

## STOCHASTIC MODELS OF INFORMATION OBSOLESCENCE

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**Abstract:** The stochastic modelling of information obsolescence is reviewed from a systematic and historical perspective. Burrell's model of the use history of the individual data item is tested statistically against the citation histories of some physics articles. Alternative modelling approaches and their implications for the design of full-text electronic databases are considered.

**Keywords:** Stochastic modelling; Obsolescence; Reliability theory.

### INTRODUCTION

The obsolescence of printed information has been inferred from the gradual decline in use frequency experienced by large collections of contemporaneous data items. In Burrell's (1985) words, "It seems to be widely accepted that any given body of library material suffers a general decline in usage with the passage of time." Where journal articles are concerned, use has been construed as citation in bibliometric studies. For books in lending libraries, circulation records are used to measure usage. If the question is how many uses a collection will experience next year, the answer is, "Probably fewer than this year"--qualified by allowance for a gestation period of a few years, especially in the case of journal citations, during which the annual number of uses will tend to rise. A recurring theme in the literature is the stronger and more explicit claim that the expected number of uses declines from year to year by the same factor. This constant obsolescence rate (COR) hypothesis will be examined in the next section.

Information obsolescence has both philosophical and practical significance. While the term has been used somewhat loosely, we prefer to say that a data item is obsolescent if and only if the number of uses it will experience in the course of time is always attained in finite time, despite its continued availability. Then the obsolescent data item, like most machines, possesses a finite useful lifespan--its age at the time of its ultimate use--beyond which the physical remnant persists with at best a sentimental value. Its useful life expired, the remnant can be discarded without practical loss to the user community. Thus, if there were a way to identify and discard obsolete items (without eliminating their surviving neighbors), a librarian or database manager could not only compress the vital information into a smaller, more economical space, but also save the user time and trouble in searches that return

increasingly long lists of gratuitous citations. Bound journals and microfilms do not lend themselves to such decomposition. Yet it seems reasonable to suppose that full-text electronic databases for scientific research will emerge in the next decades. The thousands of new papers being authored each week must demand a systematic treatment which is compatible with the emerging technologies. Verifiable stochastic models of individual item use histories could figure prominently in such a treatment.

### COR MODELS

The COR hypothesis can be regarded as a product of the "Atomic Age." Burton and Kebler (1960) recalled that the term "half-life" was much in evidence at the International Conference on Scientific Information in Washington in 1958. If  $N$  atoms of a radioisotope are present at time zero, the number remaining at time  $t$  is  $N \exp(-rt)$  when  $r$  is the decay rate. The radioactive fraction is diminished by a factor of one-half each  $\ln(2)/r = \tau$  units of time; and  $\tau$  is called the half-life. Similarly, the papers published in a given science in year zero give rise to citations in the  $t$ -th post-publication year which are about  $2^{-t/\tau}$  times as numerous as in the preceding year when the half-life of the literature in question is  $\tau$ . Burton and Kebler computed half-lives of 4.6 and 10.5 years for physics and mathematics literatures, respectively.

The COR hypothesis was canonized by Brookes (1970), who attributed an exponential distribution to the post-gestation ages of science citations of the same vintage. If  $n(t)$  is the number of articles, aged  $t$  years or more, in a sufficiently large sample of current papers, then

$$\ln n(t)/n(t_0) \approx -(t - t_0)r, \quad t \geq t_0 \geq 0, \quad (1)$$

is "Brookes' Law", in which the empirical constant  $r$  is the apparent obsolescence rate.

The minimum age  $t_0$  is the gestation time. Interpreting the age as a random variable,  $T$ , so that

$$[n(t_0) - n(t)]/n(t_0) = \Pr(T \leq t - t_0),$$

and writing  $F(y) = \Pr(T \leq y)$  for the distribution function (d.f.) of the post-gestation age ( $y = t - t_0$ ), equation (1) is

$$\ln[1 - F(y)] = -ry.$$

Thus

$$F(y) = 1 - \exp(-ry), \quad y = 0, 1, 2, \dots, \quad (2)$$

is the geometric probability that a randomly selected citation is aged  $y$  or fewer years post-gestation. Brookes and others have preferred to discuss the problem in terms of continuous time. Following this convention, equation (2) with  $y \geq 0$  defines an exponential d.f. and the obsolescence rate,  $r$ , is the reciprocal mean post-gestation age of citations. Griffith et al. (1979) tabulated five million entries from the 1974-75 **Science Citation Index** and interpreted the results in light of the Brookes model. They found an apparent obsolescence rate of about 11% per year back to 1950, but only 5% per year for older articles. The nonconstancy of the slope ( $-r$ ) in equation (1) would seem to invite the development of a more general hazard rate model which is offered in the next section.

