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Minireview

Econophysics: Past and present

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HIGHLIGHTS

- Brief historical aspects from econophysics.
- Efficient market hypothesis.
- Power laws and the emergence of the econophysics expression.
- Some applications from econophysics.
- Prospects for econophysics.

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ABSTRACT

This paper provides a brief historical review of the relationship between economics and physics, beginning with Adam Smith being influenced by Isaac Newton's ideas up to the present day including the new econophysics discipline and some of the tools applied to the economy. Thus, this work is expected to motivate new researchers who are interested in this new discipline.

Contents

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1. Introduction

Despite the connection of these two disciplines having already existed for some time, it took more than twenty years for the econophysics name to emerge. According to Schinkus [1], a new discipline has arisen making a contribution to the economy, especially the financial markets. In this paper, we intend to provide a brief discussion of the development of this new discipline from the historical aspects to its future prospects. We hope, therefore, that new researchers become interested in this topic.

The organization of this paper is structured as follows: Sections 2 and 3 provides a brief historical account of the theories of physics being applied to economics and some applications from econophysics, respectively. In Section 4 we show some of the perspectives of econophysics and finally Section 5 presents our final considerations.

2. Brief historical aspects

Econophysics can be considered a new perspective for the economy. According to Gingras and Schinckus [2] econophysics is a new discipline with a different methodology and with new tools that can greatly contribute to the advancement of the economy. However physics, from an early age, has indirectly influenced the economy.

With the discovery of the law of gravity by Isaac Newton (1643–1727), a new method of research arose, not only in physics, but in science in general, based on rationalism, experimentalism and with it an attempt to understand the universe through analysis, synthesis and application of the inductive method. Subsequently, Adam Smith (1723–1790), a philosopher regarded as the founder of economics sciences, was strongly influenced by Newton's ideas. According to Hetherington [3], "Adam Smith's efforts to discover the general laws of economics were directly inspired and shaped by the examples of Newton's success end discovering the natural laws of motion".

Another economist who was influenced by the physical sciences – mainly in relation to the equilibrium of bodies in the constitution of its laws – was Walras. His law of general equilibrium was based on the work of the mathematician Louis Poinsot (1777–1859), who produced his research on the equilibrium of forces before Walras. According to Paula [4],"there is no way to recognize the decisive influence of Poinsot on Walras and not see the Walras book as an application in the field of economy from 'science de l'équilibre des forces'". Other essential concepts in economics were also strongly influenced by physics. According to Carbonne et al. [5]:

> Adolphe Quetelet (1796–1874) corroborated the idea that physical laws could govern human behavior and also economics. His contemporary philosopher, Auguste Comte (1798–1857), first envisaged the social physics as a scientific discipline alongside astrophysics, geophysics and chemical physics. It is not by chance that these concepts raised in an intellectual climate permeated by the Newtonian ideas. The basic assumptions of neoclassical economics span a wide range of applications and concepts (utility maximization, market supply and demand, equilibrium) and are still influential.

At the beginning of the twentieth century, Louis Bachelier (1870–1940)[1] in his doctoral thesis in mathematics (*Theory de La Speculation*, published in 1900), admitted that the prices of financial assets followed a random walk.[2] She was supervised by the great mathematician Henry Poincaré. Curiously, Bachelier [8] anticipated the ideas from Einstein [9] in five years on the mathematical formalization of *random walk*. Thereby, Bachelier founded the modern theory of finance which gave rise to the efficient markets hypothesis (see [10–12] and the Black–Scholes pricing formula options).

Years later, Fischer Black and Myron Scholes managed to solve the problem of finding pricing formula options that had an application in the stock exchange. It is noteworthy that one of their ideas was to use the heat diffusion equation widely used in physics. Black and Scholes sought to publish their ideas in an article sent in October 1970 to the *Journal of Political Economy* (JPE). Editors promptly rejected the article, claiming that Black and Scholes put too much emphasis on finance while neglecting economics [13]. Another journal, *Review of Economics and Statistics* from Harvard, was also quick to return

[1] "Who was under estimated by his supervisors, perhaps because the title of his thesis was not attractive to mathematicians. Unfortunately, certain historical research follows this legend. But our study of documents shows that this simplified story is far from being correct" [6].

