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Nineteenth-Century Mathematics in the Mirror of Its Literature: A Quantitative Approach

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The point of departure of this paper is the idea that the development of mathematics is reflected in its publications. Hence, the existence of a nearly complete database renders possible general statistical accounts of the development of mathematical activities. To this end, the authors utilize the mathematical index of the *Catalogue of Scientific Papers* of the Royal Society of London dealing with the mathematical journal literature of the 19th century. The relation between the journal and book literature of that century is discussed, with the result that the size of the journal literature is presumably a valid indicator of the intensity of mathematical activities in particular areas. On the basis of this *Catalogue,* graphs of the publication activity of all of 19th-century mathematics and of 34 of its most important subareas are displayed; both the number of active contributors in each area and its share of 19th-century mathematics publications are exhibited. Furthermore, the share of mathematics of the total scientific journal literature of the 19th-century is estimated. Frequency distributions of publication activity and the specialization of 19th-century mathematicians conform to patterns well known in modern scientometrics. © 1996 Academic Press, Inc.

In dieser Arbeit wird davon ausgegangen, daß sich die Entwicklung der Mathematik in ihren Publikationen widerspiegelt. Eine annähernd vollständige bibliographische Datengrundlage gestattet daher globale statistische Beschreibungen der Entwicklung mathematischer Aktivitäten. Die Autoren werteten zu diesem Zweck den mathematischen Index des *Catalogue of Scientific Papers* der Royal Society of London aus, der die mathematische Zeitschriftenliteratur des 19. Jahrhunderts berücksichtigt. Sie diskutieren das Verhältnis von Zeitschriften- zu Buchliteratur in diesem Jahrhundert mit dem Ergebnis, daß der Umfang der Zeitschriftenliteratur vermutlich als Indikator der Intensität mathematischer Aktivitäten auf einzelnen Gebieten gelten kann. Auf der Grundlage des Catalogue werden zur gesamten Mathematik sowie zu 34 der wichtigsten Teilgebiete Verlaufskurven der Publikationsaktivitäten gezeigt, zum einen als Publikationsanteile am Gesamtgebiet, zum anderen als absolute Zahl der auf einem Teilgebiet überhaupt aktiven Mathematiker. Ferner wird der Anteil der Mathematik an der gesamten naturwissenschaftlichen Zeitschriftenliteratur des 19. Jahrhunderts geschätzt. Häufigkeitsverteilungen der Publikationsaktivität und der Spezialisierung der mathematischen Autoren des 19. Jahrhunderts ergaben in der zeitgenössischen Szientometrie bekannte Verteilungsmuster. © 1996 Academic Press, Inc.

Utgångspunkten för denna artikel är föreställningen, att matematikens utveckling återspeglas i dess publikationer. Existensen av en så gott som fullständig databas möjliggör därför allmänna statistiska beskrivningar av utvecklingen av matematiska aktiviteter. För detta ändamål utnyttjade författarna det matematiska indexet till den *Catalogue of Scientific Papers,* som utgivits av Royal Society of London och som behandler 1800-talets matematiska tidskriftslitteratur. Förhållandet mellan det åhundradets tidskrifts- och boklitteratur diskuteras med resultatet, att tidskriftslitteraturens omfång förmodligen gör, att den kan gälla som indikator på itensiteten hos matematiska aktiviteter på ensklida områden. Utgående från denna *Catalogue* visas kurvor på 34 av dess viktigaste delområden; i det senare fallet

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anges dels varje delområdes andel i publikationer av hela mathematiken, dels antalet aktiva matematiker på området. Vidare uppskattas matematikens andel av hela den naturvetenskapliga tidskriftslitteraturen under 1800-talet. Frekvensfördelningar av publikationsaktiviteten och specialiseringen hos 1800-talets matematiker följer mönster, som är bekanta i den moderna scientometrin.

AMS subject classification: 01A55.
KEY WORDS: mathematics, 19th century, history, bibliography, *Catalogue of Scientific Papers,* bibliometrics, scientometrics.

INTRODUCTION

When a reviewer of the *Catalogue of Scientific Papers* of the Royal Society of London stressed its potential usefulness for a statistical analysis of scientific activity in 1925 [14], he could not know that he would have to wait nearly 40 years to witness the emergence of new statistical methods in the historiography and theory of science, in so-called scientometrics, and another 20 years to see the first statistical analyses of the *Catalogue.* In what follows, we shall say something about both scientometrics and the *Catalogue.*

Historians of mathematics are familiar with bibliographies as a reference tool. However, they may not be familiar with a quantitative approach to bibliographies. We would like to suggest some reasons for this.

While mathematicians furnish applied sciences with formal methods of computation and analysis, they are not accustomed to applying formal methods to the study of the development of their own discipline. It is hard to find systematic attempts at constructing formal and quantitative tools for the description and analysis of the development of this formal and quantitative science *par excellence.*

The lack of suitable indicators for measurement makes the application of quantitative methods to the description of the development of mathematics difficult. Nobody knows how to "measure" the importance of a proof, the number of its consequences, or the repercussions of its refutation. How does one quantify the value or impact of a mathematical piece of knowledge or measure the scientific manpower required to prove a certain conjecture?

In their research, historians rely on the archives of administrative, ecclesiastical, and private institutions. Whereas public and private bodies are sometimes uncommunicative with respect to their records and documents, this cannot, as a rule, be said of scientists. Thus, historians of science have a more accessible source of information about their subject. Since the start of the first major scientific journals in the 17th century, authors, publishers, and reviewers increasingly tended to make scientific results available to the public. It is hard to find any serious scientific discoveries not published in the "archives" of science. Furthermore, several factors stimulate written communication in science. Thus, those public documents furnish a fairly complete picture of scientific achievements.

If those archives are taken as a mirror of scientific activities, attention must be paid to the particular ways scientists publish, which may vary from person to person, from discipline to discipline, and from one historical period to another. From a statistical perspective, however, peculiarities of individual scientists become less significant. Using publication behavior, one can quantitatively compare groups of scientists in a single discipline.

In the following quantitative analysis, we shall sometimes compare characteristics of 19-century mathematics with features of a field of modern mathematics, namely, mathematical logic, which has its roots in the second half of the 19th century. Admittedly, the two areas are embedded in quite different scientific, cultural, and social contexts. Nevertheless, they both belong to the area of professional abstract reasoning, and the number of publications in the two areas—about 36,000 in 19th-century mathematics and 47,000 in mathematical logic until 1990—makes a statistical comparison interesting. For details about data handling and results concerning mathematical logic, see [18].

THE SUBJECT INDEX OF THE ROYAL CATALOGUE

As every historian of mathematics knows, the bibliographic situation for 19th-century mathematics is not comfortable. There is no comprehensive record of the total literature from 1800 through 1900. The *Jahrbuch über die Fortschritte der Mathematik* covered monographic literature and journal literature only from 1868 on, although certainly in a quite comprehensive and reliable manner.

For the historian of mathematics, there remains a gap in the bibliographic tools for the years before 1868. With respect to journal literature, this gap can only be closed by means of the mathematical subject index to the international *Catalogue of Scientific Papers* of the Royal Society of London.[1]

There are numerous special bibliographies, of course, but they are restricted with regard to subject, content, or geographical scope. Hence, the index of the *Catalogue* is the only comprehensive guide to the journal literature in pure mathematics from the beginning of the 19th century onwards. The index was planned for all subject categories but was completed only for some categories, *inter alia* mathematics.

One of the major problems for a comprehensive analysis is the absence of book literature in the *Catalogue.* By means of some comparisons of the *Jahrbuch* and the mathematical index of the *Catalogue,* we shall demonstrate the role of this literature. At the same time, we shall test the reliability and completeness of the *Catalogue.* The *Jahrbuch* considers 96 journals in the year 1871, the *Index* 73; in the year 1900 we find 168 journals versus 137 in the *Index* (all values for the *Jahrbuch* in this and the following two sections are taken from [13], barring physics and mechanics).

A comparison of the number of journals taken into account by the *Index* and by the *Jahrbuch* reveals that the *Index* covered 76.0% of the journals in the *Jahrbuch* in 1871; 59.9% in 1880; 81.0% in 1890; and 81.5% in 1900.[2] The *Index* takes fewer

[1] Cf. [15]. The Index embraces all four series and the supplement of the *Catalogue.* For the genesis of the *Catalogue* and the *Subject Index*, see [3].

[2] Values for the *Jahrbuch* are taken from [13]. One has to bear in mind that there may be quite different assumptions about what constitutes the ‘‘identity’’ of a journal which varies its title, editing body, and publication date in a way which sometimes defies statistical description.

periodicals into account than the *Jahrbuch;* but, as is well known in information science, some journals in a special area carry the bulk of the literature while many journals have only a minor quantitative significance in this area. That is the case here, too.

To get an impression of the amount of monographic literature additionally contained in the *Jahrbuch,* the *Index*'s percentage of the *Jahrbuch* (except for the sections "History" and "Textbooks") were calculated with the following results: 1871, 71.7%; 1880, 74.6%; 1890, 69.2%; and 1900, 72.0%.[3]

Comparing the *Jahrbuch* with the *Index,* we can roughly estimate that, at most, one-third of the 19th-century literature of mathematics are monographs; as an inspection of the yearbooks shows, these include not only books but also dissertations, booklets, and many so-called *Programmschriften.* Presumably, the percentage changes over time. By considering all references in (or at the end of) articles of the *Philosophical Transactions of the Royal Society* from 1665 through 1900, it has been shown that the percentage of book citations decreased from the year 1850 on from 49% to 19% in 1900, whereas the percentage of journal citations increased from 50% in 1850 to 75% in 1900 [1].

We found that even in the development of mathematical logic from the second half of the 19th century until today, the percentage of books was relatively high during the first decades of this period and decreased afterwards to a value of about 5% at present [21]. One may speculate here about whether all fruitful areas of research exhibit relatively more monographs in the early phase. In any case, we may summarize with Allen *et al.* that "[i]n the nineteenth century, scientific journals achieved equal importance with books" [1, 293].

Nevertheless, we have to face the fact that there are many important contributions which are not to be found in the *Catalogue.* The journal literature, however, will indicate whether there was lively discussion of a subject matter. Of course, the fundamental stimuli for those discussions in journals may stem from the monographic literature. In the quantitative analysis of mathematical logic, we found that the addition of minor entries, such as reviews of books, to a bibliography tends to underline the main waves in the discipline's development [18, 55, 62]. The same will, in general, hold true even for the relation between articles and books.

