB* increases the value of the conclusion that *NA* is preferable to *NB*.

However, two further points should be noted. Firstly, *NB* performs numerically well not only under system *B* itself, but also when we evaluate it using system *A*: 60% of the effect on citation inequality of differences in citation practices at the sub-field level is eliminated via *NB* (see column 2 in Table 2). This is important because the availability of data often restricts us to a high aggregate level. The lesson is clear: whenever the only option is to normalize at a relatively high aggregate level, we should do it knowing that the reduction of the problem – even at the sub-field level – is non-negligible. Secondly, *NA* also performs better than *NB* in terms of *S*, i.e.  $\{NA_{>_{II}}(S)NB\}$  (see column 1 in Table 2), and  $\{NA_{>_I}(S)NB\}$  (results available on request). However, as illustrated in column 4 in Table 2 and in Fig. 8 (quantile interval [608, 996]), the opposite is the case when the evaluation is done in terms of *R*, i.e.  $\{NB_{>_{II}}(R)NA\}$ , and  $\{NB_{>_I}(R)NA\}$ . This serves as a warning that evaluations of normalization procedures using an independent classification system may lead us toward a conclusion that contradicts the results obtained under the two double tests and other independent evaluations.

## 6. Conclusions

In this paper, we have extended the methodology for the evaluation of classification-based-normalization procedures. For this purpose, we have used the measurement framework introduced in Crespo, Li, et al. (2013) where the effect of differences in citation practices across scientific disciplines is well captured by a between-group term in a certain partition of the dataset into disciplines and quantiles – the so-called *IDCP* term.

In the empirical part of the paper we use four classification systems: two nested systems that distinguish between 219 sub-fields, and 19 broad fields – the systems denoted *A* and *B* – as well as two polar cases in which articles are assigned to sub-fields in a systematic or a random manner – systems *S* and *R* – so as to make the differences in citation practices across sub-fields as large and as small as possible. We study four normalization procedures – denoted *NA*, *NB*, *NS*, and *NR* – that use field and sub-field mean citations as normalization factors in each case. The dataset consists of 4.4 million articles published in 1998–2003 with a five-year citation window.

We began by establishing that the *IDCP* framework is well suited for capturing the peculiarities of the two polar systems, as well as the two regular, nested systems. Then we discussed two ways of assessing any pair of normalization procedures in terms of a given classification system: a graphical and a numerical approach. Using a number of empirical examples, we established that neither of the two rankings is complete, and that they are logically independent. Next, the graphical and numerical approaches are extended to the evaluation of a pair of normalization procedures based on two different classification systems. In each case, we introduced a double test where any pair of normalization procedures is evaluated in terms of the two classification systems on which they depend. Using a number of empirical examples, we established that neither of the two new rankings is complete, and that they are independent.

The possibility that using a classification system for evaluation purposes bias the analysis in favor of the normalization procedure based in this system, makes very difficult to conclude that one classification-system-based normalization

procedure overcomes another according to the double tests. Nevertheless, this is what we obtain with the two nested classification systems *A* and *B*. Normalization at the lower aggregate level using *NA* weakly dominates normalization at the higher aggregate level using *NB* according to the first double test, a result reinforced by the dominance of *NA* over *NB* according to the second double test. Nevertheless, when the availability of data restricts us to normalize at a relatively high aggregate level, the good performance exhibited by *NB* indicates that one can still get good results using field mean citations as normalization factors.

Finally, inspiring ourselves in [Waltman and Van Eck \(2013b\)](#), we have studied the performance of any pair of normalization procedures based on different classification systems using a third, independent classification system for evaluation purposes. This strategy has proved useful to establish the weakness of the polar procedures *NS* and *NR* relative to the regular procedures *NA* and *NB*, and to establish the dominance of *NS* over *NR*. However, the dominance of *NB* over *NA* when system *R* is used for evaluation purposes illustrates the possibility that this strategy points to conclusions contradicting the results obtained with the two double tests and other independent evaluations.

Before we finish, it should be emphasized that the fact that *NA* is ranked above *NB* does not imply that classification system *A* is preferable to system *B*. Firstly, *NA* and *NB* use sub-field and field mean citations as normalization factors, but there are many other cited-side normalization procedures whose performance could be tested at different aggregate levels. Secondly, the choice of the best aggregation level is a separate problem from the comparison of normalization procedures that we have studied in this paper. To understand this point, consider the possibility of applying the percentile rank approach to systems *A* and *B*. This normalization procedure transforms every sub-field and field citation distribution into the uniform one. In this way, it completely eliminates the effect on citation inequality of differences in citation practices across sub-fields in *A* or across fields in *B*. In this sense, the percentile rank approach constitutes a “perfect normalization” scheme that drives the *IDCP* term toward zero (see [Li et al., 2013](#)). However, as indicated in [Zitt et al. \(2005\)](#) an outstanding article in a certain sub-field may get only a modest score within a larger field if the rest of this field has more generous referencing practices. Therefore, the ranking of this article after the (perfect) normalization according to the percentile rank approach will be very different depending on which aggregation level is chosen for normalization. As these authors conclude, “*The fact that citation indicators are not stable from a cross-scale perspective is a serious worry for bibliometric benchmarking. What can appear technically as a ‘lack of robustness’ raises deeper questions about the legitimacy of particular scales of observation.*” ([Zitt et al., 2005](#), p. 392; on this issue, see also [Waltman & Van Eck, 2013b](#)).

We would like to add that, once the question of the best aggregate level is somewhat settled, it will be interesting to compare less than perfect classification-based-systems normalization procedures at this and higher aggregate levels using the methods developed in this paper. Nevertheless, we should conclude recognizing that more empirical work is needed before these methods become well established. In particular, it will be illuminating to use other classification systems with interesting properties of their own apart from the two polar cases studied here.

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**Appendix**

*IDCP<sup>NK</sup>* term and overall citation inequality  $I_1(C^{K*})$  after normalization by the different procedures

Normalization procedures	Classification system used for evaluation purposes:							
	S		A		B		R	
	<i>IDCP<sup>NK</sup></i> (S) (1)	$I_1(C^{K*})$ (2)	<i>IDCP<sup>NK</sup></i> (A) (3)	$I_1(C^{K*})$ (4)	<i>IDCP<sup>NK</sup></i> (B) (5)	$I_1(C^{K*})$ (6)	<i>IDCP<sup>NK</sup></i> (A) (7)	$I_1(C^{K*})$ (8)
<b>NS</b>	0.2167	0.2173	0.1032	0.2173	0.0798	0.2173	0.0097	0.2173
<b>NA</b>	0.6385	0.7531	0.0260	0.7531	0.0122	0.7531	0.0076	0.7531
<b>NB</b>	0.6996	0.7876	0.0620	0.7876	0.0131	0.7876	0.0062	0.7876
<b>NR</b>	0.8628	0.8633	0.1678	0.8633	0.1143	0.8633	0.0021	0.8633

*IDCP<sup>NK</sup>*(G)=citation inequality attributable to differences in citation practices across the disciplines in classification system *G* after normalization by procedure *NK*, where *K, G*=S, A, B, and R.  $I_1(C^{K*})$ =citation inequality in the all-sciences case after normalization by procedure *NK*, where *K*=S, A, B, and R.

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