[2] Regarding random walk, "the best decision is made based on the present and not the past, the best example for random walk is the guy coin game or crown if someone throws a coin twice or a thousand times up the likelihood of having to face in both cases is 50%, that is, they are independent identically as in the first move der face, this does not guarantee that the second will give face again and a thousand plays der face this also does not guarantee that the thousandth first move will give face, making a completely independent event" [7].

it; neither journal bothered to ask an expert to examine it. Finally, the paper was accepted in the May–June 1973 edition in JPE, however, only after the intercession of two members from Chicago University [13].

Therefore, the Black and Scholes equation became a way to determine the "fair" price paid for an option, which from the behavioral point of view, generated greater confidence in investors at the time, since it did not only depend on fundamental analysis, but had a mathematical formula that could carry out such a calculation. Another hypothesis in economics which helps illustrate the relationship between the economy and the physical is the efficient markets hypothesis, as discussed below.

2.1. Efficient market hypothesis

The efficient markets hypothesis is, currently, one of the most discussed and studied topics in Economic Sciences [14–16]. Thus, the importance of understanding this hypothesis is fundamental to understanding the behavior of financial series and, doing so, one can get a sense of how finances work.

This hypothesis originated from the doctoral thesis in mathematics by Louis Bachelier (1870–1947), *Theorie de la speculation*, under the supervision of the eminent mathematician Henry Poincaré. In it, Bachelier [8] compared the prices of financial assets to a *random walk*. However, his thesis remained forgotten until Samuelson [10] discovered the work of the "forgotten" teacher and became fascinated because it was an attempt to give a more scientific character to financial markets.

After that, Samuelson [10] tried to give a more rigorous account of the efficient markets hypothesis. He reformulated the hypothesis of random behavior of prices while supposing that agents are rational. However, only with the help of Eugene [11] and Fama [12] was the hypothesis better developed to define "the market's ability to reflect all available information on the price of financial assets" [17]. Thus, the *random walk* model can be defined as:

> The *random walk* model is based on two different hypotheses: (a) the current price of a savings bond reflects all available information indicating that the price movements over time are a series of random numbers (**serial correlation of errors equal to zero**); and (b) price changes obey the same probability distribution [11].

The motives for economists in the finance would adopt the Efficient Market Hypothesis according to Ausloos et al. [18] Is the possibility of constructing a conceptual framework that tries to reflect the reality from hypotheses previously adopted for the implementation of computerization of financial markets. Another important point for Ausloos et al. [18] is the possibility that economists can use Gaussian stochastic processes to perform statistical tests, this allows for greater scientific rigor And the statistic used by economists was constructed using *Stable Levy Structure*implying the use of the *General Central Limit Theorem*.

A parallel between the hypothesis of efficient and econophysics markets lies in the fact that the formulator of the Eugene Fama hypothesis was guided in his Doctoral Thesis by Benoit Mandelbroat who despite being a mathematician, had a strong connection with econophysics, having started at least two discussions that today are essential to this discipline: the study of longterm memory in financial series and whether the distribution of the financial series returns are in power law format.

2.2. Power laws

According to Jovanovic and Schinckus [19] and Jovanovic and Schinckus [20], In the 60's of the 20th century, Mandelbrot, Samuelson and Fama proposed to study financial markets using non-Gaussian structures inspired by the works of Levy and the stability of the distributions of probability and extension of the central limit theorem proposed by Gnedenko et al. [21]. Mandelbrot [22]. Using two models the M1963 and M1965 initiated two new research themes in statistics applied to finance one related to the independence of information and the other related to stationarity.

According to Jovanovic and Schinckus [19] and Jovanovic and Schinckus [20] In its first model Mandelbrot proposed that the "α stable" Levy processes were entirely suited to abrupt price changes. Mandelbroat showed that the price of cotton for half a century fit a distribution in the power law format. Mandelbroat [23] realized that normal distributions could not explain the high fluctuations in the price of cotton, given that a distribution using the power law format fits the data better. The discovery by Mandelbroat was of vital importance for the emergence of econophysics early on as a large number of the studies involving econophysics are finding power laws in financial series and demonstrating its importance for financial markets.

However, according to Gleria et al. [24], Mandelbrot encountered the problem of infinite standard deviation. More precisely all moments of order greater than two are infinite. And in the financial markets the standard deviation is a measure of the volatility of the variable, Making it difficult to give meaning to this greatness in case it becomes infinite. Only in the 90's of the twentieth century with Eugene Stanley and Rosário Mantegna did Mandelbrot reconsider the possibility that Levy's distributions would again explain fluctuations in assets, This being a fundamental step for the emergence of econophysics. It should be considered that the distributions of Levy are also called Pareto–Levy, being the Pareto law a classical power law.