[3] Some random checks were performed to estimate the reliability of the *Index.* We compared the percentages of four areas in the *Jahrbuch* and in the *Index* for the years 1880, 1890, and 1900. Theory of numbers (first value *Jahrbuch,* second *Index*): 1880: 8.2%, 8.1%; 1980: 4.8%, 6.3%; 1900: 6.8%, 7.9%; Elementary and descriptive geometry: 1880: 6.9%, 6.0%; 1890: 12.1%, 6.6%; 1900: 13.3%, 7.2%. In this group we find many textbooks in the *Jahrbuch.* Without them the values would be 5.9%, 6.0%; 6.3%, 6.6%; 5.4%, 7.2%. Theory of functions, algebraic functions: 1880: 14.1%, 11.4%; 1890: 12.0%, 9.6%; 10.3%, 8.4%. Probability and statistics: 1880: 4.4%, 3.2%; 1890: 3.4%, 4.0%; 1990: 4.6%, 3.4%. Generally, we have to take into consideration the facts that (1) the percentages of monographs in the *Jahrbuch* differ in different areas; (2) there cannot be complete agreement about what belongs and what does not belong to a certain area; (3) the lists of journals in the two bibliographies do not agree completely. The random checks indicate, however, that the main trends, i.e., the ups and downs, are visible in both bibliographies in the first three cases. A trend reflected by the following figures can be assumed to be non-random if it continues for some years.

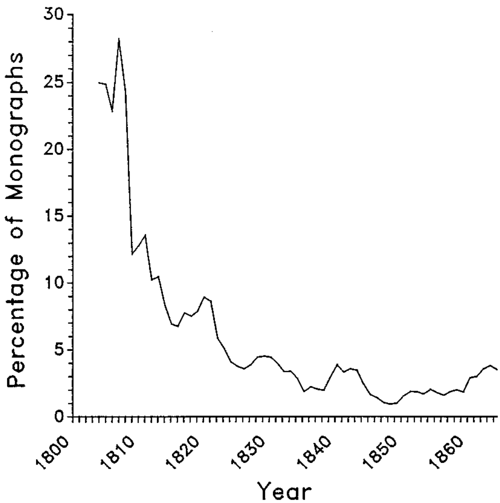

FIG. 1. Ratio of monographs to articles in pure mathematics from 1800 to 1864. The number of monographs was obtained from [4]; the number of articles came from the *Catalogue.* The ratios are based on moving five-year averages with the last year of an interval an endpoint; e.g., the five-year average ratio of monographs for 1804 is the mean of the ratios for the years 1800 through 1804.

The same may also be true for *outstanding* and important books in relation to the rest of the literature. To check this, we analyzed one of the extremely rare subject-classified records of monographic literature of modern times, namely Brunet's *Manuel du Libraire* from 1865 [4]. This bibliography contains a selection of outstanding and important books published since 1450, leaving out ordinary textbooks and similar publications but including significant collected works. Figure 1 shows the yearly output of monographic literature in the section "Mathématiques pures" for 1800 through 1864 in relation to the output of articles according to the *Index*. A comparison of Figs. 1 and 2 shows that in those years where many articles appeared, even the percentage of monographs increased.

For the sake of consistency, we shall subsequently make exclusive use of the *Index*. This has a further advantage in that this *Index* was compiled as a whole by a working group during a limited period of time.

To provide a statistical analysis of the *Catalogue of Scientific Papers,* we put all entries of the mathematical parts of the index in a machine-readable database, leaving out only some minor areas without mathematical content in a narrower sense.[4] Each entry included author(s), short title, classification code, source of publication, and publication year. Not included in the database were some anony-

[4] The areas left out are history and biography; periodicals, reports of institutions; general treatises; bibliographies; tables; addresses and lectures; pedagogy; institutions; nomenclature; aids to calculations and graphical processes. These areas have a total of about 1400 entries.

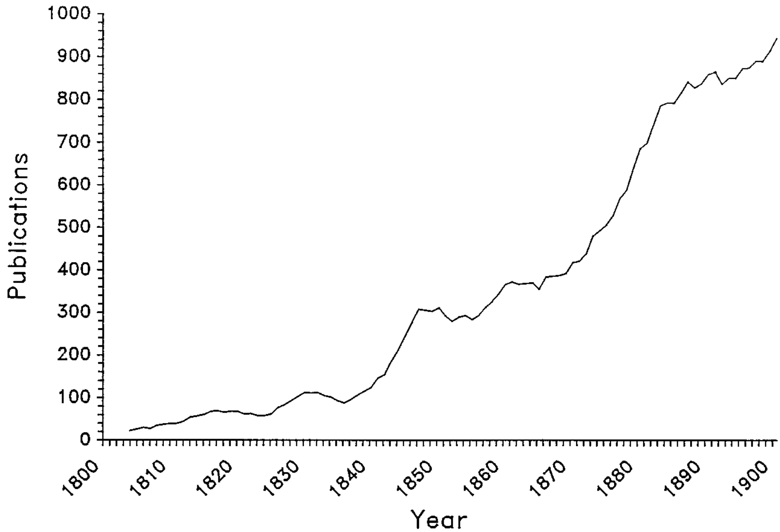

FIG. 2. Moving five-year averages of the annual number of articles in mathematics listed in the *Catalogue* from 1800 to 1900.

mous or pseudonymous entries, entries with an unknown abbreviation of the author's name, or entries under a corporate source. These entries total about 200, which does not constitute a notable deficiency from a statistical point of view. The final database consisted of 36,268 publications.[5] About 100 of them were published before 1800 (the first in 1782), and a few after 1900.

All entries had been classified by a team of qualified indexers under the guidance of a committee established by the Royal Society of London. The moderately hierarchical and relatively fine-grained subject classification scheme was set up exclusively for the purpose of the *Index.* The extent to which a paper is given more than one classification code is not quite clear; in any event, there are no more than about 500 entries in this category. All those entries were taken into account; in these cases it cannot be determined, however, whether these are different publications or single publications contributing to different fields.

In the case where articles were published in more than one part, only the last entry was considered by us because only then can the article be considered as completed. In generating an alphabetical record we standardized different spellings of names of authors and of abbreviations of journals. There were only 208 publications with second authors and no third authors. Each co-author was counted in the

[5] The typographical presentation of the *Index* prevented scanning and made manual collecting and typing necessary. In the course of the preparation of the database, about 15 entries were mutilated seriously or proved to be unfit for use. Due to the high cost of reconstructing these entries and their statistical irrelevance, they were left out.

TABLE I
AUTHORS AND PAPERS IN ALL SCIENCES AND IN MATHEMATICS

	All sciences			Mathematics				
Publication year	Authors	Papers	Papers/ authors	Authors	Fraction of math. authors to all authors	Papers	Fraction of math. papers to all papers	Papers/ authors
1800–1863	32,830	195,120	5.94	1,930	5.9	10,043	5.1	5.20
1864–1873	?	80,070	—	1,085	—	4,333	5.4	3.99
1874–1883	?	100,750	—	1,491	—	6,863	6.8	4.60
1884–1900	68,577	384,478	5.61	2,795	4.1	14,879	3.9	5.32
1800–1900	115,000	786,978	6.84	5,556	4.8	36,118	4.6	6.50

Note. The number of authors in all sciences comes from an estimate in [6, 571] based on the *Catalogue.* The number of papers in all sciences is based on [14], while the numbers of authors and papers in mathematics are our calculations. The 26,560 entries in a supplement of the *Catalogue* for the whole period, 1800–1900, were included in the data.

same manner as a first author of a publication. As a consequence the database comprises 36,476 contributions. (Because of the statistical insignificance of co-authors, we understand in the sequel by "publications" or "papers" these 36,476 contributions.)

THE PLACE OF MATHEMATICAL MANPOWER AND PUBLICATION OUTPUT IN 19TH-CENTURY SCIENCE

First, we want to estimate the percentage of mathematics in the whole scientific journal literature of the 19th century for the four periods of the alphabetical editions of the *Catalogue of Scientific Papers.* Thus, the role that mathematics played in the concert of scientific voices can be estimated.

The alphabetical author part of the *Catalogue of Scientific Papers* was published in four series plus a supplement. The statistical data for the alphabetical part were found in the literature about the *Catalogue* [6; 14]; the data for mathematics were computed by us.

From Table I the following facts are obvious. From 1874 through 1883, mathematics experiences a considerable relative increase in the whole *Catalogue;* afterwards, however, it falls back to a percentage which remains less than the values for the period from 1800 through 1873.[6] Hence, if we speak of a century of pure mathematics, we have to bear in mind that other areas of science also exhibited major expansions in that century.

[6] The percentage of mathematicians in 14 subject groups in [5] is greater. The *Historical Catalogue* compiled in [5] is selective however.

An interesting feature is that the average journal publication activity of all scientists, on the one hand, and of mathematicians, on the other, is very similar. This impression is strengthened if one recalls that concerning mathematics we considered only the last entry of an article published in more than one part. Presumably, the analyzer of the remainder of the *Catalogue* counted each entry consisting of several parts. The further development up to modern science seems to bring forth a great divergence in publication habits. At least in laboratory sciences, the average productivity in terms of publications surpasses by far the average output of mathematicians.[7]

So far we have offered some hints about the position of mathematics in the sciences of the 19th century. Now we would like to consider its development in absolute terms.

First, we will examine the development of its annual publication output. Figure 2 shows an increasing trend. This trend seems to vary from period to period, sometimes being more similar to exponential growth, sometimes more or less stagnant. Hence, even on the level of the development of a single and very autonomous discipline we see a phenomenon which was observed in physics by T. J. Rainoff and by A. Kroeber or P. Sorokin on a macrolevel in almost all cultural activities, inside and outside science, namely distinct fluctuations of "cultural growth."

We do not know, however, whether these fluctuations in 19th-century mathematics are merely a result of the instability of scientific institutions and scientific professionalism or whether they reflect, for example, the intensity of contemporary mathematical discussions.

As a first step towards an answer to this question, we have to examine whether the discontinuity still holds when we consider only the number of people active in mathematics without regard to the intensity of their publishing activity. We computed the annual amount of manpower by counting a mathematician in each year between the first and last year of his activity, inclusively. In what follows, we shall call a scientist from the first until his last publication in a scientific area an "active contributor" to this area (cf. [7]). If a mathematician has published only one article, for example, he is counted one time in the year of his publication. In Fig. 3 we show the number of active contributors in mathematics from 1800 until 1900. For the computation of the first and last years, entries of the *Catalogue* for the years from 1782 through 1799 were also used. The graph—without moving averages—shows considerably fewer fluctuations than the graph of the annual number of publications. In combination with the following results on the average strength and duration of scientific participation, we conclude that fluctuations of mathematical activities are not caused mainly by fluctuations in scientific manpower in the 19th century, for instance by interrupted scientific careers and similar discontinuities of scientific work. A similar conclusion holds for mathematical logic of the 20th century. Furthermore, these fluctuations are remarkably smaller than might be expected from assumptions in the historical literature on 19th-century mathematics (e.g., [10]). In

[7] See, for example, data in [2]; for a supplement concerning mathematicians, see [17].

