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Fuzzy Petri nets for knowledge representation and reasoning: A literature review

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ABSTRACT

Fuzzy Petri nets (FPNs) are a potential modeling technique for knowledge representation and reasoning of rule-based expert systems. To date, many studies have focused on the improvement of FPNs and various new algorithms and models have been proposed in the literature to enhance the modeling power and applicability of FPNs. However, no systematic and comprehensive review has been provided for FPNs as knowledge representation formalisms. Giving this evolving research area, this work presents an overview of the improved FPN theories and models from the perspectives of reasoning algorithms, knowledge representations and FPN models. In addition, we provide a survey of the applications of FPNs for solving practical problems in variety of fields. Finally, research trends in the current literature and potential directions for future investigations are pointed out, providing insights and robust roadmap for further studies in this field.

1. Introduction

Fuzzy Petri nets (FPNs) are a modification of classical Petri nets (PNs) for dealing with imprecise, vague or fuzzy information in knowledge based systems, which have been extensively used to model fuzzy production rules (FPRs) and formulate fuzzy rule-based reasoning automatically. An FPN is a marked graphical system containing places and transitions, where graphically circles represent places, bars depict transitions, and directed arcs denote the incidence relationships from places to transitions or from transitions to places. The main characteristics of an FPN are that it supports structural organization of information, provides visualization of knowledge reasoning, and facilitates design of efficient fuzzy inference algorithms. All these render FPNs a potential modeling methodology for knowledge representation and reasoning in expert systems (Chen et al., 1990; Liu et al., 2013a; Yeung and Tsang, 1994a).

Since the introduction of FPNs for supporting approximate reasoning in a fuzzy rule-based system (Looney, 1988), they have received a great deal of attention from academics and practitioners in the domain of artificial intelligence. However, the earlier FPNs, as indicated in the academic literature, are plagued by a number of shortcomings, and are not suitable for increasingly complex knowledge-based systems. Therefore, a variety of alternative models have been put forward in the literature to enhance the knowledge representation power of FPNs and to implement the rule-based reasoning more intelligently. Besides, FPNs have been widely used by researchers and practitioners to manage different kinds of engineering problems in many fields. To the best of our knowledge, however, no research is found to present a thorough review on FPNs as a knowledge representation formalism. This paper aims to summarize and analyze the existing approaches to enhance the performance of FPNs, and further introduce in depth the applications of FPNs to solve real-world problems. Related articles published in international journals between 1988 and 2016 are gathered and reviewed. The specific objectives of this review are:

- To establish sources of improvements around FPNs and identify those aspects that attract the most attention in the FPN literature.
- To describe the development of FPNs and find the approaches that are prevalently applied.
- To uncover gaps and trends in the current FPN literature and highlight future directions for research.

This study not only provides evidence that some alternative models are better than former FPNs, but also aids both practitioners and researchers in applying FPNs more effectively. The paper's goal is to also provide a spur to further study this area in depth and develop

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richer knowledge on FPNs to help industrialists build effective expert systems for intelligent decision making.

The rest of this paper is organized in the following way. First, some background knowledge regarding FPRs and FPNs, and the major aspects of research on FPNs are presented in Section 2. Section 3 reviews the improved FPN approaches from the perspectives of reasoning algorithms, knowledge representations and FPN models. In Section 4 we introduce the applications of FPNs in different engineering areas. Section 5 describes some general observations based on statistical analysis results of this review. Section 6 discusses the main findings of this literature survey and gives suggestions for the future work. Finally, Section 7 concludes the paper.

2. FPRs and FPNs

2.1. FPRs

FPRs have been comprehensively used to represent, capture and store vague expert knowledge in decision systems. Each rule is usually expressed in the form of a fuzzy if-then rule in which both the antecedent and the consequent are fuzzy terms expressed by fuzzy sets. If an FPR consists of either AND or OR connectors, then it is called a composite or compound FPR (Chen, 1996).

To enhance the representation and reasoning capabilities of FPRs, the weight parameter (Tsang et al., 2004; Yeung and Tsang, 1997) has been incorporated into fuzzy if-then rules, obtaining the weighted FPRs (WFPRs). Let R be a set of WFPRs, i.e.,$R = \{R_1, R_2, \ldots, R_n\}$, the form of the ith rule can be presented as

$$R_i\text{: IF } a \text{ THEN } c \text{ (CF} = \mu), \quad Th, w \tag{1}$$

where a and c are the antecedent and consequent parts of the rule, respectively, which comprise one or more propositions with fuzzy variables. The parameter $\mu\,(\mu \in [0, 1])$ is the certainty factor indicating the belief strength of the rule,$Th = \{\lambda_1, \lambda_2, \ldots,\lambda_m\}$ is a set of threshold values specified for each of the propositions in the antecedent, and $w = \{w_1, w_2, \ldots, w_m\}$ is a set of weights assigned to all propositions in the antecedent, showing the relative importance of each proposition in the antecedent contributing to the consequent.

In general, WFPRs can be divided into five types as listed below (Ha et al., 2007; Liu et al., 2013a; Yeung and Ysang, 1998):

Type 1. A simple weighted fuzzy production rule

R: IF a THEN c (μ; λ; w)

Type 2. A composite weighted fuzzy conjunctive rule in the antecedent

R: IF a_1 AND a_2 AND...AND a_m THEN c (μ; $\lambda_1, \lambda_2, \ldots,\lambda_m$; $w_1, w_2, \ldots,w_m$)

Type 3. A composite weighted fuzzy conjunctive rule in the consequent

R: IF a THEN c_1 AND c_2 AND...AND c_m (μ; λ; w)

Type 4. A composite weighted fuzzy disjunctive rule in the antecedent

R: IF a_1 OR a_2 OR...OR a_m THEN c (μ; $\lambda_1, \lambda_2, \ldots,\lambda_m$; $w_1, w_2, \ldots,w_m$)

Type 5. A composite weighted fuzzy disjunctive rule in the consequent

R: IF a THEN c_1 OR c_2 OR...OR c_m (μ; λ; w).

In many practical applications, the rules of Types 4 and 5 are not allowed to appear in a knowledge base since they can be transferred into several rules of Type 1. The following rules are several typical examples of WFPRs:

R_1: IF it is hot THEN the humidity is low (μ=0.9);
R_2: IF John is fat AND John is tall AND John is a man THEN he is heavy (μ=1.0);
R_3: IF fever is high AND cough is heavy AND blood pressure is normal THEN pneumonia (μ=0.8);
R_4: IF regulator semiconductor is broken THEN exciter is not enough (μ=0.9; λ=0.2; w =1.0);
R_5: IF frequency is higher than normal value AND double frequency is smaller than normal value AND amplitude changes obviously as the loads change THEN rotor is hot bending (μ=0.9; λ_1 =0.3, λ_2 =0.3, λ_3 =0.2; w_1 =0.5, w_2 =0.3, w_3 =0.2).

It is worth noting that R_4 and R_5 are WFPRs derived from the fault diagnosis of aircraft generator (Liu et al., 2016a).

2.2. PNs and FPNs

PNs are a graphical and mathematical modeling method used to model and analyze discrete event systems (Cassandras and Lafortune, 2008; Li et al., 2012a, 2012b) such as communication, manufacturing and transportation systems. Tokens in the places represent the state of a system (Chen et al., 2014b; Li and Zhao, 2008; Zhang et al., 2015). A PN is formally defined as a 5-tuple (Murata, 1989):

$$\text{PN} = (P, T, F, W, M_0) \tag{2}$$

where P and T are finite sets of places and transitions, respectively, the flow relation between P and T is denoted by $F \subseteq (P \times T) \cap (T \times P)$, W: $F \to \{0, 1, 2, \ldots\}$ is a weight function, and M_0: $P \to \{0, 1, 2, \ldots\}$ is the initial marking. A PN example is shown in Fig. 1(a), where $P = \{p_1, p_2\}$, $T = \{t_1\}$,$F = \{(p_1, t_1), (t_1, p_1)\}$ and its initial marking is $M_0 = [3 \quad 0]^T$ at which t_1 is enabled. After t_1 fires, one token is removed from its input place, i.e., p_1, and deposited into its output place, i.e., p_2.

To deal with uncertainty in knowledge representation and reasoning, FPNs have been developed from the PN theory, where tokens representing the state of propositions are marked by a truth value between 0 and 1. By applying a PN formalism to fuzzy rule-based systems, it is able to visualize the structure of an expert system and express its dynamic proposition logic reasoning behavior efficiently. For example, in Fig. 1(b), we have $P = \{p_1, p_2\}$, $T = \{t_1\}$, $I(t_1) = \{p_1\}$,$O(t_1) = \{p_2\}$, $f(t_1) = \mu_1$, $\alpha(p_1) = \alpha_1$, and $\alpha(p_2) = 0$ based on the basic FPN defined in Eq. (3). For an FPN, a transition is said to be enabled if all of its input places are marked by a token and its real value is greater than or equal to a threshold value. The reasoning process of an FPN is executed by firing the rules and updating the truth degree vector at each reasoning step.

Due to the features of fuzzy rule-based systems, the major differences between PNs and FPNs are as follows (Gao et al., 2003; Hanna et al., 1996; Hu et al., 2011):

(1) In FPNs, the number of tokens in a place cannot be greater than one since a token is associated with a truth value between 0 and 1. A token does not represent an "object," whereas it may likely do so in PNs.
(2) FPNs are always conflict-free nets because there is no "resource" concept in FPNs and a proposition may be shared by different rules at the same time. For example, in Fig. 2, the proposition d_3 is shared by two rules R_1 and R_2, which can utilize proposition d_3 simultaneously and reason in parallel.
(3) The tokens are not removed from the input places of a transition after it fires since the evaluation of the rules means the truth propagation of the propositions only. That is, the antecedent part remains verified although its consequent part may already be proved in the knowledge reasoning.

