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NONLINEAR DYNAMICS AND THE GROWTH OF LITERATURE

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Abstract—Progress over the last 25 years in physics has demonstrated that simple natural systems under certain conditions can exhibit very complicated behavior. Based on earlier studies of turbulence and fluid dynamics, chaotic behavior has been shown to exist in weather patterns, electric circuits, chemical reactions, physiological systems, and ecological systems. The purpose of this paper is to demonstrate that there is merit to asking and seriously examining whether information production and dissemination can be described by similar nonlinear dynamic mechanisms. First, with an example from the superconductivity literature, it is demonstrated that the argument of linear versus exponential growth in literature may be irrelevant in the short term. Second, the argument is advanced that if chaos can be found in naturally varying populations, and certain principles of population dynamics also apply to literature dynamics, then it may also be possible to find chaos in the growth of literature.

1. INTRODUCTION

One of the most exciting areas of scientific progress in the last 25 years has been in the field of nonlinear dynamics (Gleick, 1987). Also known as "deterministic chaos," it describes complicated behavior arising from simple causes. As such, it should not be confounded with the classical notions of chaos as utter disorder. First described in models of weather phenomena and fluid dynamics, deterministic chaos has been shown to occur in mechanical systems, chemical reactions, the physiology of heart cells, ecological systems, and childhood diseases (Holden, 1986). The work on ecological dynamics is particularly exciting because it demonstrates that certain behaviors and interactions of animal populations in nature can best be described by nonlinear dynamical mechanisms. Thus, recent work brings to mind the question whether the processes of information production and the growth of a body of literatures could not be described by similar dynamical mechanisms. One wonders whether what applies to natural populations could not also apply to populations of literatures. Does growth occur haphazardly (randomly) and by external influences alone or is there a deeper internal mechanism at work?

The purpose of this paper is to draw basic parallels between the growth of literatures and nonlinear dynamical mechanisms, and to illustrate its thesis with an example from the recent superconductivity literature. This examination also begs the philosophical question whether human actions can be reduced to a set of deterministic criteria, but that is well outside the scope of this paper.

2. GROWTH OF LITERATURE

2.1 *Linear versus exponential growth*

One of the central questions of information science has been the growth of literature and the consequent spread of information. What are the mechanisms by which the literature of a field grows? How is the information communicated? How does useful information in a field spread? Are there ways to model the growth, understand the basic mechanisms, and ultimately make predictions?

So far two alternatives have dominated the literature. Exponential growth, first put forward by Price (1961) in "Science since Babylon" was accepted and expanded by May

(1966) and Line (1970). Those who disagreed, chiefly Oliver (1971) and King *et al.* (1981), argued that literature grows linearly. And some, such as Bottle and Rees (1979), with liquid crystal literature, found a zig-zag pattern where growth, when it did resume, took on exponential characteristics.

The classical notion of growth, influenced chiefly by experiments on bacterial growth in a controlled medium, is that of the logistic (S) curve. Slow growth speeds up, reaches an exponential (almost explosive) stage, and then due to some external factor such as lack of food or space, reaches a saturation level. Unless the system is supplemented to enable growth to continue, saturation leads to exhaustion and eventual death.

One of the problems with observing growth in a literature population (literally, a group of published papers) has been that at the time of observation it is not possible to know what mechanism is at play and exactly which of the two above-mentioned patterns are being followed. For example, Price was criticized on those grounds by May (1966), who demonstrated that had he started his counts of growth earlier than 1900 for Physics, 1920 for Biology, or 1940 for Mathematics, he would have found a more linear pattern, approximating 2.5% per annum compounded. Thus, it seems, one can really never be too sure what kind of growth is taking place until the distant future when the growth pattern has emerged clearly and one is able to look back with perspective. It seems one is left with the historical outlook. However, the thesis of this paper is that this does not matter, and that neither linear nor exponential growth may be the valid behavior at a given time.

2.2 *The time factor*

In trying to represent the dynamic growth of a system, the time factor must be the central variable. There are comparatively few studies in information science in which the time element is taken as the key variable. One of the best known is the law of exponential growth $P(t) = ke^{at}$, where $P(t)$ is the total number of publications at time t, k is the initial size at time zero, and a is the growth rate (Tague *et al.*, 1981). However, the equation is a static one because it assumes a constant rate of growth, and over time the number of publications seems to grow to unrealistically high numbers.

Another well known study is Goffman and Newill's (1967) epidemic model for the spread of scientific ideas. In a set of nonlinear differential equations, Goffman developed a model of diffusion in terms of susceptibles, infectives, and removals, analogous to authors who may publish in the future, those currently publishing in a field, and those who published in the past. His epidemic curve was a description of the number of infectives with respect to time, with the purpose of predicting when an epidemic will occur in a given literature and when it will peak. Remarkable for its quality and prescience, the model suffered from difficulties in applications and lack of satisfactory parameters for the constants required in the equations, and the work was not followed up.