The power law or Pareto's Law was originally studied by economist Vilfredo Pareto (1848–1923) who was interested in the distribution of income in Italy in 1906. Instead of asking what would be the umpteenth higher income, he asked how many people would have a higher income than x. Thereby, Pareto defined its distribution as follows:

$$P[X > x] \sim (m/x)^{-k} \quad (1)$$

where m represents the lowest salary and $m > 0$, $k > 0$ and $x \geq m$ and k is an inequality index. The lower the index, the more unequal the distribution of income is. This expression attests that there are many millionaires and a few modest people.

For the power law, the relevant information is not how many people have a higher salary than x, but how many people receive exactly x. This is the probability distribution function associated with PDF (Probability Density Function), associated with DFC (Density Function Cumulative) given by Pareto.

$$P[X = x] \sim x^{-(k+1)} = x^{-a}. \quad (2)$$

Note that the exponent of the power law distribution, $a = k + 1$, k is the parameter of the Pareto distribution. According to Gleria [24]:

> Bouchaud and Mezard examined Pareto's law and observed that, if we consider the number of people in the United States who have 1 billion dollars, we find that four times more people have half a billion dollars and four times higher than that will have a quarter of a billion dollars, and so on.

Power laws have the property of being free of scale, they are ideal for measuring phenomena that are susceptible to extreme events, such as financial markets, for example. Therefore, the emerging econophysics gains a relevant dimension in the 90's of the 20th century, by applying power laws in the distribution of stock returns.

2.3. Emergence of the econophysics expression

The econophysics name first emerged with Stanley et al. [25] at a conference held in 1995 in Kolkata. It is a neologism used for the branch of physics of complex systems that seeks to make a complete survey of the statistical properties of financial markets, using the huge volume of data now available and working methods of statistical physics [17].

Econophysics, since its foundation, was intrinsically linked to seeking extreme events in financial series using power laws to describe them. However, this type of study was marginalized or little considered by most economists, since the main economic assumptions were built based on the normal distribution and large deviations were almost impossible. However, the results found by econophysics indicate that such events are not so rare, e.g. the 1987 crisis, where the Dow Jones index fell 22%, or the 2008 crisis.

In the 1990s, a growing number of publications began studying the distribution of financial returns using physics (in particular see [25–28]). This first period was marked, mainly, by the application of power laws in the distribution of financial returns. Other types of studies were the size of cities [29] and executive compensation [30,31].

From the 2000s, econophysics expanded rapidly and began to study various phenomena that occur not only in the financial markets, but in the economy in general; their applications range from the use of fractal analysis of the returns distributions to models based on evolutionary agents.

3. Some applications

3.1. Agentbased models

Econophysics has advanced and is no longer limited to testing financial series, thus embarking on other paths. One example is the Agent Based Model (ABM) which has been used in several areas of the economy. Its main advantages are the use of agents with limited rational adaptive behavior and the possibility of micro and macro interaction. Due to the ABM possessing these characteristics, a workshop was held in Virginia at the end of June 2010, with the organization of America's National Science Foundations and with the presence of many economists including the Fed (Federal Reserve System), Bank of England and some scientists (including physicists, economists and computer scientists), in which they showed how the ABM can be useful to Macroeconomics [32].

3.1.1. Setting agents

An agent is an entity that perceives its environment through sensors and acts on the same using the implementers; for example, on human agents, the eyes and ears are sensitive, hands and mouth are executors [33]. Furthermore, agents can evolve, adapt, learn and still have cognitive abilities [34].

Regarding the interaction between the agents and the environment, when the environment issue encounters a stimulus, the first captures it with its sensors, and then responds with an action using their executioner's elements. Based on this, Russel and Norvig [35] proposed that the basic structure of an agent is very simple: it has an internal data structure that will be updated with the arrival of new insights, this structure is used in decision making procedures, which will generate actions to be executed. Therefore, according to Stefferson Lima and Rosario [33] they have the following characteristics:

1. Agents can operate without direct human control or other agents: they are **selfemployed**.
2. Agents can act in partnership with humans and with other agents: they **communicate**.
3. Agents can react to various forms of stimulation of domains: they are **reactive**.
4. Agents can make decisions for themselves to fit the defined goals:they are **proactive**.