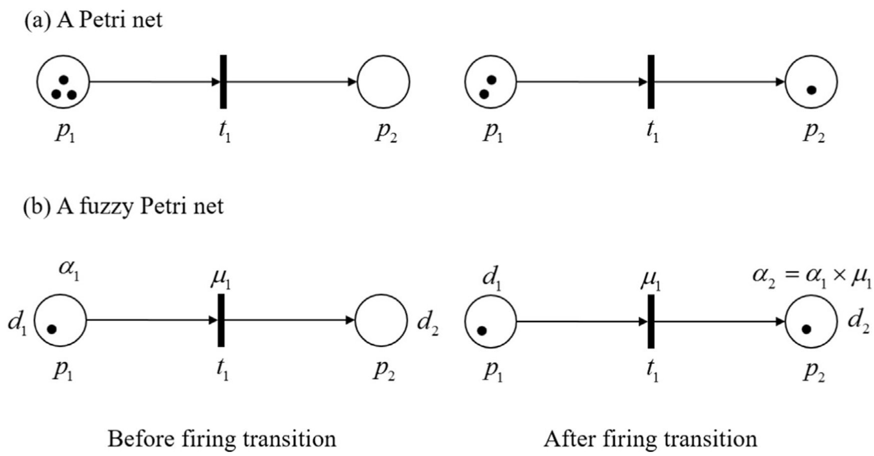

Fig. 1. An illustration of PN and FPN.

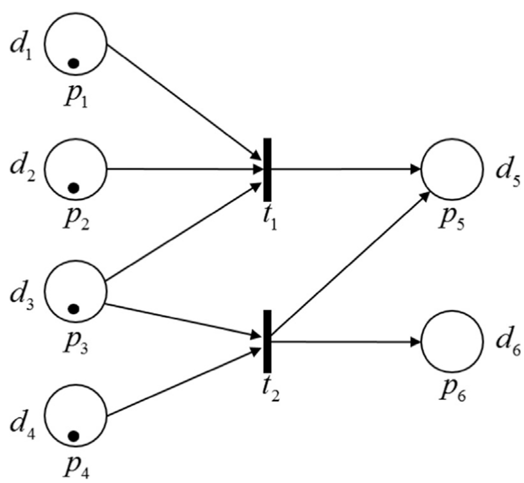

Fig. 2. An FPN example with a shared proposition.

2.3. Definitions of FPNs

In 1988, Looney (1988) pioneered the concept of FPNs to represent the FPRs of a rule-based decision making system. In a later work, Chen et al. (1990) proposed a more generic FPN model to model knowledge representation and described a fuzzy algorithm to perform knowledge reasoning automatically.

According to (Chen et al., 1990), an FPN structure is defined as an 8-tuple:

$$FPN = (P, T, D, I, O, f, \alpha, \beta) \tag{3}$$

where

$P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places,
$T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions,
$D = \{d_1, d_2, \ldots, d_m\}$ is a finite set of propositions with $P \cap T \cap D = \varnothing$, $|P| = |D|$,
I: $T \to P^\infty$ denotes the input function, a mapping from transitions to the bags of places,
O: $T \to P^\infty$ denotes the output function, a mapping from transitions to the bags of places,
f: $T \to [0, 1]$ denotes an association function, a mapping from transitions to real values between 0 and 1,
α: $P \to [0, 1]$ is an association function, a mapping from places to real values between 0 and 1,
β: $P \to D$ is an association function, a bijective mapping between places and propositions.

To capture more information of WFPRs, Yeung and Ysang (1998) improved the above FPN model by introducing the knowledge parameters of threshold values and weights, and an improved FPN can be presented as follows:

$$FPN = (P, T, D, Th, I, O, F, W, f, \alpha, \beta, \gamma, \theta) \tag{4}$$

where

P, T, D, I, O, f and β are defined as in (3).
$Th = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ is a set of threshold values,
$F = \{f_1, f_2, \ldots, f_m\}$ is a set of fuzzy sets,
$W = \{w_1, w_2, \ldots, w_m\}$ is a set of weights of WFPRs,
α: $P \to F$ is an association function which assigns a fuzzy set to a place,
γ: $P \to Th$ is an association function, a mapping from places to threshold values,
θ: $P \to W$ is an association function which assigns a weight to a place.

In an FPN, propositions are represented by places; the certainty factor of a rule is associated with its corresponding transition; the mutual causality interconnections between the propositions and reasoning rules are expressed by the arcs between places and transitions. A place may or may not contain a token associated with a truth value between 0 and 1. The token is pictorially represented by a dot. The knowledge reasoning processes are modeled through the firing of the transitions in FPNs. Based on the above specification, the three type of WFPRs can be graphically represented with FPN structures as depicted in Fig. 3.

2.4. Sources of FPN improvements

FPNs have been proven to be one of the most important knowledge representation tools; however, the original FPNs have been criticized extensively in the literature for various reasons. Generally, FPNs have been improved from the perspectives of reasoning algorithms, knowledge representations, and FPN models. First, the reasoning algorithms based on the classical FPNs were improved to be suitable for more generic FPN forms. Then, the FPN model has been enhanced to carry more knowledge information. In recent years, various versions of FPN models have been developed considering the increasing complexity of expert systems. Based on the reviewed papers, a variety of disadvantages of FPNs, depending on specific applications, are gradually recognized (See Table A1) and the most important ones are stated as follows:

(1) The fuzzy reasoning algorithms of many FPNs are implemented

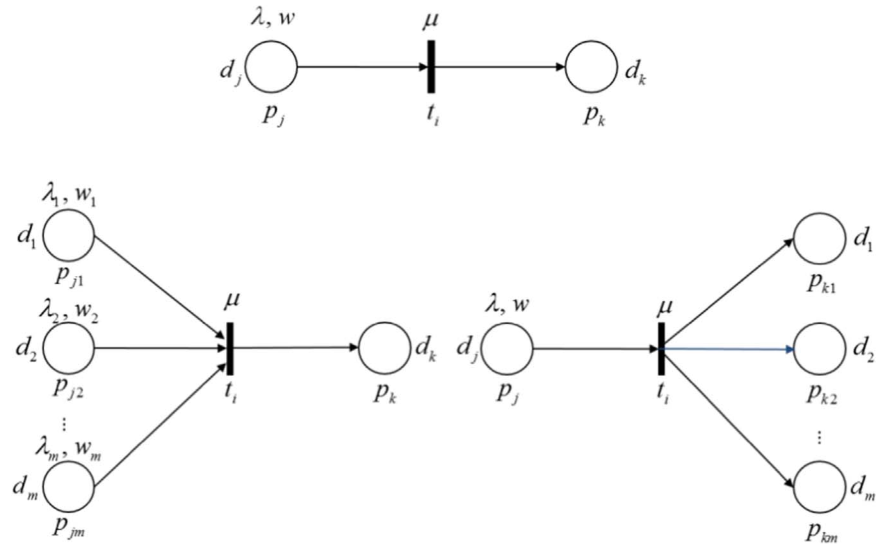

Fig. 3. FPN representations of WFPRs.

using a reachability tree-based method such that they are not suitable for parallel reasoning (Chun and Bien, 1993; Gao et al., 2003; Liu et al., 2013a, 2013c, 2016a). Such methods often require the enumeration of all possible paths in order that the final truth degrees can be properly evaluated, which, however, leads to less efficient reasoning algorithms. Large reachability sets and adjacent places tables may be generated when applying the reachability tree-based model to represent a rather complex expert system (Yeung and Tsang, 1994a).

(2) Some FPNs lack an adjustment (learning) mechanism according to the system's changes (Amin and Shebl, 2014; Feng et al., 2012; Li and Lara-Rosano, 2000; Li et al., 2000; Pedrycz and Gomide, 1994; Wang et al., 2014). The parameters (or weights) in the FPN model of (Yeung and Ysang, 1998) are fixed. The restrictions of the learning algorithm in (Li and Lara-Rosano, 2000) are overly strict for an expert system. The lack of an adjustment (learning) mechanism in FPNs cannot cope with potential changes of actual systems. FPNs are not adaptable according to the changes of the arc weight.

(3) A proposition is assigned to only one weight and the weight is assigned to its place in the FPN model (Ha et al., 2007; Liu et al., 2013a, 2013c), which is unreasonable in the situation that a proposition is shared by different rules at the same time. Accordingly, the same place with different transitions has the same weight after mapping FPRs into FPNs. The relative weight of each proposition in the antecedent contributing to the consequent is ignored or assumed to be equal (Li and Lara-Rosano, 2000; Yeung and Ysang, 1998).

(4) A proposition or rule is generally assigned to only one threshold value and the threshold value is assigned to its place or transition in the FPN model (Liu et al., 2013a, 2013c, 2016a; Yeung and Tsang, 1994b; Yeung and Ysang, 1998). Some FPN models even do not consider threshold values or only assign a single value to all FPRs.

(5) The backward reasoning is not handled (Chen, 2000; Chun and Bien, 1993; Hu et al., 2011; Liu et al., 2013b) using FPN theory. The original fuzzy reasoning methods can deal with fuzzy forward reasoning only and cannot deal with fuzzy backward reasoning.

(6) The fuzzy reasoning algorithm in (Chen et al., 1990) is unable to carry out weighted fuzzy reasoning (Chen, 2002). Fuzzy reasoning algorithms proposed in the literature are unable to perform ordered weighted linguistic reasoning (Liu et al., 2016b). The algorithm presented in (Liu et al., 2013a) does not consider the global weights of FPRs (Liu et al., 2016a, 2016b). A rule-based expert system will be more flexible if both local and global weights are considered in knowledge inference.

(7) Time factor is not introduced in previous FPNs (Liu et al., 2016a, 2016b, 2011; Suraj and Fryc, 2006), which plays a vital role in developing real-time expert systems.

(8) In the situation that the consequent part has two or more propositions in a rule, the whole rule assigns only one certainty factor to its transition in the FPN model (Ha et al., 2007; Liu et al., 2013a, 2013c). In fact, when a rule contains two or more conclusions, the influence of the transition to its output places may be different.

(9) The knowledge representation parameters, such as the truth degrees of propositions and the certainty factors of fuzzy rules, are restricted to be real values between 0 and 1 (Chen, 2002; Liu et al., 2016b). However, knowledge parameter values often take the form of intuitionistic fuzzy information (Liu et al., 2016a).

(10) The multiplication rule evaluation method may result in no conclusion for complicated expert systems (Yeung and Tsang, 1994a; Yeung and Ysang, 1998). In the reasoning algorithm proposed in (Chen et al., 1990), the dependence of the truth degree of the concluding place on the length of the reasoning paths is undesirable (Konar and Mandal, 1996).