The notion of "research fronts" was introduced by Derek DeSolla Price (1965) to describe the dynamic aspects of literature growth. However, time was not mentioned distinctly. Griffith *et al.* (1974) elaborated on it by demonstrating the existence of maps of scientific literatures in terms of literatures central to a subject area and subject clusters with papers in the periphery. Instructive as they were, the maps were developed by static techniques, with no causative explanations and no place for a time factor. Griffith and Mullins' (1972) "coherent groups" suffer from the same lack.

The idea of representing the communication process as an ecological phenomenon was introduced by Blackburn (1973), who compared "the ecology of scholars" to the behavior of biological ecosystems. He likened properties exhibited by ecological systems to dissipative structures in thermodynamics, but did not go further by examining growth with respect to time.

Recently, growth processes and the importance of time have received attention from Haitun (1982), Sichel (1986), Kunz (1987), Trofimenko (1987), Burrell (1988), and Bruckner *et al.* (1990). The majority of these studies have involved probabilistic distributions whereby growth occurs by chance phenomena, completely randomly, and, with the exception of Burrell, the time element is considered to be linear. The applications of Bruckner's simulation models to growth trends in subfields remain to be shown.

In the approaches given above, the models can be seen as reflections of an "equilibrium process," and not the dynamic, moving, and growing world of publications. They represent static, closed-system models. They do not accord with the dynamic world of information production. The primary message of this paper is that the growth of knowledge is a dynamical process, and that time-dependent phenomena should be explained in terms of time-dependent models. The new theory of nonlinear dynamics may be able to offer some explanations for the causes and mechanisms of literature growth and apparent information explosion.

3. POPULATION STUDIES

Recent work on nonlinear dynamics threatens to change a number of widely held opinions concerning linearity and population equilibrium in ecological models, ideas of competition, and population genetics. Most importantly, it states that population fluctuations are not solely based on external factors but that there are internal mechanisms at work (Schaffer, 1985; Olsen & Schaffer, 1990).

Some of the early work on population dynamics involved the Lotka-Volterra equations for predator versus prey species (Lotka, 1956). Gilpin (1979) was the first to show that the equations under certain constraints can exhibit period doubling and give rise to chaos.

The complication with natural populations is in the problem of environmental noise in the data. Much of the challenge lies in trying to obtain "clean" data that shows correlations between selected factors rather than a dominance of background noise. This means reducing measurements in a system with many variables down to a single significant, all-encompassing variable. Packard *et al.* (1980) have shown that the behavior of a multidimensional system can be exhibited through a single variable over time. Takens (1980) further showed that multidimensional data in a time series can be reconstructed by choosing a set of appropriate time steps.

Schaffer and Kot (1985) applied these techniques to study cases of measles, mumps, and chickenpox in New York and Baltimore. They reconstructed an attractor that in the case of measles shows a chaotic structure. The case for mumps and chickenpox was inconclusive. Olsen and Degn (1985) found the same for measles in Copenhagen. A significant aspect of Schaffer's work is that it demonstrates the possibility of simulating seasonal variations in the outbreaks of epidemics by nonlinear differential equations, thus associating more and more of the complex behavior with the deterministic part of the unknown factors and leaving fewer questions unanswered. It is thus possible that noisy looking systems may nevertheless be deterministic.

Schaffer performed the same analysis on the Canadian lynx population in the far north based on 200 years of data available from the Hudson's Bay Company. The lynx-hare system also exhibits chaos and is governed by a so-called "one-dimensional map" (Schaffer, 1984). The analysis indicated that the data contain chaotic regions interspersed with linear data.

Another set of problems in population biology is the interplay between environmental factors that affect individual organisms and those that affect an entire population. The problem thus becomes one of understanding how the dynamics of a population are affected from the behavior of the individuals in it. In gas dynamics, similar problems can be solved in statistical terms. However, in ecology, individuals pose more complex problems. In a given population, density-dependent mechanisms affect the population dynamics, environmental factors affect each individual, which then affects the overall population, etc. The degree of influence varies with individuals and populations. Hassell and May (1985) applied this problem to a population of whitefly in England. They found patterns ranging from stable points to stable cycles to chaos. Environmental noise (which really means factors that cannot be explained adequately) was a significant factor. However, the problem may be with the way the data were collected and not the biology of the system. In sum, simple population models suggest chaotic effects, but applications to real populations remain problematic. In information science the same problems may be posed by the citation characteristics of articles. Each publication has its own citation life history, and the combined histories of a number of publications around certain common traits form the cumulative characteristics of a literature population.

4. DEMONSTRATING NONLINEAR DYNAMICS

In contrast to theoretical systems with only a few variables, it is almost impossible to know all the variables in a natural system, let alone keep track of them and illustrate their dynamics. One distinguishing characteristic of nonlinear dynamics is "sensitive dependence on initial conditions." Imperceptible changes in the initial state of a system grow exponentially over time, until in the long run the behavior seems completely random. A serious problem in time series analysis is that it is often dependent on ARIMA models that use smoothing devices and essentially wipe out useful information about deterministic mechanisms. Recent work has demonstrated that it is in fact possible to take time series data and establish whether it arises due to chance or internal deterministic conditions (Crutchfield, 1980; Schaffer, 1984).