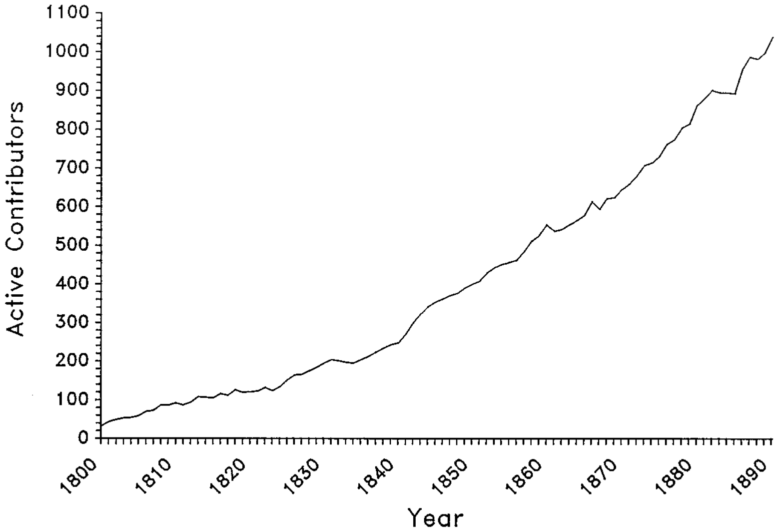

FIG. 3. Annual number of active contributors, 1800–1900. Active contributors for a given year are those mathematicians who have published their first papers but not their last by that year.

this respect there seem to be no differences at all from mathematicians working in areas of logic in the 20th century.

Hence, the fluctuations of publication graphs may be interpreted as the expression of a more or a less vivid activity of individual mathematicians, whereas the population as a whole shows considerable stability. Figure 4 emphasizes this, showing the annual number of publications per active contributor from 1800 through 1900.

One might assume that journals tend to publish more in times of general prosperity and less in times of economic depression or war. The reduction of publication activities in the two World Wars of the 20th century is apparent; the number of active contributors in an area of research, however, is less markedly influenced by these events [18].

In comparing economic indicators of prosperity in 19th-century Europe with the waves of mathematical activities, we detect an anticyclic pattern. According to Mosekilde *et al.,* for example, there were two major depressions in Europe in the 19th century and two world-wide depressions in the 20th century: 1830–1850, 1870 to the late 1890s, 1920–1940, 1974 to the present (about 1990) [12].[8] As Fig. 2 shows, in times of economic depression mathematical activity exhibits phases of exponential growth in the 19th century; in a boom, however, mathematical activity is conspicuously weaker. We cannot offer an explanation for this. One might speculate with

[8] We cannot discuss here whether there are Kitchin (3 to 5 years), Juglar (about 10 years), or Kondratieff cycles (about 50 years) even in the development of the sciences.

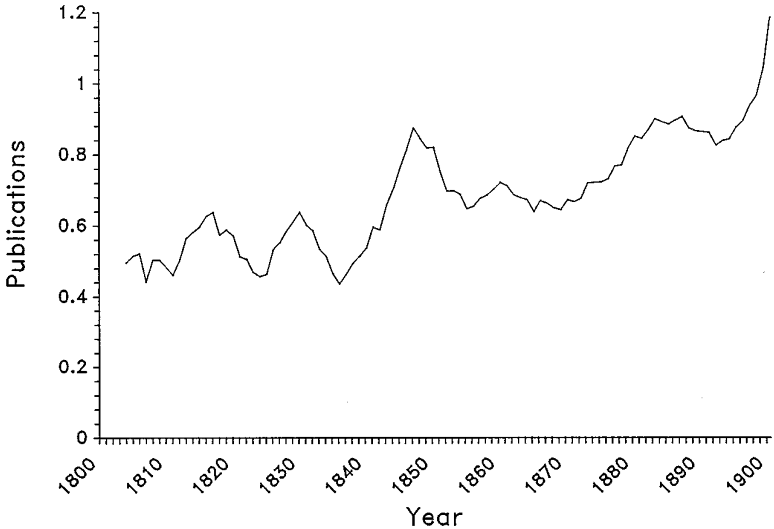

FIG. 4. Annual number of publications per active contributor, in terms of moving five-year averages. Note that the values of the last years are unrealistic since the bibliography ends for all authors in 1900. This makes the last year of publication artificial for some authors.

Mensch that in times of economic depression the theoretical and philosophical foundations of the "basic innovations" of the future are laid down [11], with mathematics as a part of these foundations. But there may also be another interpretation, namely, that the pioneering periods of new mathematical fields which later experience an influx of a major number of scientists may be observed years before the new area becomes popular; these pioneering periods could be the years of general economic prosperity. One is reminded here of the old-fashioned anticyclic budget policy of those wise governments which put aside money in a boom to spend the savings in a depression.

We now return to the level of individual scientists. What was the average productivity of scientists in terms of their journal articles in times when the battle-cry of "publish or perish" may never have been heard?

The total average number of papers per author in 19th-century mathematics (now treating co-authors as full authors) is 36,438/5,619 = 6.48. This is much more than the average logician produced from 1874 through 1990 with his 3.94 contributions [18, 66]. But the scientists' careers are cut off at the beginning and the end of the respective periods covered by a bibliography. Therefore the average productivity of all authors beginning in a certain year has to be computed in order to find out the average productivity of those mathematicians whose full period of activity is covered by the bibliography. The curve of the average total output of all mathematicians beginning to publish in 1800, 1801, . . . , 1900 showed us that the production of authors beginning later than 1888 is covered to a lesser extent. Therefore, we computed the average productivity of all authors beginning not later than 1888 with their first publication in mathematics. The average productivity is 32,253/4,138 = 7.8 papers per author. We then compared this with logicians from 1874 through 1990 beginning not later than 1972 with their first publication in logic; afterward the listing of the production of logicians is increasingly less inclusive [18, 71]. The average production of these logicians is 30,903/4,540 = 6.8.[9]

Considering the massive differences between the social and institutional circumstances of the science systems of the 19th and 20th centuries, the differences in the publication behavior of the mathematical scientists of the two centuries are remarkably small. The average productivity says nothing, however, about the development of the upper and lower levels of the productivity, which may, for example, increase in both directions in the 20th century. We also have to mention that the picture of productivity is not complete with regard to another aspect: many 19th-century mathematicians were active in other fields of science, for example, in physics. By analogy, many 20th-century logicians have been active in other parts of mathematics or in philosophy. It remains an open question whether the shares of the different activities of scientists differ in the 19th and 20th centuries.

It is a commonplace in the history of science and of mathematics that in the 19th century many laymen and nonprofessionals participated in mathematics, which developed firmer institutional structures only in the second half of the century.

Hence, we now compare the average duration of participation of 19th-century mathematicians with the average duration of participation of mathematicians working in logic mainly in our century. As a rough indicator of that duration of scientific participation, we here again take the year of the first publication of a scientist until (and including) the year of his last publication.

The inspection of the corresponding graph in Fig. 5 shows the average duration of participation for all mathematicians beginning with their first publication in the year denoted on the x-axis. We see that from 1880 on there is a steady decline, indicating that approximately from then on the duration of participation is cut off by the time span covered by the *Catalogue.* The average duration of all authors beginning before 1880 is 11.29 yr (35,868/3,178). If we take the year 1890 as the beginning of a decline in the average duration, we get for all authors beginning before 1890 an average duration of 10.07 yr (42,686/4,237).

A similar relationship holds for mathematical logic. An inspection of Fig. 25 in our *Mathematische Logik von 1847 bis zur Gegenwart* showing the average duration of participation of modern logicians [18, 74][10] tells us that beginning in 1970 (20 years before the end of the bibliography) onwards there is a decline in average duration. Up to 1969, there were 37,567 years of participation by 3,327 authors,

[9] This figure includes books. The value without books is the same: 29,026 papers/4,263 authors = 6.8 (our own computations). The percentage of authors in the 19th century producing not only papers but also books is greater than the respective percentage of logicians in the 20th century.

[10] As in the case of 19th-century mathematicians, an inspection of the values for the figures without moving averages was performed.

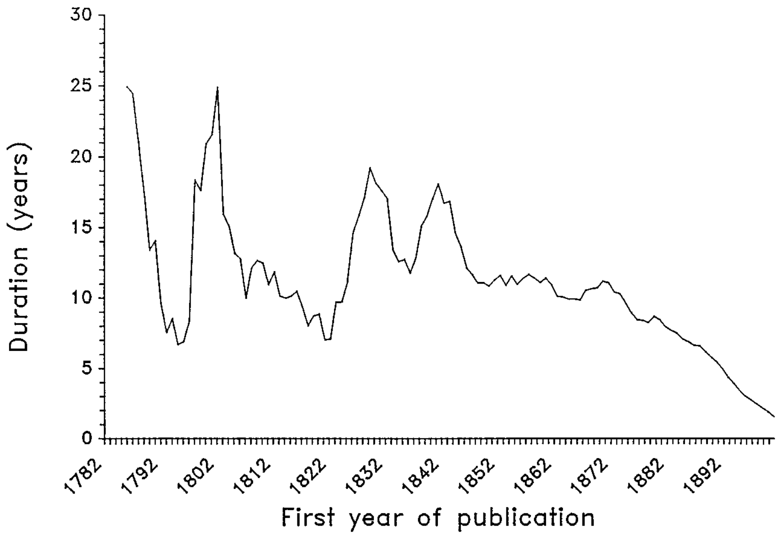

FIG. 5. Duration of activity of authors with the first year of publication shown on the x-axis, in terms of moving five-year averages. See Fig. 3 for the definition of "active contributor." The values for the last years are unrealistic; see note in Fig. 4.

which makes an average duration of 11.29 years. The similarity in size implies that the institutional and social circumstances of scientific production did not matter for the average time of participation.

All presumptions about the structure of scientific participation in the last century and today must take into account the possibility that the differences may be far smaller than supposed on the basis of mere intuitions about the 19th century. It might be the case that ideas about differences between today and former times are due to a lack of inspection of archives and a lack of data about those former times. It would certainly be of interest to know whether the average duration of participation of scientists other than mathematicians changed from the 19th to the 20th century.