3. Approaches to FPNs

In this section, we present the results of a comprehensive literature search on FPNs for knowledge representation and reasoning. The database used for our study is Scopus (http://www.scopus.com) in which the articles published between 1988 and 2016 were searched. The database search was limited to peer-reviewed articles appearing in academic journals, since the acceptance of the scientific community is most convincingly guaranteed via the peer-review process (Schmoch and Schubert, 2008). Publications in languages other than English and non-refereed professional publications, such as textbooks, doctoral dissertations and conference proceedings, are excluded from our examination. Furthermore, we only include articles that report on an algorithm or a model to represent inexact knowledge and approximate reasoning, or studies applying the existing FPN models for dealing with practical problems. This implies that articles reporting on FPN models which are not used as decision support or monitoring systems (e.g., (Gniewek, 2013; Gniewek and Kluska, 2004)) are excluded. Also, more complex, higher-level net-based structures applied to model knowledge reasoning by using high-level FPNs (HLFPNs) (Scarpelli et al., 1996; Shen, 2006) have not been considered within this study, since they are another trend of research on Petri nets. The literature analysis begins by identifying 546 published studies with reference to FPNs, which are then distilled down to 99 papers satisfying the selection criteria.

To enhance the performance of existing FPNs, a number of studies have been conducted in the literature through improving knowledge representation and reasoning abilities of FPNs or developing new versions of FPN models. Therefore we propose a framework for classifying the reviewed papers from the perspectives of reasoning algorithms, knowledge representations and new FPN models. Among the reviewed papers, some studies improved the reasoning algorithms of FPNs for modeling complex rule-based decision making systems (seven papers). Also, the ability of FPNs in knowledge representation has been enhanced by incorporating more knowledge parameters (six papers). In addition, different types of FPN models have been developed to meet the requirements of their problems (29 papers). In what follows, we more specifically go into the references and show what has been done.

3.1. Reasoning algorithms

3.1.1. Reachability tree-based algorithms

Yeung and Tsang (1994a) proposed a modified fuzzy reasoning algorithm to enhance the reasoning capability of Chen's algorithm (Chen et al., 1990). Two additional algorithms were also proposed by the authors for building reachability sets and adjacent places tables when applying the proposed fuzzy reasoning algorithm. To deal with the problems with Chen's algorithm, Manoj et al. (1998) proposed a modified fuzzy reasoning algorithm and the concept of hierarchical FPNs for data abstraction. Yeung and Ysang (1998) devised an FPR evaluation method (FPREM) by taking the weight factors into consideration, and developed a multilevel weighted fuzzy reasoning algorithm incorporating the FPREM for expert systems. In (Chen, 2000), the author extended the work of (Chen et al., 1990) to describe a fuzzy backward reasoning algorithm for knowledge based systems, where FPNs are used for representing FPRs in the knowledge base of an expert system.

3.1.2. Algebraic representation-based algorithms

In (Fryc et al., 2004), the authors proposed an algebraic (matrix) representation of FPNs and provided a parallel algorithm for the fuzzy reasoning process in knowledge based systems. Suraj (2013) extended the FPNs by introducing three operators in the form of triangular norms as substitutes of min, max and algebraic product operators, and demonstrated a new class of Petri nets called generalized FPNs (GFPNs) for knowledge representation and inexact reasoning in decision support systems. To overcome the issue of state space explosion, Zhou et al. (2015) proposed a biphasic decomposition algorithm that includes a backward search stage and a forward strategy for the FPN model. This algorithm is able to divide a large-scale FPN model into a series of completed reasoning paths (sub-FPN models) via an index function and an incidence matrix.

3.2. Knowledge representations

In Yeung and Tsang (1994b), the authors proposed an enhanced FPN model to accommodate the possibility of mapping an FPR having different threshold values in their propositions into an FPN. The model proposed is able to represent more information of an FPR. By assigning a different threshold value for each proposition, misfiring of rules can be prevented. Chen (2002) presented a weighted FPN (WFPN) model and a weighted fuzzy reasoning algorithm for rule-based systems using WFPNs. The FPRs in the knowledge base of a rule-based expert system are modeled by WFPNs, where the weights of the propositions in the rules are represented by fuzzy numbers. Ha et al. (2007) developed two types of knowledge representation parameters, i.e., input weights and output weights, and introduced a generalized FPN (GFPN) to enhance the representation capability of WFPRs in a rule-based system. Furthermore, the evaluation method of the multilevel fuzzy reasoning in (Yeung and Ysang, 1998) was improved by using these parameters and a similarity measure.

Shih et al. (2007) reported a modified PN model, associative Petri net (APN), to represent the associative production rules of a rule-based system and developed an efficient reasoning algorithm based on the APN model. In the APNs, a novel associated parameter of every transition production rule was introduced, which generalizes the notion of certainty factor of a rule by measuring the associative degree between different propositions. The work in (Liu et al., 2013c) introduced a knowledge acquisition and representation approach using a fuzzy evidential reasoning (FER) approach and dynamic adaptive FPNs (DAFPNs). In the DAFPNs, the weights of propositions are assigned to each input arc of a transition, the rule certainty factor is replaced by several output certainty factors given to the output arcs of a transition, and different threshold values are assigned to each proposition in the consequent part of a composite production rule. Based on the work in (Liu et al., 2013c), Liu et al. (2013a) further presented an improved DAFPN model for knowledge representation and reasoning, in which distinct threshold values are assigned not only to the consequent propositions of a composite production rule but also to each antecedent proposition of the rule. Furthermore, a max-algebra based parallel reasoning algorithm was proposed based on the DAFPNs to implement approximate reasoning process automatically.

3.3. New FPN models

3.3.1. FPNs combing PNs and fuzzy logic

Garg et al. (1991) investigated a new type of FPNs as a knowledge representation formalism, and reported an algorithm to check the consistency of a fuzzy knowledge base by a set of reduction rules that preserve the properties of the established FPN. Based on the PN formalism, Bugarin and Barro (1994) developed a model for the representation of fuzzy production systems with rule chaining, which permits the development of algorithms for an efficient and flexible execution of knowledge bases. Cao and Sanderson (1995a) presented a generalized definition of FPNs using three types of fuzzy variables (i.e., local fuzzy variables, fuzzy marking variables, and global fuzzy variables) to model discrete event systems with vague, random, and approximate information. Konar and Mandal (1996) discussed two distinct models of FPNs for reasoning in expert systems in presence of imprecision and inconsistency of data and uncertainty of knowledge: One model deals with the computation of precision degree of any proposition based on the fuzzy beliefs of independent starting propositions; the other is concerned with the computation of steady-state fuzzy beliefs of the propositions in the given network from their initial fuzzy beliefs. By combining PN theory and fuzzy sets, Koriem (2000) presented a formal technique called modified FPNs (MFPNs) for automated modeling and verification of ruled-based decision making systems. Bostan-Korpeoglu and Yazici (2007) proposed an FPN model to represent imprecise knowledge and the behavior of an intelligent object-oriented database environment. This model is capable of dealing with both active and deductive rules along with the compositions (either event or condition), and performing some required computations, such as fuzzification, concurrent execution, and combination, in addition to the sup-min composition.

3.3.2. FPNs considering time factor

In (Pang et al., 1995), the authors proposed a continuous FPN (CFPN) by integrating fuzzy control, PNs and real-time expert systems. The CFPN approach can handle real-time continuous inferencing for the purpose of process monitoring, control and diagnostics. In (Carinena et al., 1999), the authors introduced an enhanced model of fuzzy temporal knowledge bases (FTKBs) including time as a variable in their fuzzy temporal rules, and presented its projection and execution onto the formalism of FPNs. The model proposed can enhance the expressive capability of the rules making up the FTKB and keep the computation cost within the limits to work on-line in dynamic environments. Suraj and Fryc (2006) proposed a new class of timed approximate Petri nets (TAP-nets) by combining FPNs with time and uncertain information, which can be used for representing uncertain knowledge and for evaluating inexact reasoning in decision systems. For knowledge representation of chemical abnormality, a novel temporal version of FPNs, designated timed FPN (tFPN), was introduced in (Liu et al., 2011). In the tFPN approach, a timing factor was assigned to each transition and a reliability degree was associated with each place to capture the dynamic nature of fuzzy knowledge pertaining to abnormal events. Following a procedure towards abnormal event monitoring, two efficient algorithms for abnormality prognostication and diagnosis were also proposed by reachability analysis of tFPN.

3.3.3. FPNs based on possibility logic

Cardoso et al. (1999) combined possibility logic with PN theory and proposed possibilistic PNs (PPN) for the qualitative representation of uncertain knowledge about a system state. Lee et al. (2003) dealt with a PPN model that integrated PNs with possibilistic reasoning to lead to a tool for model uncertainty reasoning in rule-based expert systems. A reasoning algorithm based on PPNs was also outlined to improve the efficiency of possibilistic reasoning. In (Lee et al., 1998), the authors put forward a novel version of FPNs for modeling fuzzy rule-based reasoning that brings together the possibilistic entailment and the fuzzy reasoning to handle uncertain and imprecise information. A reasoning algorithm consistent with both the rule-based reasoning and the execution of PNs was presented to improve the efficiency of vague reasoning.

3.3.4. FPNs using neural networks

Li and Lara-Rosano (2000) formulated an FPN model called adaptive FPNs (AFPNs) and developed its weight learning algorithm for dynamic knowledge representation and inference. This model has learning ability like neural networks, by learning the weights from the data given by experts. In (Li et al., 2000), the authors relaxed the restrictions of the AFPN model and introduced a modified back propagation learning algorithm for knowledge learning under generalized conditions. Feng et al. (2012) proposed a learning model tool, namely learning FPN (LFPN), for the construction of knowledge systems, in which the truth degree of a proposition can be learned by adjusting the arc's weight function. Also, a leaning algorithm that enables the LFPN to obtain the capability of learning FPRs through truth degree updating was proposed. In (Amin and Shebl, 2014), the authors developed an adaptive fuzzy higher order PN (AFHOPN) considering the weight changes of the arc in fuzzy reasoning process, which has the learning ability as neural networks and can be used for knowledge representation and dynamic reasoning. Instead of using neural networks, Wang et al. (2014) proposed a dynamic representation of fuzzy knowledge (DRFK) model based on FPNs and particle swarm optimization for knowledge representation and inference. In this model, an efficient genetic particle swarm optimization learning algorithm was used for self-learning of fuzzy knowledge representation parameters. In addition, other neural network-based FPNs also appeared in other literatures such as Pedrycz and Gomide (1994), Ahson (1995), and Konar et al. (2005).