Explanations of chaos usually start with a simple deterministic model and try to illustrate the mechanisms by which apparent randomness arises. Determinism implies that there are no random inputs into the data. The search for nonlinear dynamics reverses the process. The heart of the methodology lies in taking what seem to be random data, subjecting them to a series of tests outlined below and deciding in favor of periodicity, randomness, or chaos. The following is an outline of the minimum conditions necessary to examine nonlinear behavior in time series data (Moon, 1988). For further details the reader is referred to the references given below.

4.1 *Fourier analysis*

The first step in time series analysis is to separate periodic components in the data from random ones. To investigate whether a time series is periodic, quasiperiodic, or chaotic the data are first subjected to spectral analysis. This is accomplished by doing a fast Fourier transform (FFT) of the time series data. The Fourier spectrum of a waveform reveals the presence of periodic components. In a chaotic system the spectrum shows a broad spectrum with no well defined peaks. An additional helpful step is to perform an autocorrelation of the data prior to Fourier analysis to limit spurious data (Rabinovitch & Thieberger, 1987; Kim *et al.*, 1988).

4.2 *Lyapunov exponents*

The Lyapunov exponents reflect the divergence of trajectories in phase space over a time period, and thus give a measure of sensitivity to initial conditions. Quantitative techniques exist to calculate the stretching, contracting, or folding of the orbits over different regions of phase space and to average them out. A stable system will have a negative or zero Lyapunov number where the trajectories move inward toward a stable point or limit cycle. A positive exponent indicates orbits diverging exponentially, and thus implies chaotic dynamics (Wolf *et al.*, 1985).

4.3 *Poincaré section*

The behavior of a dynamical system is best observed when one plots its phase space diagram. This involves plotting the values of the independent variables as they change over time. A series of these points produces a trajectory, the "attractor," that describes the system's movements and its speed over a time period. An attractor is basically what the behavior of the system settles down to. A point in phase space represents all the information known about the system at that point in time (its position and velocity). There cannot be more than one result emanating from the deterministic characteristics of the system at a given point. Because the trajectories cannot cross, the attractor is limited to a confined space (the limit cycle), and it must be infinitely interleaved (folded into itself) in order to contain the dynamics of all the trajectories for that system (Ruelle, 1980).

Because the attractor may be multidimensional and because of the nature of quickly varying orbits in phase space, an intersection ("a map") is taken through the trajectory of an attractor to reveal its character (Moon, 1988). This involves looking at the motion at discrete time steps such as at x_{t+1} versus x_t. The successive points must then appear to be in a functional relationship such as a line or a curve. A perfectly random system such as white noise will reveal a uniform cloud of points in its cross-section. For a chaotic system with a periodic solution (such as a population model) the portrait is a closed loop. Its orbits wander differently from periodic cycles. Furthermore, if the map repeats itself on any scale of observation (i.e., exhibits a fractal structure), then chaos is successfully demonstrated.

The basic attributes of a chaotic system can thus be summarized as follows:

1. determinism,
2. complicated dynamics,
3. sensitive dependence on initial conditions,
4. positive Lyapunov exponent, and
5. strange attractor with fractal geometry.

5. METHODOLOGY

5.1 *Examining growth patterns*

Superconductors are materials that conduct with no resistance at very low temperatures. Their potential uses extend from cheap energy production to making smaller and faster computers. The phenomenon was discovered in 1911, but the complexity and high cost of cooling confined these materials to the laboratory environment. In late 1986 a surprising paper by Bednorz and Muller (1986) on high-temperature superconductivity provided evidence for a new class of materials that retain their special characteristics at much higher temperatures and show promise of widespread commercial applications. Since 1987 the intense worldwide research effort has produced significant progress, and this interest is reflected in the explosion of the worldwide literature on superconductivity.

The major assumption made at the beginning of this work was that the extent of scientific activity in a field and the resulting transmission and communication of ideas is perfectly reflected in its literature. In full agreement with Garvey (1979), the essence of science *is* communication. It is in the communication of an idea and its eventual acceptance and absorption by the community of scholars and workers that the value of an idea is seen. However, the purpose here is not to go into the origins of ideas as Latour and Woolgar (1986) did, but to get as close to the moment of communication as possible. The goal is to state explicitly whether the dynamics of communication are random or whether there are characteristics internal to the data that result in shaping the complex picture of communication activity.

First, to establish the total size of the literature the Chemical Abstracts database on STN was searched back to its inception in 1966 for articles on superconductivity and the numbers of items abstracted. Even though no database completely carries all the articles published on superconductivity, in terms of coverage of both the theoretical and the applied, Chemical Abstracts comes very close and provides several convenient search capabilities. To get an idea of the short-term growth of the literature, the search was extended to include the number of articles added to the Chemical Abstracts database in its biweekly updates since 1984 (when it becomes searchable). This feature is not available for other databases such as INSPEC prior to the most recent couple of years. The data were not normalized for the number of new journals that have appeared since 1966, because it was assumed that the growth of ideas and publication of data precedes the growth in the number of journals, and that contributors will submit their materials to whichever publication will best advance their ideas, whether it is a small new journal or an established but expanding one. What matters is that there is a growing literature in a field of study, that avenues for publication (that is, mostly journals) will somehow absorb the new articles, and that the indexing of these publications will reflect the growth of the field in the databases.