So far only the average productivity has been examined. But in quantitative studies of science, a popular subject is the so-called asymmetric distribution of scientific productivity, named "Lotka's law" after a seminal paper by Lotka [8]. Lotka found that the number y of scientists of an area publishing x times follows a power law, $f(x) = \text{constant}/x^{\gamma}$. In the cases examined by Lotka, the exponent γ was approximately 2.0 ("inverse square law"). But later investigations showed that it lies between about 1.8 and 3.8, and that it is presumably dependent on the time period considered in the development of scientific areas [19]. With the x-values in ascending order, the function results in a hyperbolic curve or approximately in a straight line on double-logarithmic paper. This indicates that in a given time period

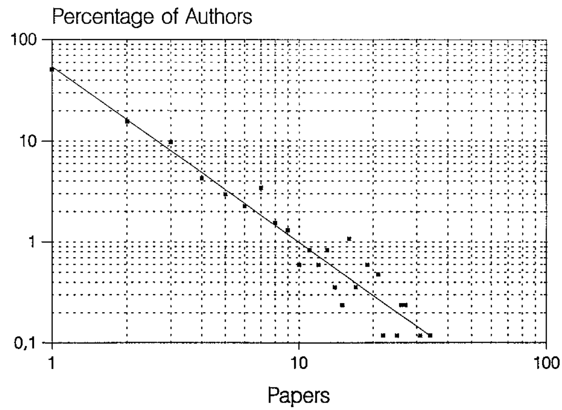

FIG. 6. Points (x, y) on this scatterplot indicate that y percent of the 842 authors publishing in mathematics between 1851 and 1860 published x papers. The graph does not include the eight most prolific authors who published at least 50 articles (according to the limit $f(1)^{1/\gamma}$ [20]). The truncated least squares regression statistics are $R^2 = 0.93$; $c = 54.37$; $\gamma = 1.739$; s.d. $= 0.096$.

for a certain scientific field there are, as a rule, many authors with few contributions and few authors with many contributions. This regularity is often misconstrued as an asymmetric structure of scientific skills, or abilities, or success. But the regularity only reflects an asymmetric structure of a mixture of scientific duration and of intensity of scientific activity in a certain field, comparable to an income distribution in economics. It is encountered in a host of different scientific areas.

We computed the Lotka distribution for two periods of time, namely, for 1851–1860 and for 1891–1900.[11] Figures 6 and 7 indicate that the regularity also holds true for mathematicians in the 19th century.

What does this type of frequency distribution tell us about the development of mathematical areas? Taking the published bulk of mathematical literature as an indicator of mathematical activity, May [9] came to the conclusion that there are quite different types of activities expressed by publications. He evaluated the total literature about determinants through 1920 (partly with the help of an expert in this area). Only 14% of 1707 titles were classified as new ideas and results. This is, according to May, probably too high a value, because in no case was the novelty of an idea beyond doubt. It is interesting to note here that an evaluation of the literature of symbolic logic from the beginning up to 1936 by Church resulted in a similar percentage of contributions evaluated as outstanding (cf. [18]). Sixty-four percent of the literature consisted of "trivia" and duplications; the rest fell roughly in equal parts into the categories applications, systematization and history, and texts and education.

[11] We did not make the calculations for more periods because of the high cost of such computations.

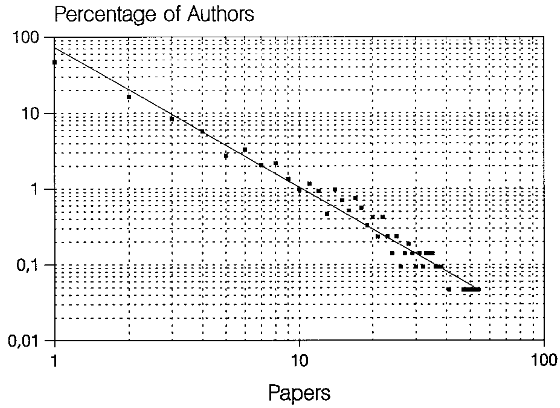

FIG. 7. Points (x, y) on this scatterplot indicate that y percent of the 2,145 authors publishing in mathematics between 1891 and 1900 published x papers. The graph does not include the three most prolific authors who published at least 57 articles (according to the limit $f(1)^{1/\gamma}$). The truncated least-squares regression statistics are $R^2 = 0.97$; $c = 73.26$; $\gamma = 1.843$; s.d. $= 0.048$. The regression does not meet the Kolmogorov–Smirnov test. Regression based on the usual truncation of more prolific authors would pass the test, but the value of such regression is extremely limited.

Different types of mathematical activity are expressed on a continuous scale by means of the computation of a frequency distribution of the Lotka form. The range of different activities is classified here only by its strength, indicated by the number of publications. From a statistical perspective, however, this number might not only be an indicator of the extent of activities of some people but also a rough indicator of influence and reception of scientific work in the scientific community (presupposing a correlation between output and reception). The most prolific authors of a certain phase in the development of an area often belong to those authors who are the developers of new ideas, contributors of new results, or the driving forces of further elaboration of those new ideas and results.

The percentage of single-item authors in an interval of about 10 years in a scientific field lies, as a rule, between 50% and 80%. These authors may typically provide what May called trivia. One should bear in mind, however, that even outstanding mathematicians are single-item authors in a range of fields. Their contributions may sometimes present views from a standpoint which is anchored in a different discipline and may be original just on that account. From quantitative studies of the history of logic, we also know that mathematicians have sometimes made basic contributions in areas they never treated again.

The regularity of the Lotka-law type entitles us to presuppose a cluster-like structure in any established area of scientific activity: there are some few “hot” stars in the center of activity surrounded at increasing distances by orbits comprising an increasing number of participants.

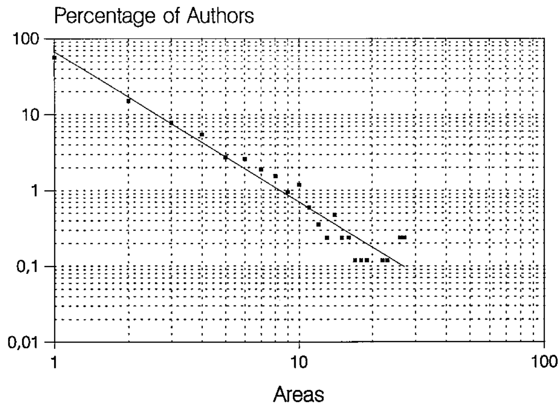

FIG. 8. Points (x, y) on this scatterplot indicate that y percent of the 842 authors publishing in mathematics between 1851 and 1860 published papers in x different areas of mathematics. The graph does not include the four most prolific authors who treated at least 29 areas (according to the limit $f(1)^{1/\gamma}$). The truncated least-squares regression statistics are $R^2 = 0.94$; $c = 67.28$; $\gamma = 1.981$; s.d. $= 0.108$.

Having investigated only the distribution of the size of scientific activities, we do not know whether it is typical for the participants to engage in only one or a few mathematical areas as specialists, or whether many engage in the whole range of areas. Characteristically, this question is completely left out of consideration in examinations concerning scientific productivity (e.g., [16]).

We found it appropriate, therefore, to calculate the number of mathematicians who treated $1, 2, \ldots, x$ different areas, respectively, in 1851–1860 and in 1891–1900.

Figure 8 shows that 56.8% of all mathematicians in 1851–1860 treated exactly one area. Only 15.2% treated two and 7.8% three areas. Figure 9 for 1891–1900 shows 51.6% who treated only one area, 16.5% with two, and 8.2% with three areas. The first distribution agrees with Lotka's "classical" law with an exponent of 2.0. For the distribution of the second period, however, there is a break between the more and the less prolific parts of the distribution. The bipartition is so obvious that we find it useless to adapt the curve to any truncated distribution before some insights are gained with respect to the causes of this difference.

The general form of the distribution permits, however, a further conclusion concerning scientific mobility. Since mathematical areas will not be treated simultaneously by a mathematician but in a step-wise order, a great number of areas a mathematician deals with will be connected, as a rule, with a great amount of cognitive mobility, i.e., the change from one research area to another. Scientific mobility is one of the main mechanisms of scientific information transfer and of interplay between research areas. It is noteworthy that, according to our results, scientific mobility is distributed in a Lotka-like manner.

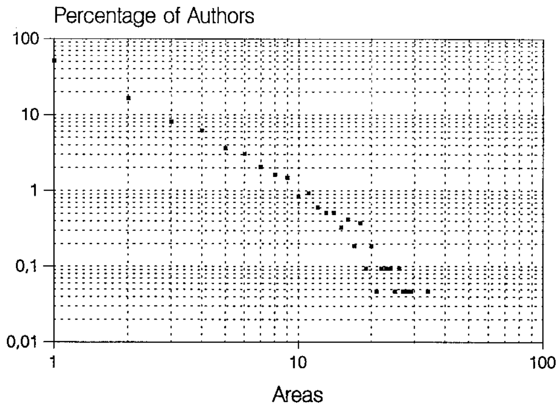

FIG. 9. Points (x, y) on this scatterplot indicate that y percent of the 2,145 authors publishing in mathematics between 1891 and 1900 published papers in x different areas of mathematics. As in Fig. 7, regression could be performed on the truncated data, but clearly the most prolific authors deviate in some systematic way.

SOME OTHER FEATURES OF THE DEVELOPMENT OF SINGLE AREAS

In the above sections, we have compared the percentage of mathematics in all scientific activities in the 19th century in a global manner. Now, we will summarize the development of single mathematical areas by inspection of the curves showing the percentage of the areas in all of mathematics as a percent of the total volume of mathematical journal articles. Our purpose is to provide a statistical overview of the development of the areas. Such insights might be of general interest for a historian of mathematics; they may, for example, be of interest for assumptions about the rise and decline of mathematical fields.

The curves are shown in Figs. 10–22. Yearly random fluctuations are smoothed by generating moving five-year averages. Experience with some 65 areas of mathematical logic in the 20th century showed us that the main trends are reflected clearly by such averages. Since no spectral analyses are made, the choice of five-year averages (instead of other averages) has no crucial importance in our context.

The first major characteristic is the vivid dynamics of the curves comparable to the dynamics shown by the areas of mathematical logic [18, Appendix]. If the annual publication output of a single area is correlated with the output of the whole discipline, the curve of the discipline would be copied by the respective area. The considerable differences between the areas demonstrate the "wave-like" form of scientific processes or their cyclic character. This means that one can always observe trends towards a concentration in time just as one can observe, for example, a geographic concentration.