3.3.5. FPNs based on matrix operations

Chun and Bien (1993) proposed an FPN model for a rule-based decision making system which contains uncertain conditions and vague rules, and presented a matrix representation and state equation for the FPN. The reasoning method introduced in this paper can not only implement both forward and backward reasoning but also perform real-time decision making under a parallel rule firing scheme. In (Wang et al., 2001), the authors defined an extended FPN model (EFPN) based on generating rules of knowledge base and presented two concurrent reasoning algorithms based on multitask schedule by considering the reasoning patterns of forward reasoning and backward reasoning. An efficient algorithm was also designed for automated reasoning and decision making. Gao et al. (2003) elaborated upon a fuzzy reasoning PN (FRPN) model to represent a fuzzy rule-based system and designed a fuzzy reasoning algorithm to perform knowledge reasoning automatically. In the proposed FRPN, the negation issues in FPNs for knowledge reasoning are addressed and the algorithm exhibits fully parallel reasoning ability via adopting the operators in max-algebra. Lehocki et al. (2008) proposed logical PNs (LPNs) and FPNs as models for knowledge representation, based on which, the authors introduced a matrix-based algorithm for knowledge propagation in decision support systems. In (Hu et al., 2011), the authors proposed a particular kind of FPNs, namely reversed PNs, for solving the backward reasoning problems, and presented a max-algebra based iterative algorithm such that the backward reasoning can be implemented efficiently and automatically.

Recently, Liu et al. (2016b) extended the FPN model presented in (Liu et al., 2013a) to propose a linguistic reasoning PN (LRPN) model and developed an ordered weighted linguistic reasoning algorithm for knowledge representation and reasoning of a rule-based expert system, where the linguistic production rules in the knowledge base were modeled by LRPNs, and the truth degrees of the proposition appearing in the rules were represented by linguistic 2-tuples. Furthermore, both local and global weights of knowledge rules are included so as to enhance the representation power of FPNs. Liu et al. (2016a) presented a new type of FPNs, namely intuitionistic FPNs (IFPNs), by using intuitionistic fuzzy sets (IFSs) and ordered weighted averaging (OWA) operators to enhance the knowledge representation and reasoning capability of FPNs. Besides, a max-algebra-based reasoning algorithm was proposed in order to execute the intuitionistic fuzzy reasoning formally and automatically. Meng et al. (2016) also constructed an IFPN model for knowledge representation and reasoning by combining IFSs with PN theory, and the reasoning process based on IFPNs was carried out by matrix operation.

4. Applications of FPNs

Due to the graphical representation and dynamic processing ability, FPNs have been extensively employed to address various engineering problems over the past several decades. Therefore, practical applications of FPNs have been comprehensively investigated, and in the following, we describe the results in detail.

4.1. Operational management

4.1.1. Disassembly process planning

Gao et al. (2004) utilized an FRPN model to represent related disassembly rules in a product with uncertainty, and the proposed model can be used to efficiently attain the next operation on the product at each disassembly step based on the product's current status and disassembly rules. Considering human factors in manufacturing systems, Tang et al. (2006) developed a fuzzy attributed PN (FAPN) model to mathematically represent the uncertainty in disassembly process due to a large amount of human intervention. An algorithm was also proposed for obtaining the optimal disassembly planning for an obsolete product. Tang and Turowski (2007) proposed a fuzzy disassembly PN (F-DPN) model for modeling uncertain product/component conditions, and designed an adaptive fuzzy system with an iterative learning mechanism to dynamically estimate their impact on a disassembly process. Building upon the works of (Tang and Turowski, 2007; Tang et al., 2006), Tang (2009) further introduced an FPN model to explicitly represent and effectively analyze involved uncertainty in disassembly. Instead of presuming that the pertinent data is already known, the authors designed a self-adaptive disassembly process planner to accumulate the past experience of predicting such data and exploit the "knowledge" captured in the data to determine the best disassembly plan. In addition, Zhao et al. (2014) also applied the FRPNs to disassembly sequence decision making for the end-of-life product recycling and remanufacturing.

4.1.2. Operation planning and process control

Cao and Sanderson (1995b) discussed the problem of representation and planning of operations in a robotic assembly system by employing an FPN mapping strategy. In the FPN representation, the objects whose properties are altered are labeled 'soft' objects, the process steps where alternations may occur are termed 'key' transitions, and a prime number marking algorithm is used to guarantee consistent sequencing of operations. Wu et al. (2002) presented a modified FPN, named OPN, for optimal operation planning with resource constraints, which can model the knowledge for operation selection and yield feasible and optimal operation plans by the T-invariant property of FPNs. Hanna et al. (1996) proposed an intelligent process control architecture based on FPNs with neural networks for the modeling of product quality from a computer numerical control (CNC)-milling machining center, in which two fuzzy input variables (spindle speed and feed rate) are utilized to monitor and control the surface roughness quality of products manufactured by a milling operation. Kasirolvalad et al. (2006) presented an AND/OR nets approach for planning of a CNC machining operation and then employed adaptive FPNs (AFPNs) with learning capability to model the activities and events and improve the product and machining process quality within CNC machine tools.

4.1.3. Rescheduling, workflow management and product ecosystem design

Qiao et al. (2011) proposed an FPN-based model for describing the rescheduling strategy problem (FPN-R) and discussed a fuzzy reasoning approach for rescheduling start-up decision making and rescheduling methodology adoption. Ye et al. (2011) reported a knowledge-based hybrid exception handling approach for workflow management using two extended knowledge models: generalized fuzzy event-condition-action (GFECA) rule and typed FPN extended by process knowledge (TFPN-PK). Based on the TFPN-PK, a weighted fuzzy reasoning algorithm was designed to realize integrated representation and reasoning of fuzzy and non-fuzzy knowledge as well as application domain knowledge and workflow process knowledge. Zhou et al. (2012) developed an FRPN framework to deal with the uncertainty, complexity, and dynamics associated with user experience (UX) modeling for product ecosystem design. Reasoning of diverse constructs of UX was embedded in the FPRs derived from self-report UX data, and a fuzzy reasoning algorithm was proposed to perform parallel inference and to simulate most likely UX under different ambient factors.

4.2. Fault diagnosis and risk assessment

4.2.1. Electric power system

Sun et al. (2004) used FPNs as a modeling technique to construct fault diagnosis models of electric power systems, which aims to accurately diagnose faults when some incomplete and uncertain alarm information of protective relays and circuit breakers is detected. Luo and Kezunovic (2008) implemented FRPNs to tackle the complexity of power system fault section estimation and addressed several key issues including optimal structure design of diagnosis models to avoid a large matrix size, utilization of fuzzy logic parameters to effectively handle uncertainties, realization of a matrix execution algorithm to achieve parallel reasoning capability, and integration of more reliable input data to enhance estimation accuracy. He et al. (2014) exposed a dynamic fault diagnosis reasoning model based on AFPNs to solve the complex power system fault-section estimation problem, in which the weights in fuzzy reasoning are decided by the incomplete and uncertain alarm information of protective relays and circuit breakers. In (Zhang et al., 2016), the authors investigated the temporal constraint between event occurrences in power systems and introduced a temporal reasoning FPN (TRFPN) approach for fault diagnosis. The work in (Cheng et al., 2015a) presented a fault diagnosis method based on FPNs considering service feature of information source devices and applied it to diagnose faults of power supply system devices. To efficiently detect fraudulent and abnormal consumption, Chen et al. (2015) employed fractional-order self-synchronization error (FOSE)-based FPNs to locate nontechnical losses and outage events in microdistribution systems dealing with power utilities. Yang and Huang (2002) introduced an FPN knowledge representing approach to achieve the on-line service-restoration plan of distribution systems. In their study, an FPN model was built to represent the knowledge and inference scheme about the service restoration and tested on a practical distribution system of Taiwan Power Company.

4.2.2. Mechanical and manufacturing systems

To address the impact of solar array anomalies, Wu et al. (2011) established a model using fault tree analysis (FTA) and FRPNs to perform reliability analysis of a solar array mechanical system, which can be used to find the most important root causes and put forward propositions to improve reliability of the solar array. Wu et al. (2012) also developed a reliability apportionment approach which combines fuzzy comprehensive evaluation with FRPNs to accomplish the reliability apportionment of spacecraft solar array. In (Wu and Hsieh, 2012), the authors explored a real-time FPN (RTFPN) approach to diagnose progressive faults in programmable logic controllers (PLC)-based discrete manufacturing systems. In this approach, a real-time PN (RTPN) model was used to monitor a running status on the manufacturing plant and the FPN diagnoser was utilized to isolate the fault root causes when a fault happens. An and Liang (2013) put forward an FPN framework with unobservable transitions to resolve the fault diagnosis problem of discrete event systems with inaccuracy and unobservable events such as Hall thruster. A two-directional reasoning strategy was proposed for computing certain factor values of diagnosis results, which are forward reasoning and backward reasoning. Liu et al. (2013b) presented a fault diagnosis and cause analysis approach based on the FER approach and DAFPNs, which is able to capture all types of abnormal event information provided by experts, and identify the root causes and determine the consequences of the identified abnormal events by combining forward reasoning and backward reasoning.

4.2.3. ERP implementation and pipeline transportation

Guo et al. (2016) established a comprehensive risk evaluation framework based on an FPN model for long-distance oil and gas transportation pipelines, in which the analytic hierarchy process (AHP), entropy method (EM), and cloud model are adopted to improve the evaluation accuracy. Pramod et al. (2014) brought out an FPN-based risk assessment model by selecting certain critical pitfalls in the implementation of enterprise resource planning (ERP) in small and medium enterprises.

4.3. Wireless sensor networks

To increase the reliability during routing selection, Hu et al. (2005) proposed a reliable routing algorithm in mobile ad hoc networks (MANETs) based on FPNs with their reasoning mechanism. The algorithm allows the structured representation of network topology and can compute the most reliable route by comparing the degree of reliability in the routing sprouting tree. In (Yu et al., 2011), the authors presented a reliable energy-efficient multi-level routing algorithm for wireless sensor networks using FPNs. The algorithm considered the residual energy, number of the neighbors and centrality of each node for cluster formation, which not only balances the energy load of each node but also provides global reliability for the whole network.