5.2 *Examining nonlinear dynamics*

Imagine turning on a water tap and watching the drops fall one by one. At first the drops fall slowly and regularly. As the tap is opened more, the speed of the falling drops increases until the regularity of the flow disappears and the drops appear to be falling randomly. At that moment the system has crossed into the world of deterministic chaos (Shaw 1984). With all other factors being equal and the only parameter of variation being the rate of flow, what mechanisms are at work that push the flow from regularity into chaos? Is it, similarly, possible to find a literature that grows periodically until forcing mechanisms push it to exhibit chaotic behavior? This is the very question that motivated the search for nonlinear dynamics in the growth of literature. The appearance of the article by Bednorz and Muller (1986) provided just the ideal tool of study.

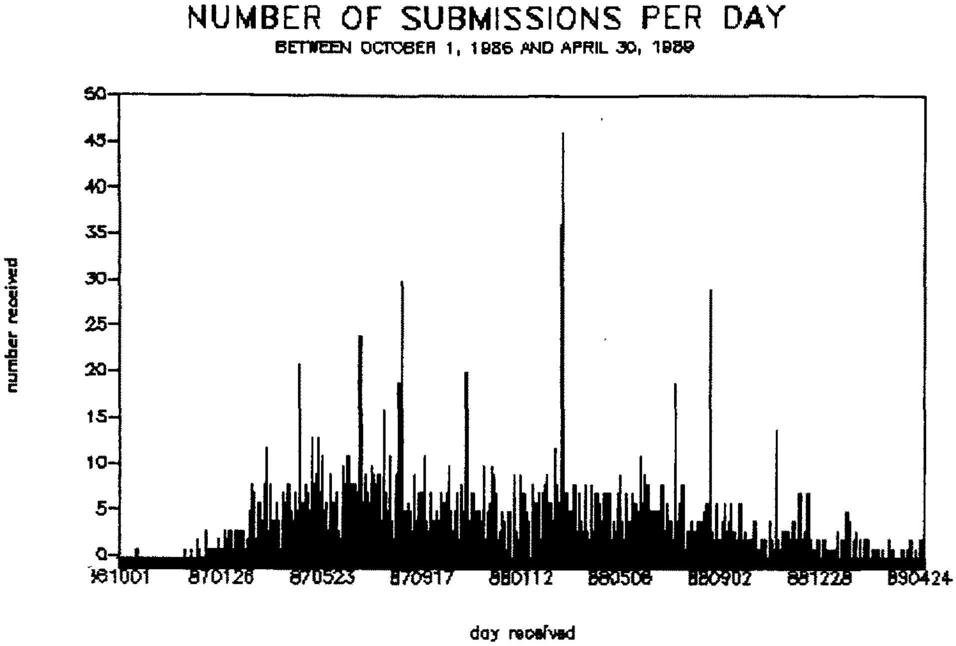
The Science Citation Index® database was searched for articles citing the seminal article by Bednorz and Muller (B&M). The assumption here was that all those influenced by their work would be citing the B&M article. As it turns out, a number who *should* have cited them did not, and were lost to the analysis. The resulting 2712 citing articles were consulted to find out their dates of submission. The assumption here was that, especially in the physics literature, the submission of an article for publication is accompanied by the wide circulation of preprints to interested colleagues in the hope of establishing priority as well as communicating results of work in progress. Thus, the submission dates for articles at the same time act as dates of communication of ideas. In the case of conference papers appearing in special issues of periodicals, the submission data was taken as the day of the conference held and the paper given. A small number of journals do not carry the date of submission of their articles. As well, a number of articles citing B&M were either review papers or invited papers, and as such did not carry a date of submission. In view of the limited new technical contributions review papers bring to theory, the date of submission would not be expected to be congruent with the date of the communication of a new idea in a technical paper. Overall, of the 2712 papers identified, only 2444 carried dates of submission. This 10% error may very well introduce serious limitations on the validity of this study. Another piece of missing evidence is the number of papers rejected by journal editors and conference organizers. These may very well constitute communications that cannot be covered here. Finally, the very number of journals covered by the Science Citation Index® puts serious limits on the number of articles covered by this study in the first place.

6. RESULTS

The argument whether scientific literature grows linearly or exponentially still depends on one's point of reference. As Fig. 1 shows, the number of superconductivity articles published each year started slowly in 1966 and grew until the mid 1970s, after which it followed a constant pattern until the explosion in 1987. This may still indicate a temporary exponential phase, since the contributions for 1990 are below the number for 1989. From this perspective, it may be impossible to judge until a number of years have gone by.

The number of cumulative articles for the Chemical Abstracts database (Fig. 1b) confirms the above argument. A linear pattern from 1968 is followed by a sudden burst in 1987 and possibly the beginning of an exponential phase. However, that is still uncertain because the number of articles abstracted in 1990 is smaller than that in 1989. Between 1971 and 1986 the compound growth in the number abstracted was 8.7% a year. Between 1986 and 1989 it was 28.5% per year (it doubled from 10841 to 40868). For 1989–1990 the growth rate went down to 7.3%, closer to the historic norms. The major problem with the B&M and subsequent findings is that there is no theoretical basis for the behavior of materials exhibiting the high temperature superconductivity. The field may be entering a period of exhaustion or at least a respite until a theoretical breakthrough produces a new burst of publications worldwide.