The COR model has also been employed in the study of aging among books in lending libraries, in which case use is construed as borrowing. P.M. Morse (1968) introduced a simplistic Markovian model of book use in a science library which, after elaboration by Morse and Elston (1969), holds that the number of circulations experienced by a collection in the  $t$ -th year, denoted  $n(t)$ , is Poisson with a mean of  $a + bn(t-1)$ . Parameter  $a \geq 0$  is the "asymptotic circulation" and parameter  $b < 1$  describes the obsolescence phenomenon. (If the books in question are obsolescent, according to our definition above, the asymptotic circulation must be zero.) Beheshti and Tague (1984) studied the circulation of 56,000 monographs in the University of Saskatchewan's library system from 1968 to 1978 in light of this model. In apparent contradiction of the originators' intent, they found the  $a$ -parameter decreasing with time and the  $b$ -parameter "fluctuating randomly."

#### THE HAZARD RATE MODEL

The gradual decline in use frequency experienced by large collections of data items can be explained in at least two ways on the "microscopic" level of the individual item. The first is by strict analogy with radioactive decay, in which  $N$  items (like atoms) undergo abrupt and irreversible transition from an active state to an inactive state at ages  $(X_1, \dots, X_N)$  which are independent and identically distributed with density  $p(x)$ . In the limit of large  $N$ , the fraction still active at time  $t$  is

$$N(t)/N(0) = \int_t^\infty p(x) dx. \quad (3)$$

The instantaneous rate of transition at time  $t$

is the hazard rate,

$$h_1(t) = -d \left( \ln \int_t^\infty p(x) dx \right) / dt. \quad (4)$$

If the population of items is replenished by the introduction of  $\lambda$  new items per unit time, all of which are active at the time of introduction, the active number will approach a limiting value of  $\lambda \bar{X}$ , with  $\bar{X} = EX$  (the life expectancy at introduction), which is known to queueing theorists as Little's result. A random sample of the active population in steady state now elicits a time-invariant age distribution, the d.f. being  $F(y)$ , with  $f(y)$  the corresponding density, which is related to  $p(x)$  by Feller's (1971) law of self-renewing aggregates,

$$f(y) = (1/\bar{X}) \int_y^\infty p(x) dx. \quad (5)$$

The apparent obsolescence rate, which is obtained as

$$r(t) = -d [\ln f(t)] / dt, \quad (6)$$

is the same as  $h_1(t)$  defined in (4) when equation (5) pertains. Comparing (6) to (2) it is evident that the COR hypothesis  $h_1(t) = r$  is a special case.

Thus the gradual decline in usage of the entire collection is seen as the macroscopic effect of a large number of sharp transitions by individual items; and the metaphor of radioactive decay leads to a hazard rate model in which a database can, at any time, be divided into two parts: an "information base" of active (i.e., useful) items and a remainder portion which, if it could be separated, could be discarded without loss to the user community. In the same spirit, Bostic (1985) asserts that two parts of a library's serials collection "can be identified with confidence." These are the "core collection", which will satisfy 95 to 99 per cent of demands, and a "weedable part" which accounts for the balance of the total uses.

Line (1970) discerned a weakness of the empirical frequency-of-citation approach to quantifying obsolescence. The number of papers published in a given science grows from year to year. A citation sample at present time gives rise to an age distribution which is influenced by this growth as well as by the presumed obsolescence of the literature. If the cumulative number of published papers has been increasing exponentially for a long time, and one draws an age  $Y$  from this population, the distribution of  $Y$  will be exponential under the assumption that all papers are cited equiprobably, even with  $h_1(t) = 0$ . Moreover, the empirical constant  $r$  will in this case be the same as the rate constant describing the exponential growth of the population. Brookes (1980) was still "at a loss to understand what a rate of aging corrected for growth really measures." We shall return to this problem later.