Khoukhi et al. (2014) proposed a model, namely FuzzyWMN, which uses FPNs to realize traffic adaptation in wireless mesh networks characterized by information uncertainty and imprecision. Based on the secure ad hoc on-demand distance vector (SAODV), Pouyan and Yadollahzadeh Tabari (2015) used FPNs to propose a secure routing protocol in MANET, which is called FPN-SAODV. In the FPN-SAODV routing protocol, a type of bidirectional node-to-node fuzzy security verification was carried out for sending and receiving packets between each pair of nodes. A through route security verification was used for selecting the most secure route among candidate path through sources to destination. In (Chiang et al., 2009), the authors focused on a dynamic knowledge inference approach using AFPNs to discover the best routing path for multicast routing protocols in a highly bandwidth-scarce environment. The work in (Tan et al., 2015) found a trust based routing mechanism to defend against attacks in both data plane and routing plane in optimized link state routing (OLSR)-based MANET, in which a trust reasoning model based on FPNs is used to evaluate trust values of mobile nodes and avoid malicious or compromised nodes, and a trust based routing algorithm is utilized to select a path with the maximum path trust value among all possible paths.

4.4. Transportation systems

4.4.1. Bridge damage assessment

Based on the FPNs presented in (Lee et al., 1998), Lee et al. (1999) proposed a framework of integrated expert systems, called FPN-based expert system (FPNES), and applied it to damage assessment of a bridge in Taiwan. Major features of the FPNES are: knowledge representation through the use of hierarchical FPNs, a reasoning mechanism based on FPNs, and transformation of modularized fuzzy rule bases into hierarchical FPNs. Chiang et al. (2000) also developed an expert system for bridge damage assessment through the FPNES.

4.4.2. Railway operation control and traffic congestion control

Fay (2000) developed a fuzzy knowledge-based system for use in railway operation control systems and described an FPN notion to model rule-based expert knowledge in the dispatching support system. Cheng and Yang (2009) utilized an FPN approach to formulate the decision processes based on the train dispatching rules transformed from dispatchers in the case of disturbance, so as to obtain any possible dispatching option in railway traffic control. In (Milinković et al., 2013), an FPN model with characteristics of hierarchy, color, time, and fuzzy reasoning was proposed to simulate traffic processes and train movements in a railway system for estimating train delays. In this research, the data detected from the real system was used to train the neuro-fuzzy adaptive network fuzzy inference system (ANFIS) model, which was then replicated by an FPN after the results of the fuzzy logic system were verified. Yin et al. (2015) improved the FPNs to strengthen their ability of knowledge expression and reasoning, and then established an intelligent decision making model for traffic congestion control.

4.5. Biological and healthcare systems

4.5.1. Gene regulatory networks

Hamed et al. (2010a) proposed an FPN approach to design genetic regulatory networks and describe the dynamical behavior of gene, and introduced an efficient reasoning algorithm based on the FPN model to automatically reason about imprecise and fuzzy information. The proposed approach is able to obtain results with fuzzy intervals rather than point values, thus offering more flexible reasoning capability. Hamed et al. (2010b) further presented a fuzzy reasoning model based on FPNs for modeling gene regulatory networks, which considers the regulatory triplets by means of predicting changes in expression level of the target gene based on input expression level. In (Hamed and Ahson, 2011), an FPN approach for modeling fuzzy rule-based reasoning was proposed to predict the confidence values for each base called in DNA sequencing, and was validated by comparing the results produced with the FPN model and fuzzy logic using the MATLAB Toolbox.

4.5.2. Disease assessment and diagnosis

Hamed (2015) developed an AFPN reasoning algorithm as a prognostic system to determine the predictive value of risk degree for esophageal cancer based on the serum concentrations of C-reactive protein (CRP) and albumin as a set of input data. Chen et al. (2014a, 2014b) used FPNs to propose a rule-based decision making diagnosis system to evaluate arteriovenous shunt (AVS) stenosis for long-term hemodialysis treatment of patients. Chiang (2015) created a rule-based reasoning model by combining fuzzy computing and APN for electrocardiograms (ECG)-based mental stress assessment, and Chiang and Pao (2016) described an EEG-based fuzzy probability model using fuzzy and APN methodologies for early diagnosis of Alzheimer's disease.

4.6. Others

4.6.1. System control

Andreu et al. (1997) developed an FPN-based PLC based on a combination of PNs with possibility theory (PNs with fuzzy markings). The proposed approach is able to directly integrate continuous control within discrete-event models, and to implement symbolic specifications. Dimirovski (2005) proposed a novel approach to FPN reasoning, generating a solution to initial or another state in Markov-chain models. In this work, the reasoning was performed by an FPN supervisory controller by using a fuzzy-rule production system design and a fuzzy reasoning algorithm. Maeda (1998) dwelled on a method for the evaluation of ambiguity in a fuzzy algorithm using FPNs, which makes possible the discrete expression of ambiguity changing, the global recognition of ambiguities in the whole algorithm with a marking matrix and easy tracing of state transition by matrix operations.

4.6.2. Computing with words, web learning, and service composition

Since impreciseness and uncertainty are often involved in computing with words (CWs), Cao and Chen (2010) developed a concurrency computational model of CW by exploiting FPNs, which is called FPNs for CWs (FPNCWs). To make the model robust for CWs, a faithful extension was further made by employing the methodology of fuzzy reasoning. Chen et al. (2005) applied a dynamic FPN (DFPN) model to web learning systems to increase the flexibility of the tutoring agent's behavior and thus provide an appropriate dynamic learning content structure for a lecture course. In (Huang et al., 2008), a complete course generation platform was developed to facilitate efficient course design and management in e-Learning, in which the DFPN was adopted to dynamically organize courses for lecturers. Cheng et al. (2015b) presented a fuzzy semantic-based automatic Web service composition method, in which a fuzzy predicate PN (FPPN) is applied to model the Horn clause set, and a T-invariant technique is used to determine the existence of composite services fulfilling the user input/output requirements.

4.6.3. Formal analysis, real-time decision making, and image annotation

Shen and Lai (1998) proposed a hybrid model composed of FPNs and marked PNs for formal specification and verification of digital systems, in which fuzzy reduction rules and a consistency checking algorithm are employed to check the consistency of a knowledge base. Peters et al. (1999) presented an approach to construct PNs for a real-time decision system and used rough FPNs to create highly parallel programs to simulate reasoning system computations. The constructed nets are able to evaluate the design of decision system tables and trace computations in rules derived from decision tables. Ivasic-Kos et al.

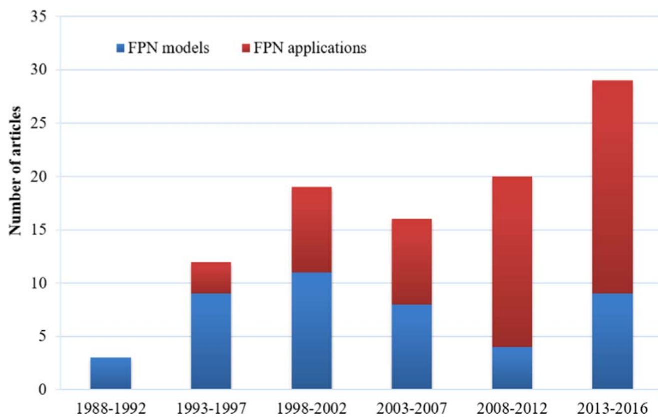

Fig. 4. Distribution of the reviewed articles by published years.

(2015) defined a fuzzy knowledge-representation based on FPN (KRFPN) formalism to represent knowledge concepts in an image and put forward an intelligent system for multi-layered image annotation.

5. Bibliometric analysis

Based on the collected papers on FPN improvements and applications, a bibliometric analysis is conducted in this section regarding the quantity of articles published per year and the journals in which the articles appeared. First, the distribution of the 99 reviewed articles is shown in Fig. 4.

As we can see, there is a significant growth in the studies concerning FPNs from the first 5 years (1988–1992) to the recent 5 years (2008–2012), 3 vs. 20. Particularly, in the last 4 years (2013–2016), 29 papers have been published on the topic. The growth could also mark a movement away from the proposition of new FPN models and towards the use of FPNs for solving real-life engineering problems. Specifically, the above review indicates that FPNs have been widely implemented in the fields of fault diagnosis, operational management, wireless sensor networks, transportation system, and biological and healthcare systems. Moreover, the FRPN model proposed by Gao et al. (2003) is found to be the most frequently used FPN approach. It is anticipated that the trend of FPN publications may be expected to continue increasing in the coming years because of the distinguished power and efficiency of FPNs in knowledge representation and reasoning and the increased interest in knowledge management and artificial intelligence by both researchers and practitioners.

To arrive at an understanding of what outlets most artificial intelligence scholars publish their work in—that is, academic peer-reviewed journals—we have further conducted an analysis of the reviewed articles based on their published journals (see Fig. 5). Note that only journals in which three or more articles appeared are analyzed and marked in Fig. 5. The result shows that, there are eight main journals publishing studies on FPNs within the artificial intelligence community: Expert Systems with Applications (10, 10.1%), IEEE Transactions on Systems Man and Cybernetics Part B (9, 9.1%), IEEE Transactions on Systems Man and Cybernetics Part A (8, 8.1%), IEEE Transactions on Knowledge and Data Engineering (5, 5.1%), and IEEE Transactions on Fuzzy Systems (4, 4.0%). Apart from these main journals, 48 journals covered the remaining 54.5% reviewed papers.