What does the form of the growth in the biweekly updates to Chemical Abstracts look like? From the bar-graph in Fig. 2, it is growing, but very irregularly. The number of articles published per two-week period since 1984 indicates that a phase of increased activity in 1986 was followed by a lull in 1987 before the significant growth started in 1988. Although an accelerated phase is clear from the graph, it is also true that the growth is not regular, but that it comes from bursts in activity. These bursts may be the result of certain publication trends, such as conference proceedings published as special issues in periodi-



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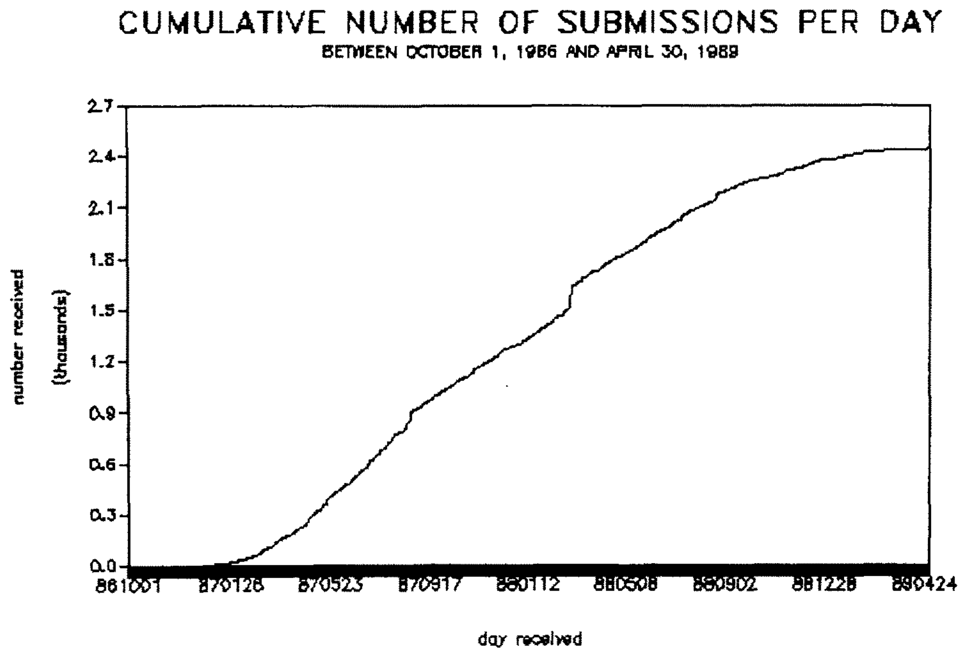
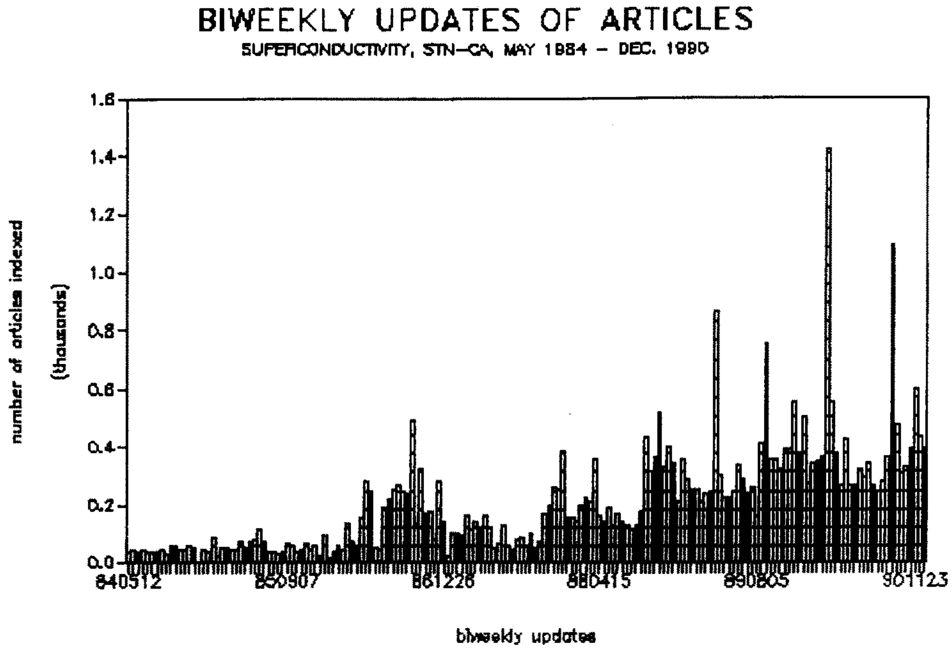


(b)

Fig. 1. The number of superconductivity articles abstracted by Chemical Abstracts for 1966–1990. (a) The number abstracted each year. (b) The cumulative growth since 1966. A period of accelerated growth for 1970–74 was followed by linear growth until 1986. The sudden rise for 1987–89 and the subsequent drop in 1990 indicates an exhaustion phase or temporary break.

cals, or they may be due to irregularities in indexing activity from contributors to Chemical Abstracts. The causative mechanisms remain to be identified.