One of the most salient features of the 19th century is the well-known decline

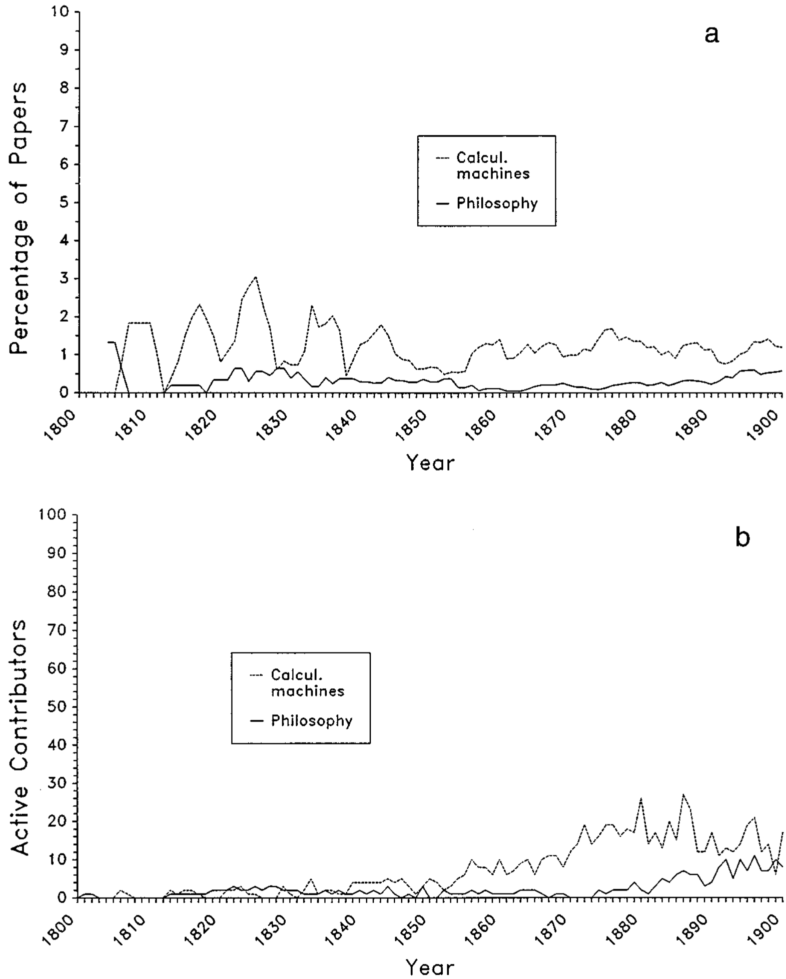

FIG. 10. In this figure and those following, (a) displays the ratio of articles in the given area to all articles in mathematics for each year, based on moving five-year averages; (b) gives the number of active contributors, as defined in Fig. 3, to the given area. All numbers come from the *Catalogue of Scientific Papers.* (a) Percentage of papers and (b) number of active contributors in philosophy (0000) and calculating machines and other instruments (0080).

of some geometric fields, for example trigonometry (6830) (numbers in parentheses indicate the classification codes of the *Catalogue*), stereometry (6820), and elementary geometry (6800–6840) in general, a dominant field in the first half of the century, and to a lesser degree foundations of geometry (6400–6430) (recovering at the end of the century) and geometry of conics and quadrics (7200–7260) since 1860. A relatively moderate decline can be observed in descriptive geometry (6840)

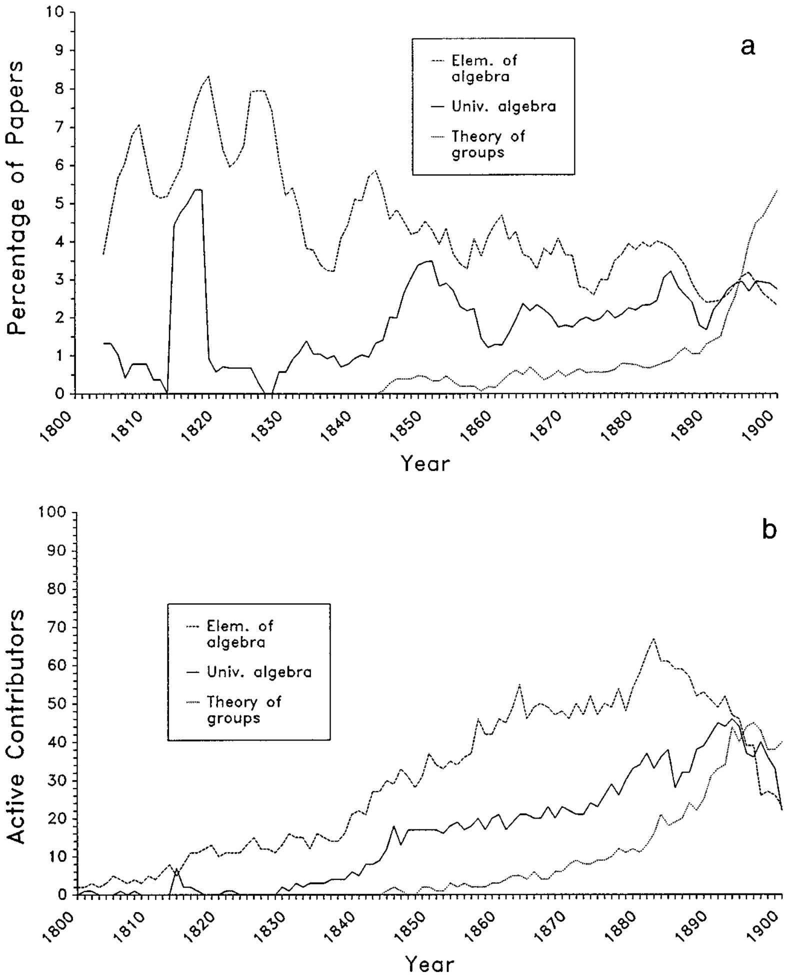

FIG. 11. (a) Percentage of papers and (b) number of active contributors in universal algebra (0800–0870), theory of groups (1200–1230), and elements of algebra (1600–1625, 1640).

and planimetry (6810). An expansion can be seen in the following areas: algebraic curves and surfaces of degree greater than 2 (7600–7660), differential geometry (8800–8870), transformations (8010, 8020), and transformations and algebraic configurations (8000–8100). A stable trend (including more or less distinct fluctuations) is exhibited by kinematic geometry (8420) and infinitesimal geometry (8400–8490).

Two of the winners of the century are differential equations (4800–4880) and linear substitutions (2000–2070). A moderate expansive trend was shown by the theory of functions of complex variables (3600–3640) and other special functions

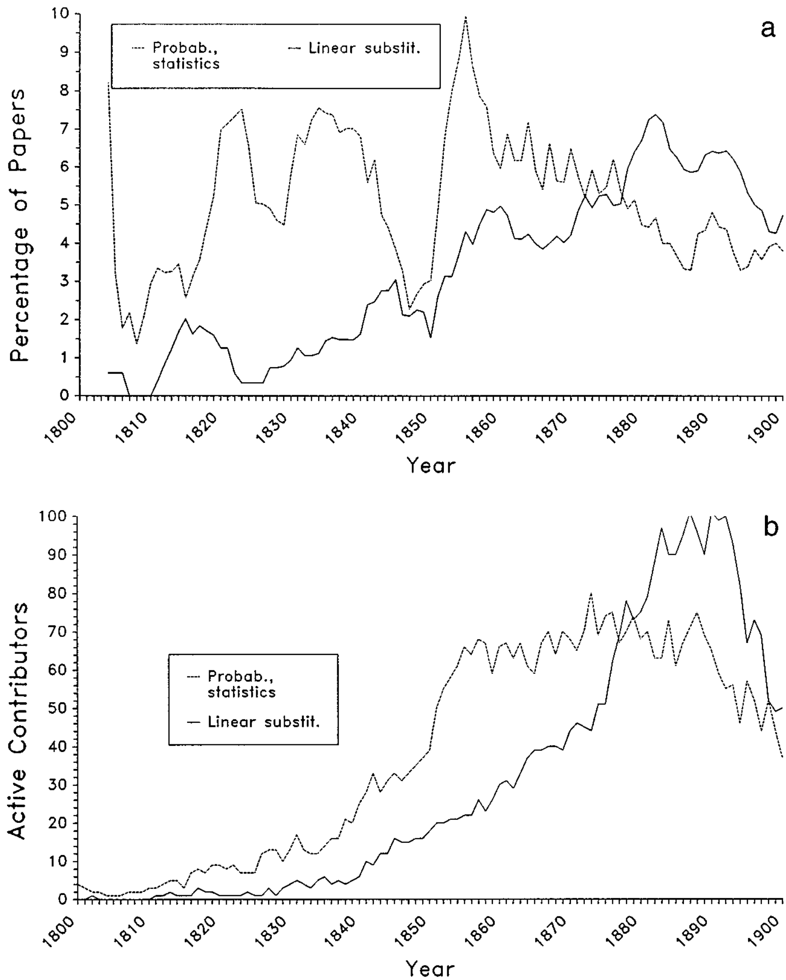

FIG. 12. (a) Percentage of papers and (b) number of active contributors in probability and statistics (1630, 1635) and linear substitutions (2000–2070).