6. Observations and future work

6.1. Main findings

6.1.1. Reasoning algorithms

For the convenience of reading and comparison, we summarize the reviewed reasoning algorithms in Table A2, where the knowledge

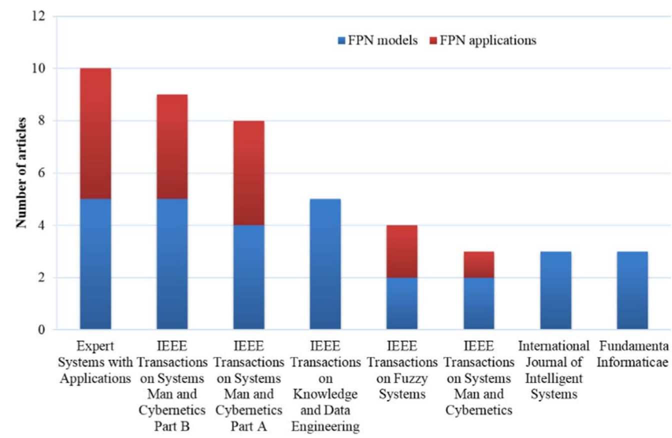

Fig. 5. Distribution of articles in terms of published journals.

reasoning algorithms are generally classified into two types, i.e. reasoning based on reachability tree and that by algebraic representation. The two reasoning mechanisms have different advantages and disadvantages and the details for each one are given in Table 1. Note that both of the two reasoning mechanisms suffer the state space explosion issue. That is, dimensions of the sprouting tree or related matrices depend on the scale of the created FPN model and, with the increased scale of FPNs, algorithm complexity will increase rapidly.

6.1.2. Knowledge representations

To facilitate the reading and understanding, Table A3 summarizes the knowledge representation methods discussed in the cited studies. From Table A3, we can observe that the representation ability of FPNs is normally improved by considering more knowledge parameters and assigned them to the arcs of an FPN, e.g., the work by Ahson (1995) and Carinena et al. (1999). Besides, as far as knowledge acquisition is concerned, extracting knowledge from domain experts is a time-consuming and painstaking job; it is very difficult to determine accurately the knowledge representation parameters in the first place. Therefore, several studies suggested using uncertainty theories, such as fuzzy numbers (Chen, 2002), IFSs (Liu et al., 2016a) and linguistic 2-tuples (Liu et al., 2016b), to acquire vague or imprecise expert knowledge more naturally.

6.1.3. New FPN models

To improve readability, the comparison table of the different categories of new FPN models are summarized in Table A4. As shown in Table A4, lots of alternative FPN modes have been developed for enhancing knowledge representation and logic reasoning of FPNs, and each approach has its own characteristics. Especially, four of the reviewed studies introduced time factors into FPNs so as to represent the dynamic nature of uncertain knowledge (Carinena et al., 1999; Liu et al., 2011; Pang et al., 1995; Suraj and Fryc, 2006), and eight papers present dynamic FPN frameworks which have the learning ability in real-world applications (Ahson, 1995; Amin and Shebl, 2014; Feng et al., 2012; Konar et al., 2005; Li and Lara-Rosano, 2000; Li et al., 2000; Pedrycz and Gomide, 1994; Wang et al., 2014).

6.2. Future research

Based on this particular literature review, we conclude the following possible directions for future research:

First, it is observed that the FPNs based on reachability tree analysis is not supportive and robust enough in knowledge representation and reasoning. Especially, the sources of improvements of FPNs that have received great attention from the literature are inference algorithm and knowledge representation related issues. For example, the fuzzy reasoning algorithms are not suitable for parallel reasoning

Table 1
Advantages and disadvantages of the two types of knowledge reasoning algorithms.

	Advantages	Disadvantages
Reachability tree	A complex expert system reasoning path can be reduced to a simple sprouting tree A graphical representation of the inference process can be given for visual appraisal Easy to follow and find inference path	Large reachability sets and adjacent places tables may result for a complex expert system The sprouting tree becomes complex as the number of places and transitions increase Reasoning speed and efficacy is low Hard to be stored and processed by computer
Algebra representation	A complex fuzzy expert system reasoning path can be reduced to simple matrix operations Ease in computer processing Reasoning speed and efficacy is high	The dimensions of these matrices and vectors are increased with the growing of the scale of the FPN model

and lack of learning mechanism; the knowledge representation parameters (e.g. weight and threshold value) are unable to express various kinds of expert knowledge accurately. Therefore, developing more advanced vague reasoning algorithms which can not only learn from the data but also perform knowledge reasoning efficiently should be explored in the future. For example, additional research on the reasoning algorithm to detect and recover errors in practical knowledge-based systems is a worthwhile effort. To handle complex systems, hierarchical structure based reasoning algorithms are expected to explore such that the complexity for the whole system can be reduced through decomposition and reuse. Besides, for solving the knowledge representation issues of FPNs, consideration of more knowledge representation parameters, such as time factors, is another important topic that should be tackled. By incorporating the FPN paradigm into a real-time expert system environment, it is possible to describe a continuous industrial process.

Second, it has been found that a lot of research efforts have been devoted on the modification of FPNs in knowledge representation and reasoning. However, in most FPNs, the knowledge parameters are restricted to be crisp values between 0 and 1. Only a few uncertainty methodologies have been incorporated into FPNs in order to arrive at conclusions based on imprecise rules and facts (Chen, 2002; Liu et al., 2016a, 2016b). Thus, investigating how to deal with inexact, uncertain and vague nature of knowledge information is another avenue of potential research. For instance, the combination of FPNs with other uncertainty theories (e.g., type-2 fuzzy sets, hesitant fuzzy sets, and cloud model, etc.) should be examined in the further such that the FPN model is more powerful in representing domain expert knowledge. In addition, although many FPN approaches have been adopted to solve practical problems, there are still some FPN models proposed in the literature which have not been applied in practice. The applications of more theoretical models to real-word problems should be investigated for future research. Furthermore, researchers have mainly applied FPNs to disassembly process planning, electric power systems, and mobile ad hoc networks. There are ample opportunities to implement the FPNs or develop more generic FPNs to solve complex issues in other fields.

Third, although some comparison results of FPN models have been presented in preceding studies, different FPN methods have different advantages and drawbacks. The existing studies did not make sufficient comparisons among various FPN models. As a result, it is difficult for the practitioners and engineers to select a suitable FPN model in real applications. Therefore, in future research, it is necessary to conduct a detailed comparative study to adequately evaluate and compare the advantages and disadvantages of different FPNs or to develop some programs to carry out the evaluation and comparison to aid the practitioners and scholars in finding out the most suitable ones for the problem to be solved. Moreover, the development of computer-aided tools to execute the FPNs given in the literature is certainly an important direction in order that FPNs can be easily utilized by practitioners to manage complex problems at higher speed.

Fourth, the continuous increase in the amount and detail of data captured by organizations nowadays has produced an overwhelming flow of data in either structured or unstructured format, which are referred to as big data. Big data are characterized by three aspects (Hashem et al., 2015): (a) data are numerous, (b) data cannot be categorized into regular relational databases, and (c) data are generated, captured, and processed rapidly. However, existing FPN models have limitations and are inapplicable to the representation and management of big data. Therefore, novel FPN models that can deal with big data and real-time applications should be developed in the future such that the expert system is able to perform large-scale and complex knowledge inference for decision support. The new opportunity for researchers is to match the accuracy of state-of-the-art models while reducing reasoning efficiency and computational cost.

7. Conclusions

FPNs are one of the most popular and applicable class of PNs in the domain of artificial intelligence, which have been widely studied by researchers and practitioners. In this paper, we have conducted a systematic literature review of the state of the art literature on FPN models and their applications from 1988 to 2016 to present the available body of knowledge and to analyze the trends in considering FPNs as expert systems involving fuzzy-based reasoning. To our best knowledge, this is the first comprehensive study reviewing the literature that improves the capabilities of FPNs and apply FPNs to solve real-world engineering problems. This paper has set out to provide a framework of the FPN literature as an aid to the categorization of studies in this area. Overall, the FPN-based artificial intelligence field is growing and maturing. Significant room still exists for development given the small number of reviewed articles and that there are only 99 papers relatively close-related. We believe that this number will continue to increase given the solid foundation provided by the existing research, a foundation that did not exist a decade ago. Particularly, opportunities abound for additional research in formal modeling of FPNs with practical applications.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.engappai.2017.01.012.