The appearance of the B&M article was so sudden (in fact, prior to publication it was largely unknown even to the team's superiors) that at first it was regarded with skepticism, as just another unsupportable claim, until further work showed its significance (Hazen, 1988). The time series data shown in Fig. 3 reflects the number of articles submitted (and



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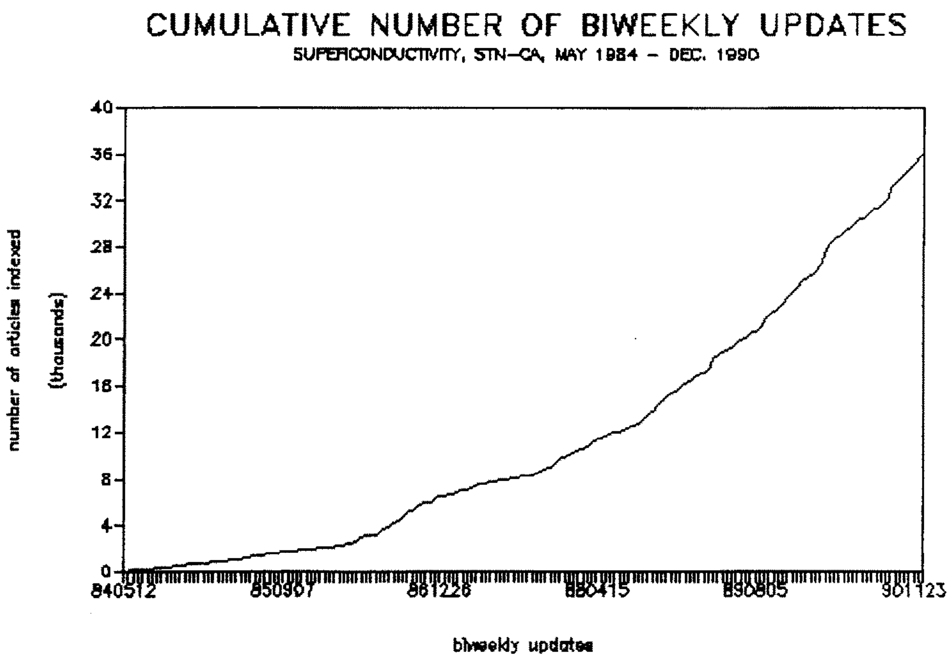
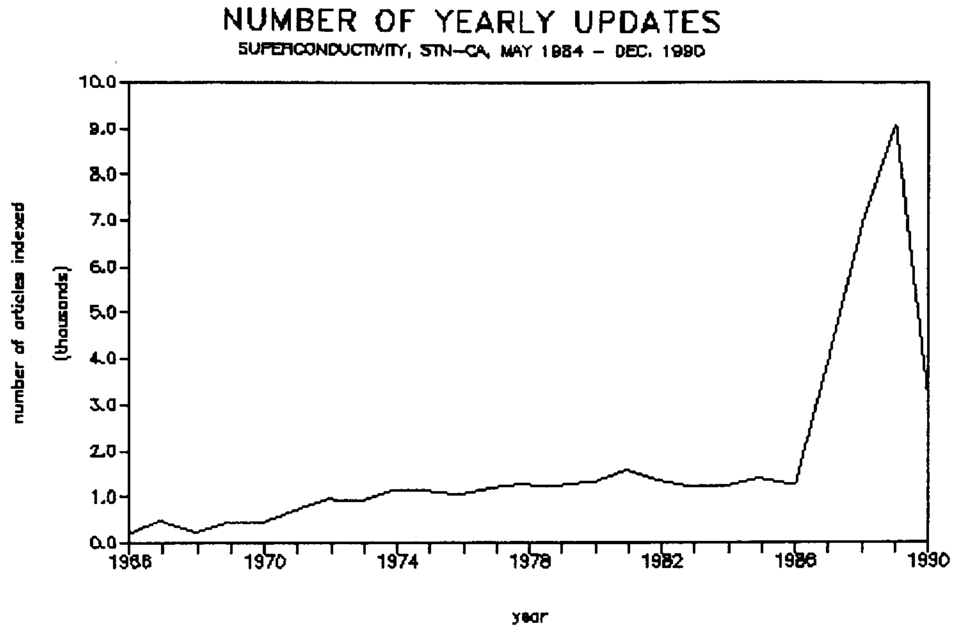


(b)

Fig. 2. Inclusion of new superconductivity articles into the Chemical Abstracts every two weeks between 1984 and 1990. (a) Biweekly updates. (b) Cumulative numbers abstracted, reflecting the specific growth for superconductivity. Continuous growth is marked by exponential bursts in late 1986, early 1988, and 1989. Such bursts are not discernible from the cumulative graph.

eventually accepted) for each day between October 1, 1986 and April 30, 1989. The choice of October 1 is somewhat arbitrary. The Bednorz and Muller article came out in the September 1986 issue of *Zeitschrift für Physik*, and by October 1 it is expected that it reached all its subscribers. At the beginning of this work thought was given to removing Saturdays and Sundays from the time series data so as not to introduce a false periodicity into the data, as these constitute weekend days when publication offices would not be expected to