An alternative view of collective obsolescence is to suppose that each individual data item undergoes a gradual, even graceful passage from greater to lesser utility as its age

advances. In this view, the actual use history of an obsolescent data item represents a stochastic realization of some decreasing parameter sequence (or "trajectory").

THE BURRELL MODEL

A model which exemplifies this alternative has been proposed by Burrell (1985), whose fundamental assumptions are (i) that the times at which a particular item is used constitute a Poisson process of rate  $m$ , (ii) that  $m(t)$  for any particular item is an exponentially decreasing function of time, as

$$m(t; A, B) = A \exp(-Bt),$$

and (iii) that the parameter pair  $(A, B) = \Theta$  varies from item to item and may itself be regarded as a random variable. Under these assumptions, the individual item experiences  $K(t)$  uses, up to time  $t$  post-publication, where

$$\Pr[K(t) = n | \Theta = (a, b)] = \exp[-M(t; a, b)] [M(t; a, b)]^n / n!$$

and

$$M(t; a, b) = \int_0^t m(s; a, b) ds = (a/b) [1 - \exp(-Bt)].$$

The total lifetime uses are POISSON( $a/b$ ) in number; and the number of uses in an interval  $[t, t+s]$  is Poisson with parameter

$$M(t+s; a, b) - M(t; a, b) = (a/b) \exp(-bt) [1 - \exp(-bs)].$$

If time is divided into regular intervals of length  $s=1$ , indexed by  $j=0, 1, 2, \dots$ , the uses in the  $j$ -th interval, denoted  $U_j$ , are Poisson with parameter  $q_0 q^j$ , where

$$q_0 = (a/b) [1 - \exp(-b)]$$

and

$$q = \exp(-b)$$

define the pair  $(q_0, q) = \underline{\psi}$ . Hence

$$\Pr[U_j = n | \underline{\psi} = (q_0, q)] = \exp(-q_0 q^j) q_0^n q^{nj} / n! \quad (7)$$

Thus the numbers of uses in successive years are conditionally independent Poisson random variables whose means decline by the same annual factor  $q$  from an initial use rate of  $q_0$  per year. The total lifetime uses are now Poisson  $q_0 / (1-q)$  as can readily be proved by summing the logarithms of the generating functions over all  $j$ .

Given a use history  $Z = (Z_0, \dots, Z_{J-1})$ , in which  $Z_j$  is the observed number of uses of the item in the  $j$ -th year, the maximum likelihood estimators (MLEs) of the Burrell model parameters are

$$\hat{q}_0 = (\sum Z_j) / (\sum \hat{q}^j) \text{ and } \hat{q} = (\sum_j j Z_j) / (\sum Z_j) = (\sum_j j \hat{q}^j) / (\sum \hat{q}^j),$$

in which the sums range across the  $J$  years in question. Given the MLEs  $\hat{\underline{\psi}} = (\hat{q}_0, \hat{q})$ , the predicted lifetime uses are

$$\hat{M} = \hat{q}_0 / (1 - \hat{q})$$

in number when  $j=0$  truly marks the date of origin. The "lack of memory" property of the geometric sequence permits computation of the MLEs when the item was truly  $y$  years old at  $j=0$ . For in this case one merely solves the last equations and invokes the transformation  $q_0 \leftarrow q_0 q^{-y}$ .

In order to test a model such as (7), one computes the posterior likelihood of the observations, with the MLEs inserted in the equations, and compares the result to the expected value of the likelihood under the same assumptions. A BASIC program was written for the IBM PC-AT to

- (i) compute  $\hat{\underline{\psi}}(Z)$  and  $\hat{M}$ ;
- (ii) calculate the posterior log-likelihood of  $Z$ , denoted  $L(Z | \hat{\underline{\psi}})$ ;
- (iii) generate simulated use histories  $\{\tilde{Z}\}$  based on the assumption of  $\hat{\underline{\psi}}$  (a Monte Carlo method); and
- (iv) calculate the log-likelihoods  $L(\tilde{Z} | \hat{\underline{\psi}})$ , the mean ( $\bar{L}$ ), and the standard deviation ( $\sigma$ ).