(4400–4470). Their years of expansive prosperity in the first half of the century embraced infinite series (3220), probability and statistics (1630, 1635), elements of algebra (1600–1625, 1640), foundations of arithmetic (0400–0430) (which seemed to stabilize again), integral calculus (3250–3270), theory of equations (2400–2470), and foundations of analysis (3200–3280). No clear increasing or declining trend is obvious in solutions of partial differential equations (4830, 4840), application of analysis to physics (5600–5660) (both expanding considerably at the end of the century), algebraic functions (4000–4070) (an area which was always strong and

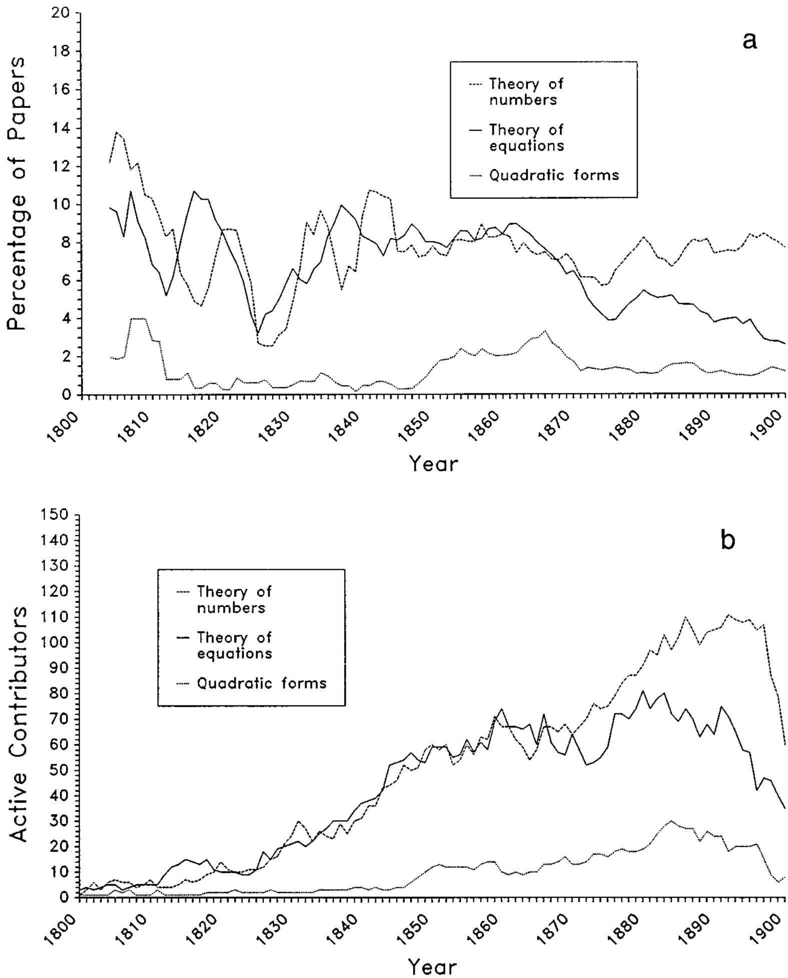

FIG. 13. (a) Percentage of papers and (b) number of active contributors in theory of equations (2400–2470), theory of numbers (2800–2920), and quadratic forms (2830, 2840).

showed some major "explosions"), and theory of numbers (2800–2920). The subject of calculating machines and other instruments (0080) attracted some interest in the first half of the century and thereafter seems to have received moderate attention without major events. Some decades had to pass until the revival of this subject occurred in the 20th century.

Another area remained some 40 years in the shadow of its neighbors; then it spread in an epidemic manner, in this respect comparable to, for example, fuzzy logic and fuzzy set theory in the 20th century [18, 197, 231]: we are talking about

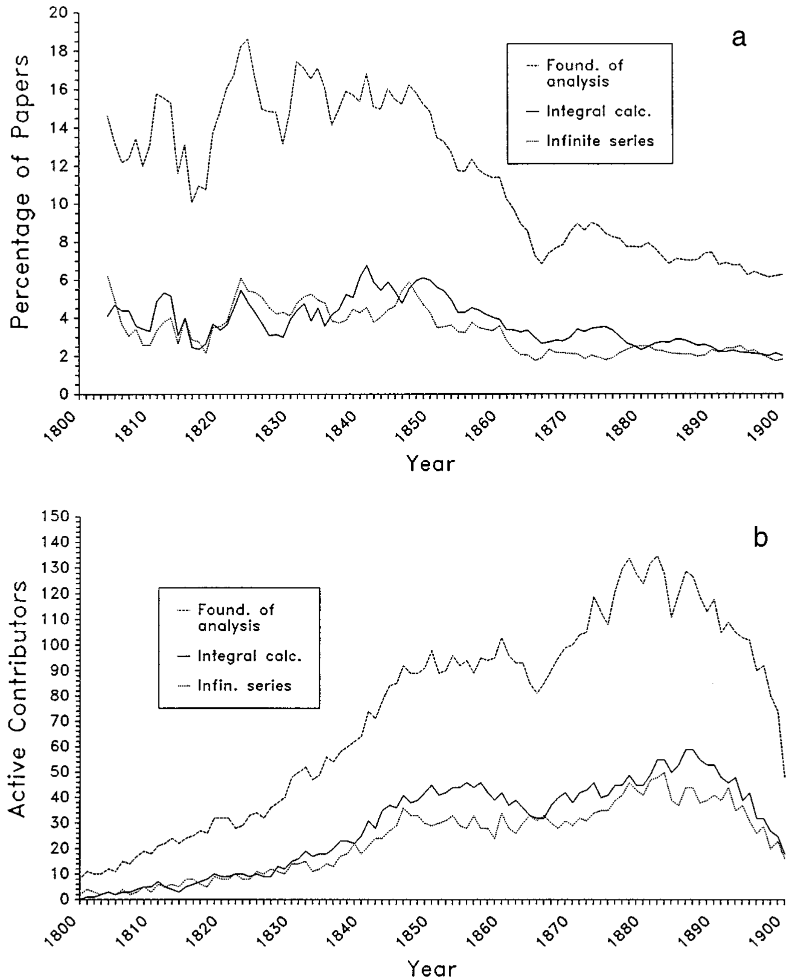

FIG. 14. (a) Percentage of papers and (b) number of active contributors in foundations of analysis (3200–3280), infinite series (3200–3220), and integral calculus (3250–3270).

the theory of groups (1200–1230). Despite its fashion-like appearance, this theory would contribute to one of the most revolutionary theories of the 20th century, namely, quantum mechanics.

There was a field of general mathematics which appeared to be almost invisible in those days: the "philosophical" section (0000), embracing *inter alia* certain aspects of logic. Presumably, a major portion of "philosophical" work in mathematics appeared as monographs and not as articles, and therefore the number of annual journal publications is extremely low. Nevertheless, the second half of the 19th

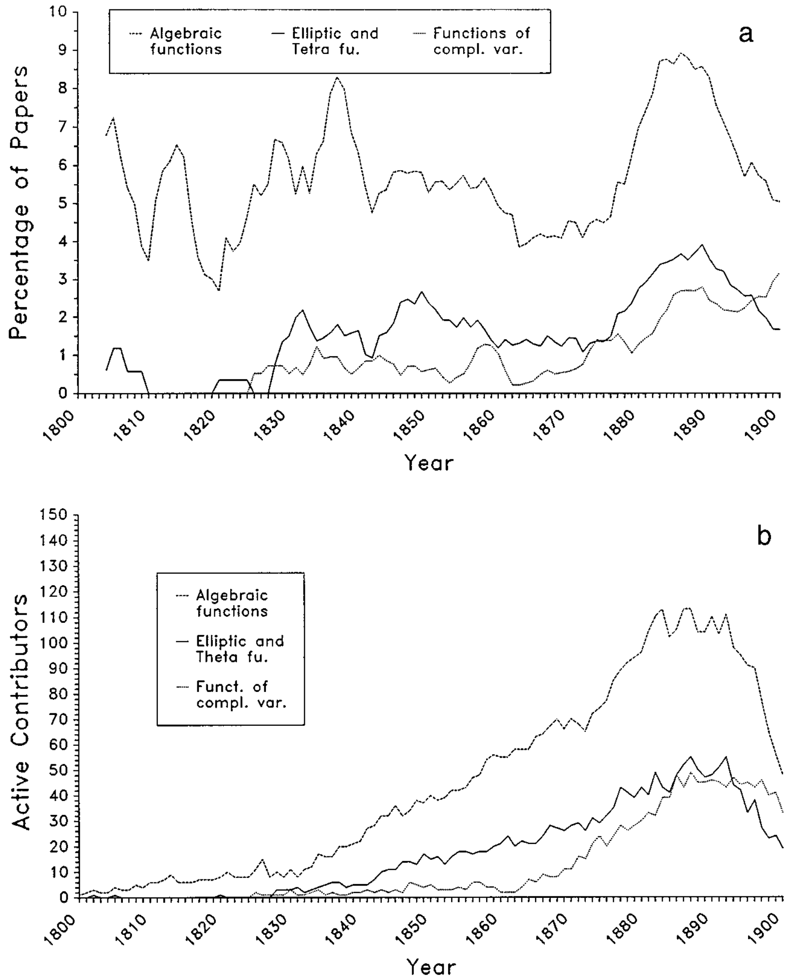

FIG. 15. (a) Percentage of papers and (b) number of active contributors in theory of functions of complex variables (3600–3640), algebraic functions (4000–4070), and elliptic and theta functions (4040).

century can be seen in hindsight as a period of prosperity for logic, marked by the pioneering contributions of Boole, Cantor, Dedekind, Frege, and many others, only leading in the 20th century to a considerable expansion in the number of interested scientists.

In view of the development of the theory of groups and mathematical logic, it seems that the number of participants is not essential for the future importance of certain mathematical results. Whereas many ideas in 19th-century geometry have

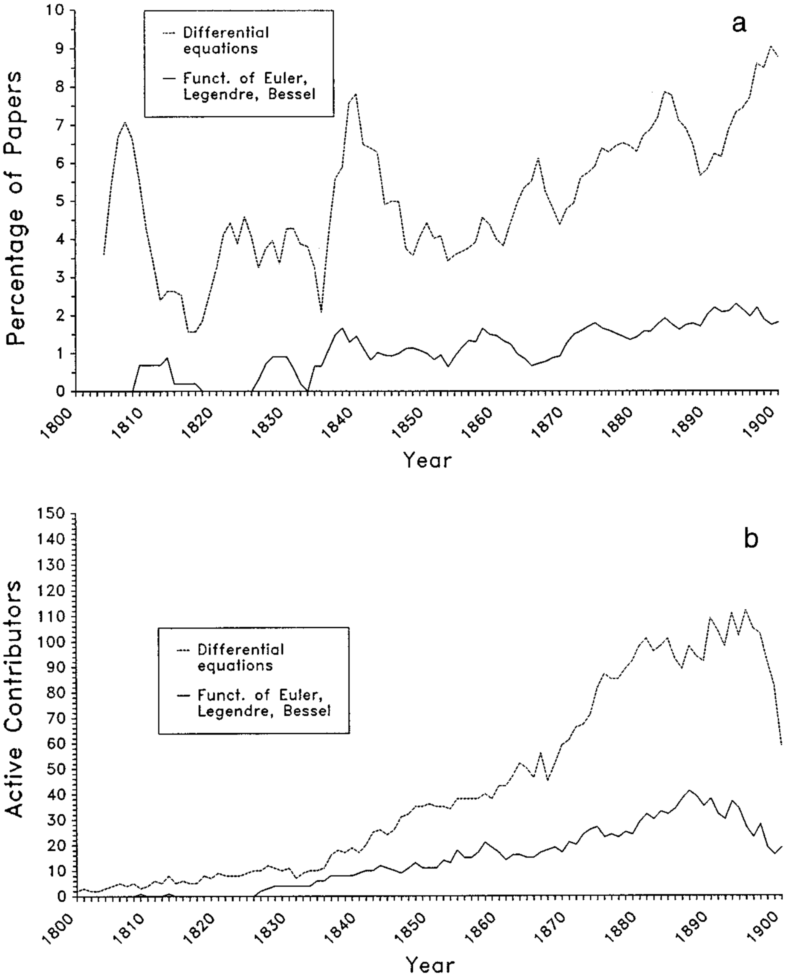

FIG. 16. (a) Percentage of papers and (b) number of active contributors in differential equations (4800–4880) and functions of Euler, Legendre, and Bessel (4410, 4420).

lost any substantial meaning for the mathematics of the 20th century, other activities such as a "philosophical" treatment of mathematics, at that time almost invisible in relation to geometry and other major mathematical fields, have established the foundations of the important subject of modern abstract mathematics, including its vast potential for computer science. On the other hand, it is hard to find an area that attained considerable importance without ancestors, be they a quite dispersed number of "eccentrics" or marginal persons or, on the contrary, a "visible college" with a manifest number of publication activities as in the theory of groups.