References

Ahson, S.I., 1995. Petri net models of fuzzy neural networks. IEEE Trans. Syst. Man Cybern. 25, 926–932.
Amin, M., Shebl, D., 2014. Reasoning dynamic fuzzy systems based on adaptive fuzzy higher order Petri nets. Inf. Sci. 286, 161–172.
An, R., Liang, W., 2013. Unobservable fuzzy Petri net diagnosis technique. Aircr. Eng. Aerosp. Technol. 85, 215–221.
Andreu, D., Pascal, J.C., Valette, R., 1997. Fuzzy Petri net-based programmable logic controller. IEEE Trans. Syst. Man Cybern. B 27, 952–961.
Bostan-Korpeoglu, B., Yazici, A., 2007. A fuzzy Petri net model for intelligent databases. Data Knowl. Eng. 62, 219–247.
Bugarin, A.J., Barro, S., 1994. Fuzzy reasoning supported by Petri nets. IEEE Trans. Fuzzy Syst. 2, 135–150.
Cao, T.H., Sanderson, A.C., 1995a. Representation and analysis of uncertainty using fuzzy Petri nets. J. Intell. Fuzzy Syst. 3, 3–19.
Cao, T.H., Sanderson, A.C., 1995b. Task sequence planning using fuzzy Petri nets. IEEE Trans. Syst. Man Cybern. 25, 755–768.
Cao, Y., Chen, G., 2010. A fuzzy Petri-nets model for computing with words. IEEE Trans. Fuzzy Syst. 18, 486–499.
Cardoso, J., Valette, R., Dubois, D., 1999. Possibilistic Petri nets. IEEE Trans. Syst. Man Cybern. B 29, 573–582.
Carinena, P., Bugarin, A., Fraga, S., Barro, S., 1999. Enhanced fuzzy temporal rules and their projection onto fuzzy Petri nets. Int. J. Intell. Syst. 14, 775–804.
Cassandras, C.G., Lafortune, S., 2008. Introduction to Discrete Event Systems. Springer-Verlag, New York.
Chen, J.N., Huang, Y.M., Chu, W.C.C., 2005. Applying dynamic fuzzy petri net to web learning system. Interact. Learn. Environ. 13, 159–178.
Chen, S., Zhan, T., Huang, C., Chen, J., Lin, C., 2015. Nontechnical loss and outage detection using fractional-order self-synchronization error-based fuzzy Petri nets in micro-distribution systems. IEEE Trans. Smart Grid 6, 411–420.
Chen, S.M., 1996. A fuzzy reasoning approach for rule-based systems based on fuzzy logics. IEEE Trans. Syst. Man Cybern. B 26, 769–778.
Chen, S.M., 2000. Fuzzy backward reasoning using fuzzy Petri nets. IEEE Trans. Syst. Man Cybern. B 30, 846–856.
Chen, S.M., 2002. Weighted fuzzy reasoning using weighted fuzzy Petri nets. Ieee. Trans. Knowl. Data. Eng. 14, 386–397.
Chen, S.M., Ke, J.S., Chang, J.F., 1990. Knowledge representation using fuzzy Petri nets. IEEE Trans. Knowl. Data. Eng. 2, 311–319.
Chen, W.L., Kan, C.D., Lin, C.H., Chen, T., 2014. A rule-based decision-making diagnosis system to evaluate arteriovenous shunt stenosis for hemodialysis treatment of patients using fuzzy Petri nets. IEEE J. Biomed. Health Inform. 18, 703–713.
Chen, Y.F., Li, Z.W., Barkaoui, K., Uzam, M., 2014b. New Petri net structure and its application to optimal supervisory control: interval inhibitor arcs. IEEE Trans. Syst. Man Cybern. Syst. 44, 1384–1400.
Cheng, H., He, Z., Wang, Q., Yang, J., Lin, S., 2015a. Fault diagnosis method based on Petri nets considering service feature of information source devices. Comput. Electr. Eng. 46, 1–13.
Cheng, J., Liu, C., Zhou, M., Zeng, Q., Ylä-Jääski, A., 2015b. Automatic composition of semantic web services based on fuzzy predicate petri nets. IEEE Trans. Autom. Sci. Eng. 12, 680–689.
Cheng, Y.H., Yang, L.A., 2009. A fuzzy Petri nets approach for railway traffic control in case of abnormality: evidence from Taiwan railway system. Expert. Syst. Appl. 36, 8040–8048.
Chiang, H.S., 2015. ECG-based mental stress assessment using fuzzy computing and associative Petri net. J. Med. Biol. Eng. 35, 833–844.
Chiang, H.S., Pao, S.C., 2016. An EEG-based fuzzy probability model for early diagnosis of alzheimer's disease. J. Med. Syst. 40, 125–134.
Chiang, T.C., Tai, C.F., Hou, T.W., 2009. A knowledge-based inference multicast protocol using adaptive fuzzy Petri nets. Expert. Syst. Appl. 36, 8115–8123.
Chiang, W.L., Liu, K.F.R., Lee, J., 2000. Bridge damage assessment through fuzzy Petri net based expert system. J. Comput. Civil. Eng. 14, 141–149.
Chun, M.G., Bien, Z.N., 1993. Fuzzy Petri-net representation and reasoning methods for rule-based decision-making systems. IEICE Trans. Fundam. E76A, 974–983.
Dimirovski, G.M., 2005. Fuzzy-Petri-net reasoning supervisory controller and estimating states of Markov chain models. Comput. Inform. 24, 563–576.
Fay, A., 2000. A fuzzy knowledge-based system for railway traffic control. Eng. Appl. Artif. Intel. 13, 719–729.
Feng, L.B., Obayashi, M., Kuremoto, T., Kobayashi, K., 2012. A learning fuzzy Petri net model. IEEJ Trans. Electr. Electr. Eng. 7, 274–282.
Fryc, B., Pancerz, K., Peters, J.F., Suraj, Z., 2004. On fuzzy reasoning using matrix representation of extended fuzzy Petri nets. Fund. Inform. 60, 143–157.
Gao, M.M., Zhou, M.C., Huang, X.G., Wu, Z.M., 2003. Fuzzy reasoning Petri nets. IEEE Trans. Syst. Man Cybern. A 33, 314–324.
Gao, M.M., Zhou, M.C., Tang, Y., 2004. Intelligent decision making in disassembly process based on fuzzy reasoning Petri nets. IEEE Trans. Syst. Man Cybern. B 34, 2029–2034.
Garg, M., Ahson, S., Gupta, P., 1991. A fuzzy Petri net for knowledge representation and reasoning. Inform. Process. Lett. 39, 165–171.
Gniewek, L., 2013. Sequential control algorithm in the form of fuzzy interpreted Petri net. IEEE Trans. Syst. Man Cybern. Syst. 43, 451–459.
Gniewek, L., Kluska, J., 2004. Hardware implementation of fuzzy Petri net as a controller. IEEE Trans. Syst. Man Cybern. B 34, 1315–1324.
Guo, Y., Meng, X., Wang, D., Meng, T., Liu, S., He, R., 2016. Comprehensive risk evaluation of long-distance oil and gas transportation pipelines using a fuzzy Petri net model. J. Nat. Gas. Sci. Eng. 33, 18–29.
Ha, M.H., Li, Y., Wang, X.F., 2007. Fuzzy knowledge representation and reasoning using a generalized fuzzy Petri net and a similarity measure. Soft Comput. 11, 323–327.
Hamed, R.I., 2015. Esophageal cancer prediction based on qualitative features using adaptive fuzzy reasoning method. J. King Saud. Univ. -Comput. Info. Sci. 27, 129–139.
Hamed, R.I., Ahson, S.I., 2011. Confidence value prediction of DNA sequencing with Petri net model. J. King Saud. Univ. -Comput. Info. Sci. 23, 79–89.
Hamed, R.I., Ahson, S.I., Parveen, R., 2010a. Designing genetic regulatory networks using fuzzy Petri nets approach. Int. J. Autom. Comput. 7, 403–412.
Hamed, R.I., Ahson, S.I., Parveen, R., 2010b. A new approach for modelling gene regulatory networks using fuzzy petri nets. J. Integr. Bioinform. 7, 113.
Hanna, M.M., Buck, A., Smith, R., 1996. Fuzzy Petri nets with neural networks to model products quality from a CNC-milling machining centre. IEEE Trans. Syst. Man Cybern. A 26, 638–645.
Hashem, I.A.T., Yaqoob, I., Anuar, N.B., Mokhtar, S., Gani, A., Ullah Khan, S., 2015. The rise of “big data” on cloud computing: review and open research issues. Inform. Syst. 47, 98–115.
He, Z.Y., Yang, J.W., Zeng, Q.F., Zang, T.L., 2014. Fault section estimation for power systems based on adaptive fuzzy Petri nets. Int. J. Comput. Intell. Syst. 7, 605–614.
Hu, H.S., Li, Z.W., Al-Ahmari, A., 2011. Reversed fuzzy Petri nets and their application for fault diagnosis. Comput. Ind. Eng. 60, 505–510.
Hu, Z.G., Ma, H., Wang, G.J., Liao, L., 2005. A reliable routing algorithm based on fuzzy Petri net in mobile ad hoc networks. J. Cent. South Univ. T. 12, 714–719.
Huang, Y.M., Chen, J.N., Huang, T.C., Jeng, Y.L., Kuo, Y.H., 2008. Standardized course generation process using dynamic fuzzy Petri nets. Expert. Syst. Appl. 34, 72–86.
Ivasic-Kos, M., Ipsic, I., Ribaric, S., 2015. A knowledge-based multi-layered image annotation system. Expert. Syst. Appl. 42, 9539–9553.
Kasirolvalad, Z., Motlagh, M.R.J., Shadmani, M.A., 2006. An intelligent modeling system to improve the machining process quality in CNC machine tools using adaptive fuzzy Petri nets. Int. J. Adv. Manuf. Tech. 29, 1050–1061.
Khoukhi, L., El Masri, A., Sardouk, A., Hafid, A., Gaiti, D., 2014. Toward fuzzy traffic adaptation solution in wireless mesh networks. IEEE Trans. Comput. 63, 1296–1308.
Konar, A., Mandal, A.K., 1996. Uncertainty management in expert systems using fuzzy Petri nets. Ieee. Trans. Knowl. Data Eng. 8, 96–105.
Konar, A., Chakraborty, U.K., Wang, P.P., 2005. Supervised learning on a fuzzy Petri net. Inf. Sci. 172, 397–416.
Koriem, S.M., 2000. A fuzzy Petri net tool for modeling and verification of knowledge-based systems. Comput. J. 43, 206–223.
Lee, J., Liu, K.F.R., Chiang, W.L., 1998. Fuzzy Petri nets for modeling rule-based reasoning. Int. J. Artif. Intell. Tools 7, 463–486.
Lee, J., Liu, K.F.R., Chiang, W.L., 1999. A fuzzy Petri net-based expert system and its application to damage assessment of bridges. IEEE Trans. Syst. Man Cybern. B 29, 350–370.
Lee, J., Liu, K.F.R., Chiang, W.L., 2003. Modeling uncertainty reasoning with possibilistic Petri nets. IEEE Trans. Syst. Man Cybern. B 33, 214–224.
Lehocki, F., Juhas, G., Lorenz, R., Szczerbicka, H., Drozda, M., 2008. Decision support with logical and fuzzy petri nets. Cyber. Syst. 39, 617–640.
Li, X., Lara-Rosano, F., 2000. Adaptive fuzzy petri nets for dynamic knowledge representation and inference. Expert. Syst. Appl. 19, 235–241.
Li, X., Yu, W., Lara-Rosano, F., 2000. Dynamic knowledge inference and learning under adaptive fuzzy Petri net framework. IEEE Trans. Syst. Man Cyber. C 30, 442–450.
Li, Z.W., Zhao, M., 2008. On controllability of dependent siphons for deadlock prevention in generalized Petri nets. IEEE Trans. Syst. Man Cybern. A 38, 369–384.
Li, Z.W., Liu, G.Y., Hanisch, H.M., Zhou, M.C., 2012a. Deadlock prevention based on structure reuse of Petri net supervisors for flexible manufacturing systems. IEEE Trans. Syst. Man Cybern. A 42, 178–191.
Li, Z.W., Wu, N.Q., Zhou, M.C., 2012b. Deadlock control of automated manufacturing systems based on Petri nets: a literature review. IEEE Trans. Syst. Man Cybern. C. 42, 437–462.
Liu, H.C., Lin, Q.L., Mao, L.X., Zhang, Z.Y., 2013a. Dynamic adaptive fuzzy Petri nets for knowledge representation and reasoning. IEEE Trans. Syst. Man Cybern. Syst. 43, 1399–1410.
Liu, H.C., Lin, Q.L., Ren, M.L., 2013b. Fault diagnosis and cause analysis using fuzzy evidential reasoning approach and dynamic adaptive fuzzy Petri nets. Comput. Ind. Eng. 66, 899–908.
Liu, H.C., Liu, L., Lin, Q.L., Liu, N., 2013c. Knowledge acquisition and representation using fuzzy evidential reasoning and dynamic adaptive fuzzy Petri nets. IEEE Trans. Cybern. 43, 1059–1072.
Liu, H.C., You, J.X., You, X.Y., Su, Q., 2016a. Fuzzy Petri nets using intuitionistic fuzzy sets and ordered weighted averaging operators. IEEE Trans. Cybern. 46, 1839–1850.
Liu, H.C., You, J.X., You, X.Y., Su, Q., 2016b. Linguistic reasoning Petri nets for knowledge representation and reasoning. IEEE Trans. Syst. Man Cybern. Syst. 46, 499–511.
Liu, Z.J., Li, H.G., Zhou, P.J., 2011. Towards timed fuzzy Petri net algorithms for chemical abnormality monitoring. Expert. Syst. Appl. 38, 9724–9728.
Looney, C.G., 1988. Fuzzy Petri nets for rule-based decision-making. IEEE Trans. Syst. Man Cybern. 18, 178–183.
Luo, X., Kezunovic, M., 2008. Implementing fuzzy reasoning Petri-nets for fault section