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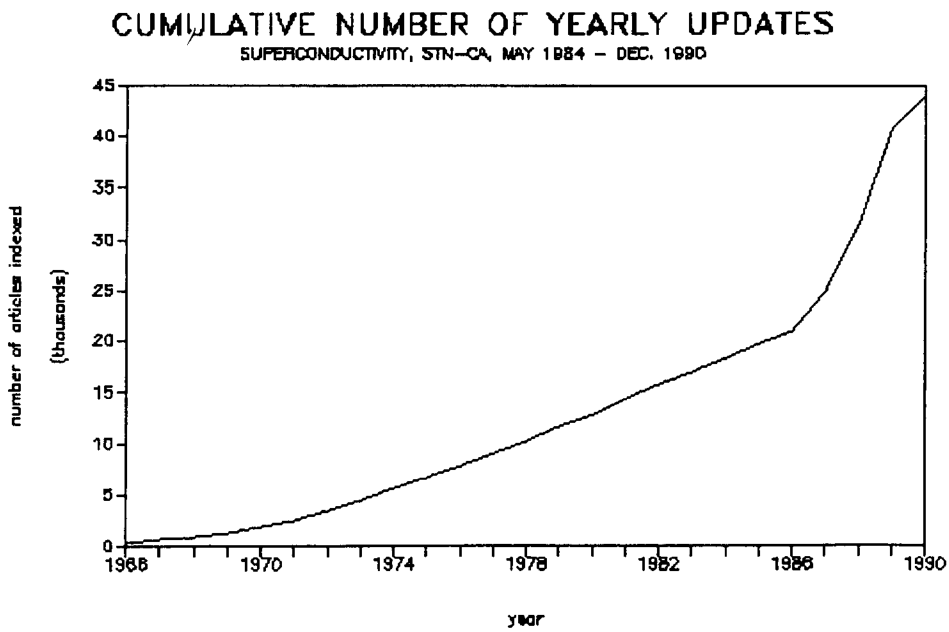

(b)

Fig. 3. The number of articles citing B&M and submitted to journals every day between October 1986 and April 1989. (a) The number of articles received by editors each day. (b) Growth of submissions during the same period. Except for occasional bursts due to conference submissions, exhaustion and obsolescence marked the end of a typically logistic growth period.

be open. As it turned out, especially between April 1987 and October 1988, virtually every day saw the submission of a paper somewhere in the world, East or West. This further reflects the global character of science and the incessant activity within its communication networks.

The data in Fig. 3 further reflect the difficulties with taking sides in favor of linear or exponential growth. By mid 1989 the growth of articles citing B&M found a plateau. The graph reflects how much science is still heavily driven by events. It is dominated by sudden bursts of activity which coincide with major conferences held on those days (e.g., 26 April 1987, 24–25 August 1987, 29 February–2 March 1988). Otherwise, it is difficult to see any regularities in the graph.

Another question that may arise here is what external forces affect the growth of research literature. One widely accepted answer so far has been that the major driving force for research is funding and manpower. However, the uncritical acceptance of this statement is now suspect on two grounds. First, the suddenness with which laboratories switched to high temperature superconductivity research demonstrated that the lack of time for formal applications for funding was not necessarily a hindrance to work on urgent new ideas. Nor was there a sudden influx of new PhDs into superconductivity research. Funding statistics, therefore, cannot reflect the sudden switch. On the other hand, it may be instructive to find out which research area was left in a vacuum due to the sudden shift toward high-temperature superconductivity. Second, the data presented here indicate that the more important consideration may be in terms of the events that spur on that growth, that is, the activities of professional societies and conferences (Eisemon, 1990). Conferences (close, intense communication) produce a flood of papers into the journal literature that increase the number of items ultimately covered by the abstracting services. In fact, one popular avenue to encourage growth in a field of activity still is to hold international conferences and to ensure the wide dissemination of their proceedings.

To test whether the time series exhibits periodic or chaotic characteristics, the data were submitted to the tests mentioned above. The spectral graph (correlogram) showed three distinct peaks coincident with weekend periodicity. On the other hand, the phase space diagram and the Poincaré section (Fig. 4) reflected a pattern previously encountered in population studies (Olsen & Degn, 1985). The section on the left-hand side of the figure is neither linear nor circular. Most of the points seem to fall on a unimodal curve. In other words, the magnitude of each point is predictable from each one preceding it, and the day-to-day variation in the numbers is deterministic. On the other hand, without question, the data contain a substantial amount of noise. If the dynamics of the system are inherently stable, then the presence of noise can push it to become unstable, turn the Lyapunov exponent positive, and produce a strange attractor (Crutchfield *et al.*, 1982). The fact that the Poincaré section here is rather thin is consistent with low-level dynamics. The identity and interplay of the salient variables remain to be determined.