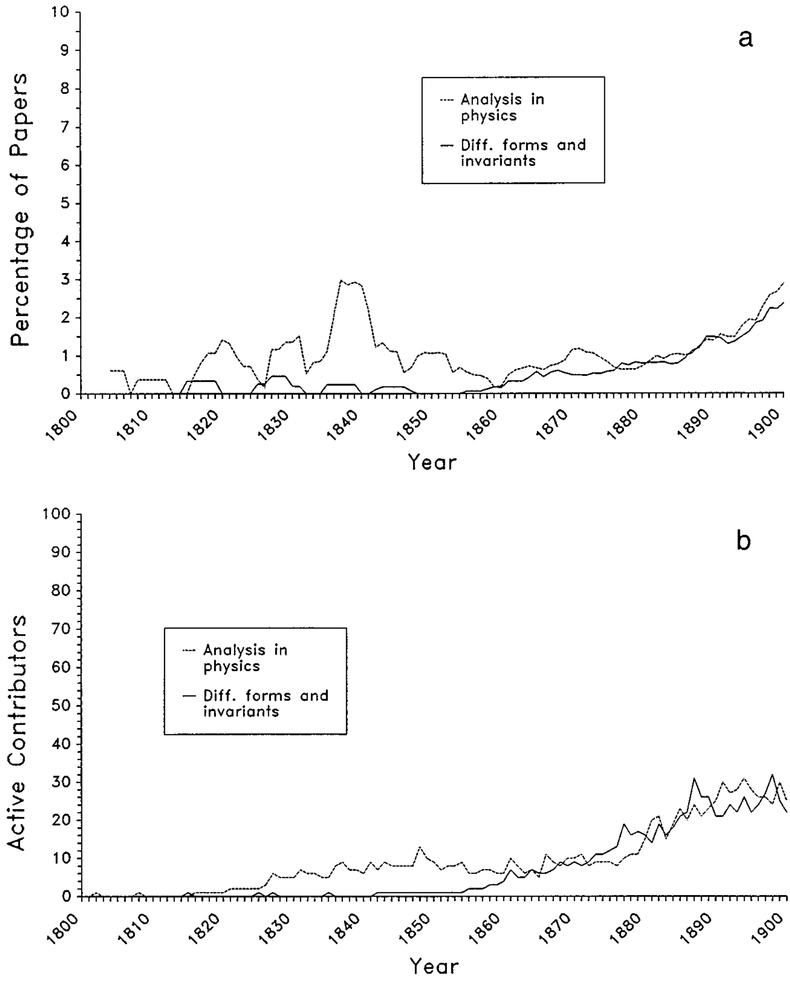

FIG. 17. (a) Percentage of papers and (b) number of active contributors in differential forms and invariants (5200–5240) and applications of analysis to physics (5600–5660).

We now turn to the second indicator of mathematical activity, that is, the manpower that an area is able to attract. By manpower, we understand any scientist working in an area. As the reader will remember, a rough indicator of the commitment of a mathematician to an area is the time interval between the first and the last year of his contribution to the area. Whether there is activity within that interval is not taken into account, however. The annual number of active contributors is plotted in Figs. 10–22 for every area without any moving averages. In general, the

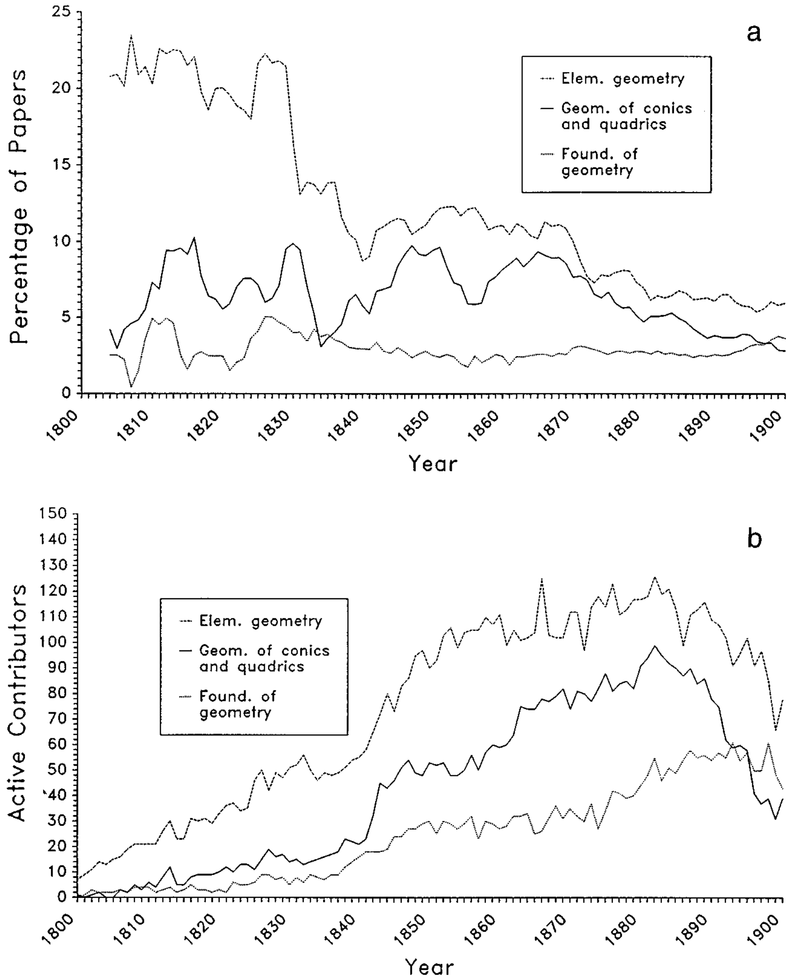

FIG. 18. (a) Percentage of papers and (b) number of active contributors in foundations of geometry (6400–6430), elementary geometry (6800–6840), and geometry of conics and quadrics (7200–7260).

curves oscillate much less than the publication curves or the curves of the percentage of an area in relation to all publications without moving averages. The same holds for mathematical logic in the 20th century. But as in mathematical logic, some areas show "premature" activity: the scientists involved here do not find themselves in a continuing flow of work or are unable to lay down the foundations for such a flow. We can only speculate about how many areas have definitely failed; the retrospective observer remembers only the survivors. In about one-third of the

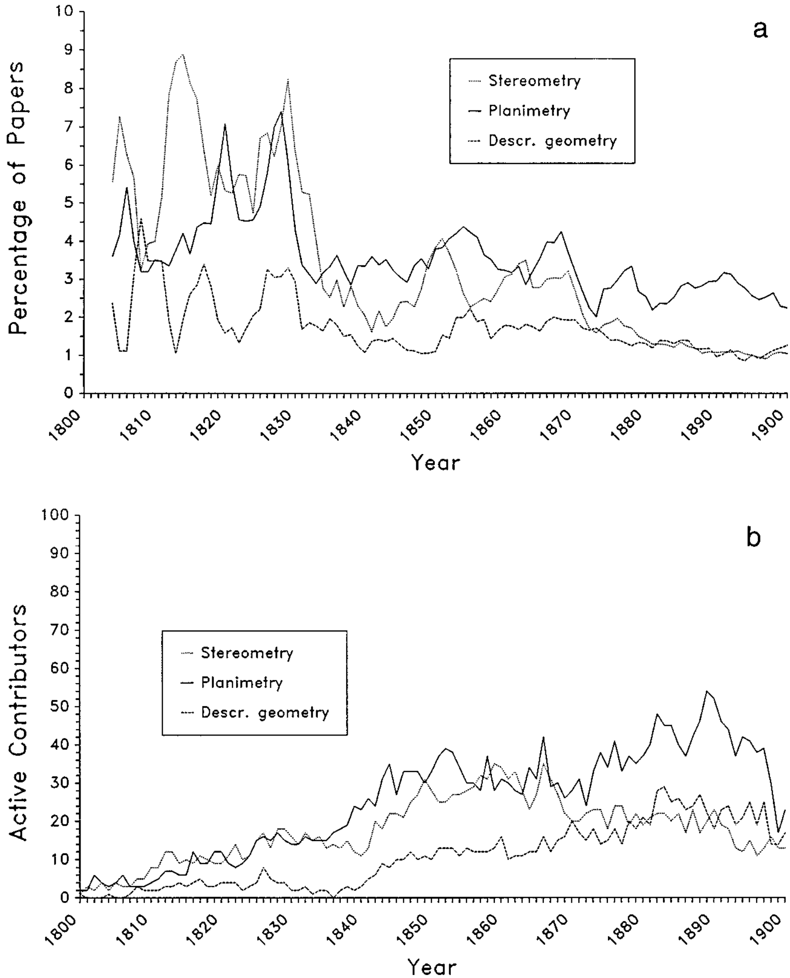

FIG. 19. (a) Percentage of papers and (b) number of active contributors in planimetry (6810), stereometry (6820), and descriptive geometry (6840).

examples, one can observe "premature" activity in the above-mentioned sense. It may be because of the novelty of mathematical logic that the percentage of that kind of activity was higher there.