With reference to (7), the log-likelihood is explicitly

$$L(Z | \underline{\psi}) = \sum_{j=0}^{J-1} \ln \Pr(U_j = Z_j | \underline{\psi} = (q_0, q)).$$

Regarding the generation of Poisson random variables in step (iii), recall that the BINOMIAL( $N, \xi/N$ ) random variable is asymptotically POISSON( $\xi$ ) for large  $N$ ; and binomial trials of size  $N$  are readily performed with computer-generated uniform (unit) pseudorandom numbers.

DATA ACQUISITION AND ANALYSIS

We sought to test the Burrell model using the citation histories of selected scientific papers as recorded in the **Science Citation Index** (1975-84). The 1 December 1973 issue of **Physical Review B** was the source of the sample of size 63. Five of the articles therein received no citations through 1984; six received only one and two were cited twice. The ten year citation histories of the remaining 50 articles are listed (by first author) in Table 1. The MLEs of  $q_0$  and  $q$  are shown in columns 12 and 13. In six cases,  $\hat{q}$  was found to equal or exceed 1.0 in contradiction of the obsolescence hypothesis on which the Burrell model is based. The predicted lifetime uses ( $\hat{M}$ ) are listed in column 14 in the 44 obsolescent cases. In computing the MLEs and the lifetime uses, the assumed first post-gestation year is indicated by the bar below the citation count. The use history to the left of the year in question is disregarded in the estimation. The computed numbers pertain to the post-gestation years only. Figure 1 shows four of the citation histories and the fitted trajectories.

The right-most columns of the table show the

TABLE 1.

Citation histories and trajectory estimates for the papers appearing in the **Physical Review B** of 1 December 1973.