estimation. IEEE T. Power Deliv. 23, 676–685.
Maeda, Y., 1998. Evaluation of ambiguity in fuzzy algorithm represented with fuzzy petri net. Int. J. Uncertain. Fuzzy Knowl.-Based Syst. 6, 225–240.
Manoj, T.V., Leena, J., Soney, R.B., 1998. Knowledge representation using fuzzy Petri nets-revisited. IEEE Trans. Knowl. Data Eng. 10, 666–667.
Meng, F.X., Lei, Y.J., Zhang, B., Shen, X.Y., Zhao, J.Y., 2016. Intuitionistic fuzzy Petri nets for knowledge representation and reasoning. J. Digit. Inform. Manag. 14, 104–113.
Milinković, S., Marković, M., Vesković, S., Ivić, M., Pavlović, N., 2013. A fuzzy Petri net model to estimate train delays. Simul. Model. Pract. Th. 33, 144–157.
Murata, T., 1989. Petri nets: Properties, analysis and applications. Proceedings of the IEEE 77, pp. 541–580.
Pang, G.K., Tang, R., Woo, S.S., 1995. A process-control and diagnostic tool based on continuous fuzzy Petri nets. Eng. Appl. Artif. Intel. 8, 643–650.
Pedrycz, W., Gomide, F., 1994. A generalized fuzzy Petri-net model. IEEE Trans. Fuzzy Syst. 2, 295–301.
Peters, J.F., Skowron, A., Suraj, Z., Pedrycz, W., Ramanna, S., 1999. Approximate real-time decision making: concepts and rough fuzzy Petri net models. Int. J. Intell. Syst. 14, 805–839.
Pouyan, A.A., Yadollahzadeh Tabari, M., 2015. FPN-SAODV: using fuzzy petri nets for securing AODV routing protocol in mobile Ad hoc network. Int. J. Commun. Syst.. http://dx.doi.org/10.1002/dac.2935.
Pramod, D., Bharathi, S.V., Raman, R., 2014. A fuzzy Petri-net model for predicting the post-implementation risks of ERP in small and medium enterprises. Int. Rev. Comput. Softw. 9, 1852–1860.
Qiao, F., Wu, Q.D., Li, L., Wang, Z.T., Shi, B., 2011. A fuzzy Petri net-based reasoning method for rescheduling. Trans. Inst. Meas. Control 33, 435–455.
Scarpelli, H., Gomide, F., Pedrycz, W., 1996. Modeling fuzzy reasoning using high level fuzzy Petri nets. Int. J. Uncertain. Fuzzy Knowl.-Based Syst. 4, 61–85.
Schmoch, U., Schubert, T., 2008. Are international co-publications an indicator for quality of scientific research? Scientometrics 74, 361–377.
Shen, V.R.L., 2006. Knowledge representation using high-level fuzzy Petri nets. IEEE Trans. Syst. Man Cybern. A 36, 1220–1227.
Shen, V.R.L., Lai, F.P., 1998. Requirements specification and analysis of digital systems using fuzzy and marked Petri nets. IEEE Trans. Syst. Man Cybern. B 28, 748–754.
Shih, D.H., Chiang, H.S., Lin, B., 2007. A generalized associative Petri net for reasoning. Ieee. Trans. Knowl. Data. Eng. 19, 1241–1251.
Sun, J., Qin, S.Y., Song, Y.H., 2004. Fault diagnosis of electric power systems based on fuzzy Petri nets. IEEE Trans. Power Syst. 19, 2053–2059.
Suraj, Z., 2013. A new class of fuzzy Petri nets for knowledge representation and reasoning. Fund. Inform. 128, 193–207.
Suraj, Z., Fryc, B., 2006. Timed approximate Petri nets. Fund. Inform. 71, 83–99.
Tan, S.S., Li, X.P., Dong, Q.K., 2015. Trust based routing mechanism for securing OSLR-based MANET. Ad Hoc Netw. 30, 84–98.
Tang, Y., 2009. Learning-based disassembly process planner for uncertainty management. IEEE Trans. Syst. Man Cybern. A 39, 134–143.
Tang, Y., Turowski, M., 2007. Adaptive fuzzy system for disassembly process planning with uncertainty. J. Chin. Inst. Ind. Eng. 24, 20–29.
Tang, Y., Zhou, M., Gao, M., 2006. Fuzzy-Petri-net-based disassembly planning considering-human factors. IEEE Trans. Syst. Man Cybern. A 36, 718–726.
Tsang, E.C., Yeung, D.S., Lee, J.W., Huang, D.M., Wang, X.Z., 2004. Refinement of generated fuzzy production rules by using a fuzzy neural network. IEEE Trans. Syst. Man Cybern. B 34, 409–418.
Wang, H., Jiang, C., Liao, S., 2001. Concurrent reasoning of fuzzy logical Petri nets based on multi-task schedule. IEEE Trans. Fuzzy Syst. 9, 444–449.
Wang, W.M., Peng, X., Zhu, G.N., Hu, J., Peng, Y.H., 2014. Dynamic representation of fuzzy knowledge based on fuzzy petri net and genetic-particle swarm optimization. Expert. Syst. Appl. 41, 1369–1376.
Wu, J., Yan, S., Xie, L., 2011. Reliability analysis method of a solar array by using fault tree analysis and fuzzy reasoning Petri net. Acta Astronaut. 69, 960–968.
Wu, J., Yan, S., Xie, L., Gao, P., 2012. Reliability apportionment approach for spacecraft solar array using fuzzy reasoning Petri net and fuzzy comprehensive evaluation. Acta Astronaut. 76, 136–144.
Wu, R.R., Ma, L., Mathew, J., Duan, G.H., 2002. Optimal operation planning using fuzzy Petri nets with resource constraints. Int. J. Comput. Integr. Manuf. 15, 28–36.
Wu, Z.H., Hsieh, S.J., 2012. A realtime fuzzy Petri net diagnoser for detecting progressive faults in PLC based discrete manufacturing system. Int. J. Adv. Manuf. Tech. 61, 405–421.
Yang, H.T., Huang, C.M., 2002. Distribution system service restoration using fuzzy Petri Net models. Int. J. Elec. Power 24, 395–403.
Ye, Y., Jiang, Z., Diao, X., Du, G., 2011. Extended event-condition-action rules and fuzzy Petri nets based exception handling for workflow management. Expert. Syst. Appl. 38, 10847–10861.
Yeung, D.S., Tsang, E.C.C., 1994a. Fuzzy knowledge representation and reasoning using Petri nets. Expert. Syst. Appl. 7, 281–289.
Yeung, D.S., Tsang, E.C.C., 1994b. Improved fuzzy knowledge representation and rule evaluation using fuzzy Petri nets and degree of subsethood. Int. J. Intell. Syst. 9, 1083–1100.
Yeung, D.S., Tsang, E.C.C., 1997. Weighted fuzzy production rules. Fuzzy Sets Syst. 88, 299–313.
Yeung, D.S., Ysang, E.C.C., 1998. A multilevel weighted fuzzy reasoning algorithm for expert systems. IEEE Trans. Syst. Man Cybern. A 28, 149–158.
Yin, J.S., Liu, L., Chen, K., Lin, J., 2015. Intelligent decision making model for traffic congestion control based on fuzzy petri net. J. Comput. Methods Sci. Eng. 15, 91–98.
Yu, Z.H., Fu, X.A., Cai, Y.L., Vuran, M.C., 2011. A reliable energy-efficient multi-level routing algorithm for wireless sensor networks using fuzzy Petri nets. Sensors 11, 3381–3400.
Zhang, J.F., Khalgui, M., Li, Z.W., Frey, G., Mosbahi, O., Salah, H.B., 2015. Reconfigurable coordination of distributed discrete event control systems. IEEE T. Contr. Syst. T. 23, 323–330.
Zhang, Y., Zhang, Y., Wen, F., Chung, C.Y., Tseng, C.L., Zhang, X., Zeng, F., Yuan, Y., 2016. A fuzzy Petri net based approach for fault diagnosis in power systems considering temporal constraints. Int. J. Elec. Power Energ. Syst. 78, 215–224.
Zhao, S.E., Li, Y.L., Fu, R., Yuan, W., 2014. Fuzzy reasoning Petri nets and its application to disassembly sequence decision-making for the end-of-life product recycling and remanufacturing. Int. J. Comput. Integr. Manuf. 27, 415–421.
Zhou, F., Jiao, R., Xu, Q., Takahashi, K., 2012. User experience modeling and simulation for product ecosystem design based on fuzzy reasoning Petri nets. IEEE Trans. Syst. Man Cybern. A 42, 201–212.
Zhou, K.Q., Zain, A.M., Mo, L.P., 2015. A decomposition algorithm of fuzzy Petri net using an index function and incidence matrix. Expert. Syst. Appl. 42, 3980–3990.