Almost all areas could participate in the expansion of mathematics as measured by the absolute number of contributors. The size of the areas in terms of contributors is quite similar to areas of logic in the 20th century. Equally divergent is the appearance of the curves. In many cases only a linear growth takes place, in other

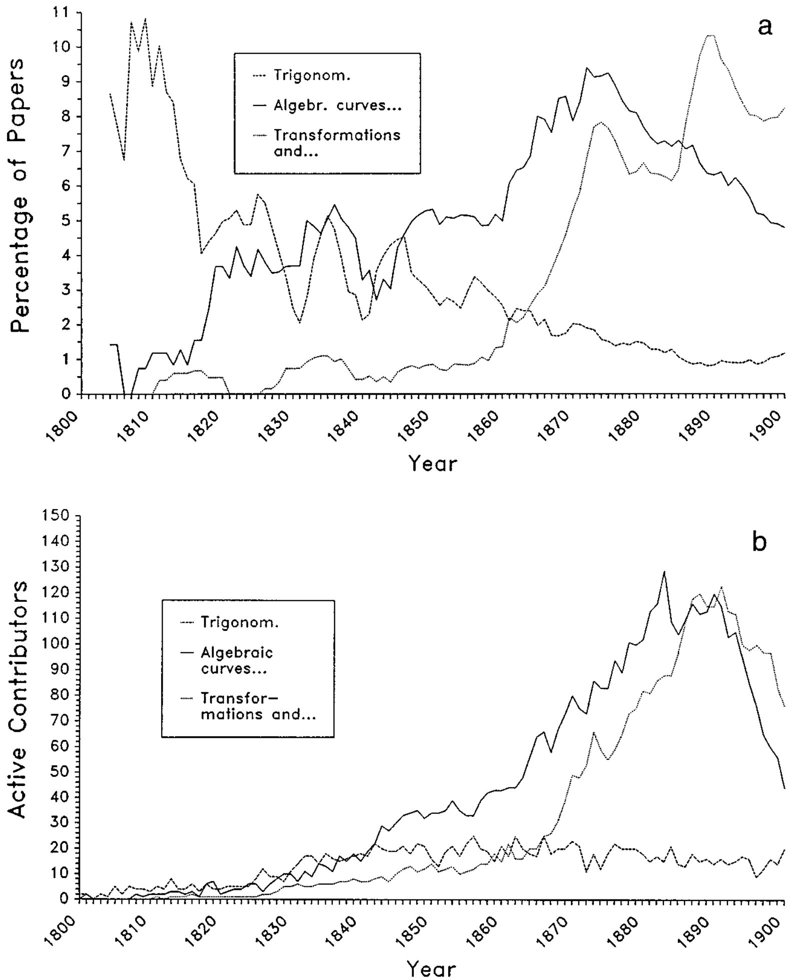

FIG. 20. (a) Percentage of papers and (b) number of active contributors in trigonometry (6830), algebraic curves and surfaces of degree >2 (7600–7660), and transformations and algebraic configurations (8000–8100).

cases the form of the curve is reminiscent of the classical S-form of logistic growth. In some cases a phase of linear growth is succeeded by a phase of exponential growth, which again is succeeded by a linear phase and so on, a feature which holds true even for the whole discipline.

A comparison between the growth of publications in an area and the growth of active contributors shows that there are great differences. A curve of contributors

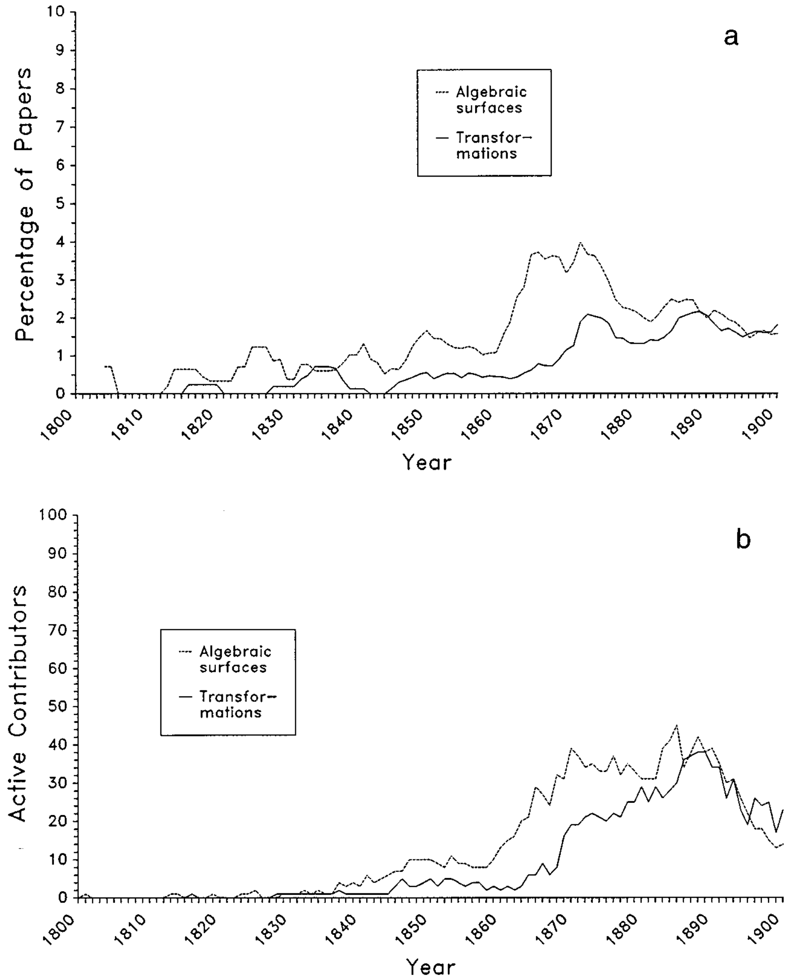

FIG. 21. (a) Percentage of papers and (b) number of active contributors in algebraic surfaces (7640, 7650) and transformations (8010, 8020).

may indicate no major event while the corresponding publication curve may show sudden eruptions of activity. From a statistical perspective, inside those waves of activity a quite regular pattern of scientific output can be detected; in such a wave there seem to exist, as a rule, typical frequency distributions of scientific activity. The frequency distributions considered above were computed as averages comprising only different states of the areas of the whole discipline. But distributions of that type, computed for moving 10-year intervals in the development of an area,

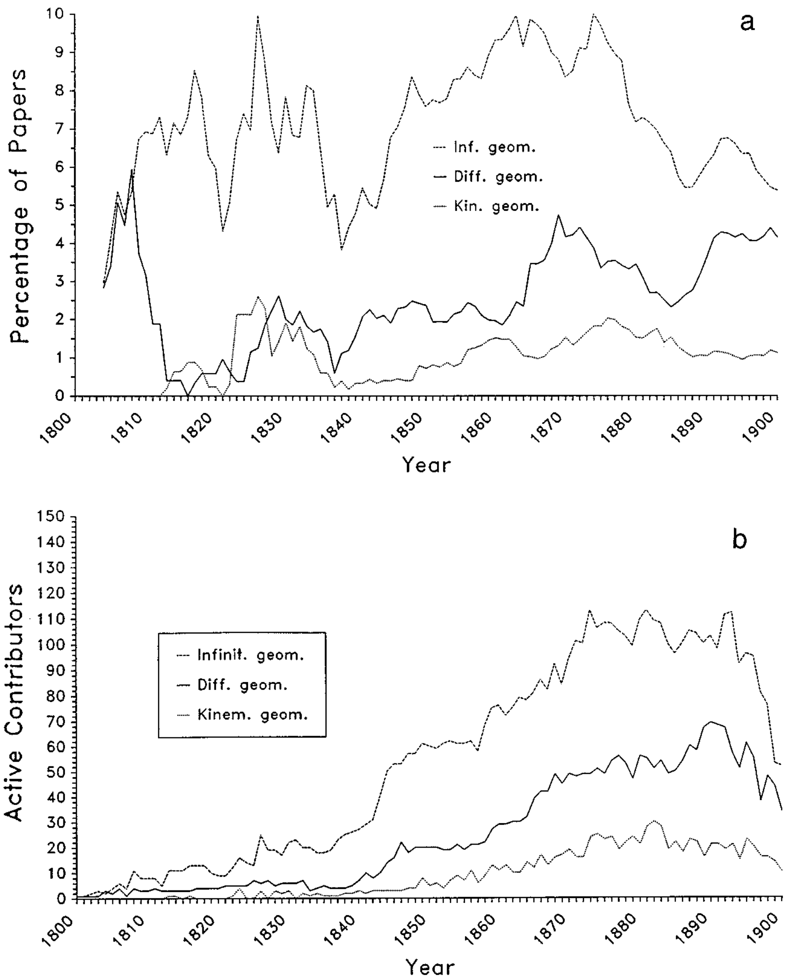

FIG. 22. (a) Percentage of papers and (b) number of active contributors in infinitesimal geometry (8400–8490), kinematic geometry (8420), and differential geometry (8800–8870).

can be used as a diagnostic instrument to distinguish between developments based on the isolated activity of some individuals and developments embedded in a multitude of related activity connected with the nucleus of a prolific elite.[12] It is this cluster-like structure which makes a process of scientific innovation comparable to technological innovation.

[12] Detailed analyses are under preparation by the authors [20].

APPENDIX

TABLE II

ANNUAL NUMBER OF PUBLICATIONS, MATHEMATICS 1800–1900

Year	Papers	Year	Papers	Year	Papers	Year	Papers
1800	16	1826	127	1851	270	1876	626
1801	28	1827	100	1852	280	1877	641
1802	33	1828	105	1853	311	1878	703
1803	17	1829	104	1854	272	1879	721
1804	18	1830	126	1855	281	1880	739
1805	35	1831	121	1856	322	1881	693
1806	48	1832	105	1857	371	1882	857
1807	20	1833	67	1858	381	1883	917
1808	54	1834	90	1859	362	1884	758
1809	29	1835	83	1860	393	1885	735
1810	45	1836	95	1861	350	1886	808
1811	50	1837	147	1862	345	1887	988
1812	45	1838	116	1863	390	1888	851
1813	102	1839	136	1864	373	1889	806
1814	45	1840	129	1865	314	1890	844
1815	59	1841	201	1866	494	1891	841
1816	84	1842	187	1867	356	1892	841
1817	60	1843	274	1868	398	1893	923
1818	79	1844	269	1869	397	1894	806
1819	59	1845	288	1870	440	1895	957
1820	58	1846	363	1871	514	1896	849
1821	53	1847	342	1872	441	1897	922
1822	66	1848	264	1873	606	1898	919
1823	55	1849	255	1874	462	1899	924
1824	58	1850	328	1875	504	1900	1107
1825	77						

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