First Author	1975	76	77	78	79	80	81	82	83	84	$\hat{q}_0$	$\hat{q}$	$\hat{M}$	-L	$-\bar{L}$	$\sigma$
Andersson	2	3	0	0	3	2	0	1	0	1	1.8	.87	14	14.7	12.4	3.7
Barak	2	2	2	3	1	2	2	2	0	1	2.5	.91	13	13.4	15.6	4.3
Bartel	1	1	3	3	0	1	0	0	1	0	3.1	.63	8	6.6	9.1	3.2
Berlinsky	1	0	0	2	2	1	0	1	0	2	1.6	.88	13	9.4	8.9	2.5
Bhide	2	3	0	1	4	2	0	1	3	1	1.9	.98	55	12.4	14.0	4.1
Brezin	4	6	4	2	1	1	1	1	1	0	5.4	.69	18	11.8	13.2	4.0
Buschow	2	3	6	2	3	2	1	0	0	0	6.4	.54	14	9.6	10.4	3.1
Butler	0	0	3	1	0	0	0	0	0	0	3.2	.20	4	1.8	3.7	2.6
Caudron	3	4	4	1	0	1	0	2	1	1	3.2	.80	16	11.1	13.1	4.4
Chandler	1	1	0	0	0	0	0	1	0	0	0.7	.78	3	4.9	6.5	2.4
Deutscher	2	2	0	2	0	0	2	1	0	0	1.9	.82	10	11.4	11.5	3.5
Drake	0	0	1	2	1	0	0	0	0	0	2.0	.51	4	4.6	5.6	2.6
Ebner	2	2	1	1	0	0	0	0	0	0	2.8	.54	6	6.1	7.5	3.7
Elias	0	4	3	5	1	3	2	1	0	2	4.3	.83	26	14.8	14.9	4.4
Fedders	3	1	2	0	2	0	0	0	3	1	1.7	.92	21	14.5	14.1	4.2
Ferer	4	4	4	3	4	4	1	3	4	3	4.2	.96	97	13.8	19.9	5.5
Fiory	2	0	2	2	0	1	2	2	2	2		$\geq 1$				
Flaherty	1	1	0	1	1	1	1	1	0	2		$\geq 1$				
Garland	0	0	4	3	4	1	5	1	0	1	4.5	.80	23	13.3	14.3	4.0
Hardy	1	2	1	2	1	0	0	1	3	1	1.3	.98	69	11.9	12.7	3.6
Huck	1	0	1	3	2	1	0	0	0	0	1.6	.82	9	11.1	11.5	3.4
Jullien	0	7	1	2	3	1	0	1	0	0	5.6	.63	15	11.1	10.8	3.9
Kasuya	2	1	0	0	0	0	0	0	1	0	1.1	.73	4	7.0	7.2	3.0
Klemm	5	1	2	1	1	1	0	0	0	0	3.8	.57	9	7.8	8.9	3.2
Koidl	6	6	5	5	4	1	0	1	0	0	8.8	.68	28	14.2	15.4	4.2
Koon	2	0	0	1	0	1	0	1	0	0	1.0	.82	6	8.4	8.9	3.4
Kornblit	5	9	5	6	1	5	2	1	1	1	7.2	.85	46	19.2	18.2	4.9
Leonardi	3	1	1	0	0	0	0	1	0	0	2.5	.57	6	6.6	7.8	3.0
Lick	0	0	1	1	0	1	1	0	0	0	1.2	.73	4	5.9	7.0	2.3
Lo	3	2	1	0	0	0	0	0	0	1	2.3	.67	7	8.7	8.3	2.7
Migliori	2	2	1	1	0	0	1	0	0	0	2.5	.65	7	7.1	8.8	3.9
Misra	0	1	0	0	0	0	0	1	0	1		$\geq 1$				
Rado	0	1	0	0	1	0	0	0	0	1	0.4	.95	8	6.2	6.4	3.0
Rainer	1	2	3	1	3	3	3	2	4	2		$\geq 1$				
Rath	10	9	7	4	3	13	7	5	3	1	9.9	.89	89	24.7	22.5	6.2
Ray	5	2	3	3	1	2	3	1	0	1	4.2	.83	25	14.4	16.5	4.9
Reiter	1	0	0	0	0	1	1	0	0	0	0.5	.90	5	6.5	6.0	3.0
Rettori	3	1	0	2	1	2	2	0	0	0	2.3	.82	13	12.3	12.4	4.0
Riedl	3	1	4	3	1	2	4	1	7	1		$\geq 1$				
Szczurek	0	4	1	2	1	0	0	0	1	0	3.2	.65	9	8.8	9.6	3.1
Tedrow	3	2	3	3	3	4	1	5	4	2		$\geq 1$				
Vincent	1	1	1	1	0	0	0	0	0	0	1.6	.61	4	5.1	6.5	3.6
Tanner	2	3	0	0	0	0	0	2	0	0	1.8	.75	7	10.7	10.5	3.7
Wang	3	2	1	0	0	0	0	0	0	0	3.6	.40	6	4.4	5.7	2.8
Weber(p.5093)	10	9	10	5	7	2	3	1	1	0	12.9	.75	51	17.8	18.9	5.3
Weber(p.5082)	13	16	15	8	13	8	7	4	3	6	16.8	.86	121	23.1	23.9	6.3
Wender	0	0	2	3	0	1	0	0	1	0	2.4	.67	7	8.1	8.7	3.5
Wurfel	1	1	1	0	0	2	1	0	0	0	1.2	.84	7	9.1	9.6	3.0
Yamaguchi	5	1	1	1	2	2	2	1	0	0	3.3	.80	17	13.2	13.6	3.7
Zitkova-Wilcox	1	2	1	0	0	2	2	1	1	0	1.3	.93	19	10.9	11.6	3.5

Note: The thirteen papers which received two or fewer citations during the years in question are not included in the table.