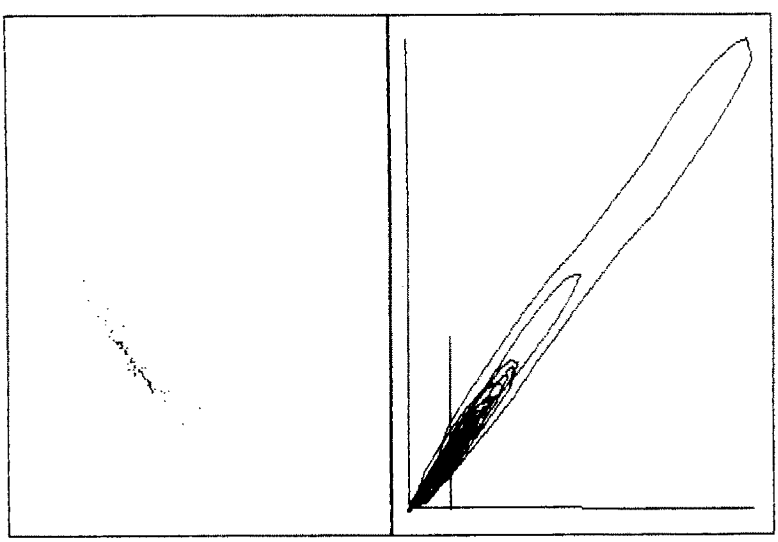

Fig. 4. Stretching and folding in the growth of superconductivity articles submitted each day. View of the phase space is from above the attractor with a slicing plane normal to the page and cutting through the center. The slice through the orbit yields an almost one-dimensional curve – the Poincaré section (left). The fact that the motion is confined to a finite region in phase space and that the slice is linear rather than a diffuse mass is indicative of nonlinear dynamical motion.

7. IMPLICATIONS FOR INFORMATION SCIENCE

An examination of articles published on superconductivity over the last 25 years has demonstrated that growth takes place by waves and spurts, and that it is futile to argue in favor of either linear or exponential growth. Science is inherently a spontaneous activity, and the above work attests to the argument. In that light, it may be more valuable to identify ideas and publications that may provoke new bursts and waves of activity. Work is currently under way to identify just such waves in publication, as well as the origins of their dynamic mechanisms.

There are several avenues open to further work in information science. For one, it would be valuable to take groups of articles in various subjects over a period of time and subject them to time series analysis and try to determine whether they exhibit nonlinear dynamic characteristics. It may also be that there are chaotic regions in the data between periodic regions consistent with sudden bursts of activity. The work reported on superconductivity above provides one way to test the hypothesis. Previous work introduced nonlinear dynamics to information science and demonstrated a broad spectrum of frequencies in polymer chemistry data (Tabah & Saber, 1989). Work is now under way to lengthen the data set and complete the analysis. In terms of individual articles affecting the future output in a given literature, it may be instructive to examine further the effect of an influential paper on the subsequent growth of literature by finding more examples from science.

Second, Goffman's equations should be tested to see whether they exhibit chaos under certain parameters. While it may be impossible to find solutions to nonlinear differential equations, computer software now exists to allow modelling and testing by varying the different parameters in the equations and observing results on a computer screen (Schaffer *et al.*, 1988). It would be instructive to reexamine schistosomiasis literature (on which Goffman based much of his work) as time series data and apply it to the chaotic analysis described above. The same should be repeated for other literatures. Recent work on epidemics gives a mixed picture of the results (Olsen, 1988).

It would be instructive to find fractality in a given literature as well. A large number of co-citation maps exhibit similarities among themselves. In tune with observing self-similar pictures in co-citation maps, it may be possible to find chaotic dynamics behind their fractal appearances (Egghe, 1990). Or in Van Raan's words, "Like fractal distributions in ecological systems . . . we could consider co-citation clusters as a representation of the ecosystem of scientists" (Van Raan, 1991). There is value in following up recent theoretical work with concrete demonstrations.

Lotka-Volterra equations and their modifications provide a mathematical model for examining two-species competition in nature. It may be possible to draw analogies to information science by following the resolution of differences between two populations of articles on a subject with opposing theses. Ecological models pose a potential problem, in that they deal with density-dependent populations. Once a natural population grows beyond a certain level, lack of food supply and disease become limiting factors. In the growth of literature such limits are difficult to find, as worldwide output in science and publication continues to grow incessantly. One other problem is finding data with seasonal variations. In that respect, it may be instructive to extend this analysis to library circulation data, where seasonal fluctuations do exist, and find out whether it is possible to explain the data in nonlinear dynamical terms.

One serious shortcoming of the work reported here is that the theoretical basis for its approach remains to be established within the domain of information science. Since there is no theoretic reason to believe that all information production activities progress linearly, the above approach at least provides empirical support for further research into nonlinear dynamics in the growth of literature. It is hoped that the ideas put forward in this paper will encourage future work in this domain. While a certain amount of work on the growth of literature exists, it is worth adopting a new approach with the purpose of finding causative mechanisms that have been lacking in informetrics. Ecological and epidemic phenomena are a function of cyclicality in nature. What cyclic mechanisms exist in information production and literature growth is an open question. It may be that the growth of literature occurs by chance (stochastic processes) as well as by necessity (deterministic processes). Nonlinear dynamics and deterministic chaos may offer new avenues of investigation.

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