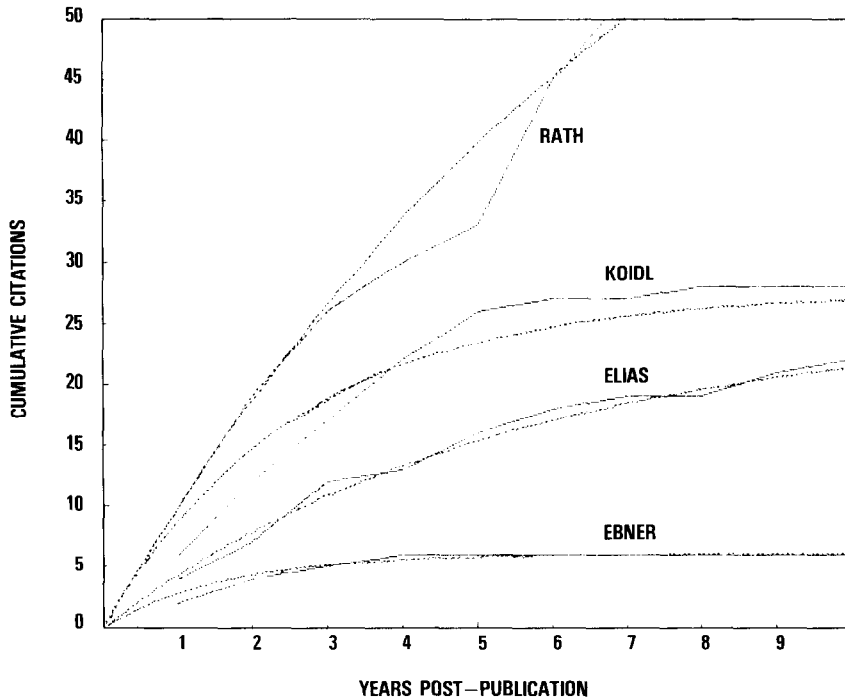


FIG. 1. CITATION HISTORIES (SOLID) AND FITTED TRAJECTORIES (DOTTED) OF SOME PHYS. REV. ARTICLES.

posterior likelihood of the observations ( $L$  times  $-1$ ) and the mean and standard deviation found in 16 Monte Carlo runs per case, with binomial trials of size 64 being used to generate the Poisson random variables. In general we find  $L > \bar{L}$ ; and  $L \geq \bar{L} - \sigma$  in every instance. These results provide strong support for the Burrell model in the 44 obsolescent cases.

#### OTHER INHOMOGENEOUS POISSON PROCESS MODELS

Having attributed the COR hypothesis to a nuclear analogy, and having found it expressed on the level of the individual item in the exponential trajectory of the Poisson process parameter in the Burrell model, we seek alternative analogies in the hope that they can better account for nonconfirming observations. As Burrell has noted, the inhomogeneous Poisson process seems quite flexible as a modelling tool; and a substantially better fit might be obtained by a more realistic (and doubtlessly more complicated) parametric formula for the use trajectory, i.e., the Poisson process parameter as a function of time ( $t$ ) or the index ( $j$ ).

Consider, for example, an epidemic analogy which asserts that the propagation of ideas is like the propagations of disease. Some papers present "contagious" ideas and give rise to subsequent papers which cite the originator. An entire community of specialists may become "infected" as the citation count increases year by year. Eventually an "immunity" is built up as the idea undergoes a metamorphosis from "revolutionary concept" to "textbook knowledge." Of all the models employed by epidemiologists, the logistic model is perhaps the simplest. The differential equation

$$d\pi(t)/dt = B\pi(1-\pi), \quad \pi(0) = \pi_0$$

for  $\pi(t)$ , the infected fraction of the population at time  $t$ , is solved by

$$\pi(t) = 1/[1 + (1/\pi_0 - 1)\exp(-Bt)].$$

If the population consists of  $A$  individuals, the infected number is  $A\pi(t)$ . Now the appearance of a paper which cites the work in question is regarded as the "symptom" of contagion; so the rate at which new citations appear is

$$\begin{aligned} m(t) &= d[A\pi(t)]/dt \\ &= AB\pi(t)[1 - \pi(t)] \end{aligned}$$

when  $A$  is constant. Figure 2 suggests a least squares fit of the last formula, with the parameters  $(A, B, \pi_0) = (230, 0.39, 0.02)$ , to the citation history of a survey paper. The figure indicates a fit which, though superficially poor, must surely capture the trend more faithfully than any COR trajectory. Note that the logistic trajectory is asymptotically COR.

Trajectory development may be motivated by metaphor or it may rely on some inductive insight which arrives at a parametric family which conforms to the observations. If  $M(t, \Theta)$  is the accumulated trajectory, the obsolescence assumption requires that it have the form  $MG(t, \Theta)$  when the parameters are  $\Theta$ , where  $M$  is the mean number of lifetime uses and  $G(t, \Theta)$  is a nondecreasing function of  $t$  which rises from zero to one as  $t$  goes from zero to infinity. I.e.,  $G(t, \Theta)$  is always equivalent to the d.f. of a nonnegative random variable; and the trajectory itself can be written  $Mg(t, \Theta)$  in terms of  $g = dG/dt$ , the corresponding density. Any elementary text which lists some non-

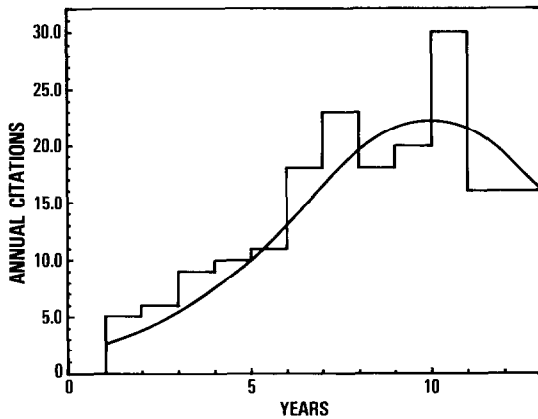


FIG. 2. Logistic fit to the citation history of R.G. Miller (1974). The jackknife - a review. *Biometrika* 61(1), 1-15.

negative random variables and their distributions provides a source of candidate trajectories which might be evaluated in the manner of the preceding section.

RECONCILIATION OF THE MICROSCOPIC PERSPECTIVES

The lifespan  $X$  of an obsolescent item was distributed with density  $p(x)$  in the context of the hazard rate model. Since  $X = x$  iff  $U_x > 0$  but  $U_j = 0$  for each  $j$  thereafter, clearly the discrete density  $p(x)$  is

$$\Pr(X=x) = \Pr(U_x > 0) \prod_{j=x+1}^{\infty} \Pr(U_j = 0) .$$

Substituting the Poisson density, expressed in terms of trajectory  $Mg(t)$ , in which the parameters are suppressed, this is

$$p(x) = [1 - \exp(-Mg(x))] \prod_{j=x+1}^{\infty} \exp(-Mg(j)) \\ = [1 - \exp(-Mg(x))] \exp[-M(1 - G(x))] ,$$

$x = 0, 1, 2, \dots$  In this way the lifespan distribution is determined by the trajectory.

Yet the mathematical relation between lifespan and trajectory fails to fully reconcile the views of Line and Brookes on the question of data interpretation. Rejecting "asynchronous" studies of citation frequency versus age in favor of "diachronous" studies of contemporaneous items, Line implicitly posits the use history as the proper subject of analysis. Our equation (3), in this view, is an artificial assumption. Let that equation be reinterpreted so that  $N(t)$  is the number of latent uses belonging to a collection of items originating at  $t=0$ . This number is  $M - M(t)$  times the collection size, the overbars indicating an average over all individual trajectories. In the limit of large size we have

$$(1/N(0))dN(t)/dt = 1 - \bar{M}(t)/\bar{M} \\ = 1 - \bar{G}(t)$$

instead of (3) whence the generalized trajectory based hazard rate,

$$h_2(t) = -d \log N(t)/dt = \bar{g}(t) [1 - \bar{G}(t)]^{-1} ,$$

has the same form as  $h_1(t)$  after substituting the composite trajectory  $\bar{g}$  for the lifespan density  $p$ .

Moreover, if the arrival rate of new items is nonconstant, there is no need to redefine the trajectory-based hazard rate. If obsolescence is factual, the trajectory will vanish despite literature growth and the influx of new researchers into a field. Though the latent uses of an item may be just as conjectural and difficult to predict as its lifespan (on the basis of "right-censored data"), there is no basis for estimating the latter without adopting a trajectory model, which serves to predict the former.

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