ABM is the creation of a population of agents, with the capacity of perception and action similar to the actual components to be simulated. So they can act as if they own the components of a system, and one should provide them with behaviors and rules that define the possible actions [35–37]. This is done by modeling behaviors, by analyzing the variables of a system and extracting its main features so that they can be incorporated into the respective individuals. According to Pykas and Fagiolo [38], ABM has the following characteristics:

(a) Time: Usually it is a model evolving overtime in discrete steps, $t = 1, 2, \ldots$
(b) Agents (or actors): The system is populated by a number of agents $I_t = \{1, 2, \ldots, N_t\}$. In many examples, but not necessarily all, the population size is assumed to be constant over time ($N_t = N$).
(c) Microstates (or actions): Each agent $i \in I_t$ is characterized by a L vector of Microstates (or variable micro) $X_{i,t} = (X_{i,t}^1, \ldots, X_{i,t}^L)$, these variables are easy to handle, can make them endogenous, modifying the agents' decisions (as a product of the company, number of shares that individuals have etc.).
(d) Microparameters: Each agent is also characterized by a microparameter vector H $\theta_i = (\theta_i^1, \ldots, \theta_i^h)$, which are slow variables, that is, they cannot be quantified without a time scale in a dynamic process. Therefore, θ_i typically contains information about the behavior and characteristics of the agents i (Company productivity factors, elasticity of consumption, etc.).
(e) Macroparameters: The system can be characterized as a whole instead of being characterized by an independent technological vector M (macroparameters) $\Theta = (\Theta_1, \ldots, \Theta_M)$ governing all direct attitudes; again, Θ these variables are slow and cannot be modified by agents.
(f) Structure Interaction: In each agent t, the way that information is channeled between agents is governed by a chart (directly and possibly weighted) G_t containing all links ij_t currently in place of an agent for j agent. The existence of a link ij_t means that the agent updates its microvariables $x_{i,t}$. It is affected by the choice made in the past by j agent.
(g) Microdecision rules: Each agent is provided with a set of decision rules $\Re_{i,t} = \{R_{i,t}(\bullet|\bullet),\ b = 1, \ldots, B\}$ mapping the observable variables and the next microchanging period $x_{i,t}$. An example might be: production function, innovation rules.
(h) Aggregate Variables: Clustering (average, sum, etc.) from microvariables can create a vector K macrovariables $x_t = (x_{1t,}, \ldots, x_{kt})$ containing all relevant information to analyze in the system. Examples include GDP, aggregate demand, unemployment, etc.

After the definition and characterization of ABM, as already shown, we will show its use in econophysics demonstrating its importance to the economy.

3.1.2. Agent based econophysics and agent based modeling

According to Schinckus [39] *agent-based econophysics*is based on the micro-approach, comes from computational physics and is an area that has developed models of *order-driven markets or models using Kinetic Theory*. Whereas *agentbased modeling*Focuses on the atomistic approach of agents, but is different from the neoclassical approach based on methodological individualism and certain assumptions (utility function, risk aversion and rationality) the *agent-based modeling*Provide microfundamentals for statistical regularities that emerge at a macro level of a socioeconomic system.

3.1.3. The importance of ABM in economics

One of the first studies involving adaptive agents and economic theory was Holland and Miller's [40]. *Artificial Adaptive Agents in Economic Theory* emphasizing the many advantages of applying this new methodology, among which we mention two: the ability to control all system variables and environmental conditions as well as working with adaptive agents. Since that time, this new research agenda has grown considerably and currently the number of publications in economics has been significant in relation to other sciences [41].

In this context, a famous work involving ABM and financial markets was the Santa Fe Artificial Market [42], in which the authors innovated implementing agents that can modify their strategies overtime or had the ability to learning. In Lux and Marchesi [43] heterogeneous agents were used to find high frequency returns of extreme events, *clustered* volatility and power laws.

Using the ABM methodology in the options market has also found anomalies in a study by Suzuki et al. [44] using *traders* with aversion to loss and using the pricing formula of Black–Scholes options. Therefore, ABM studies in finances have found several stylized facts in financial series and it represents a new look, since these anomalies are now treated as events that can occur, approaching most of the stock exchanges, in which there are crises, extreme events and panics, unlike the world of the Efficient Market Hypothesis [11] or the Black–Scholes equation [45] wherein the financial series approaches a normal curve, with the possibility of almost zero crises.

Furthermore, in the words of Le Baron [46] financial markets are particularly attractive for applications based on an agent model for several reasons:

- First, the main debates in finance on market efficiency and rationality are still unresolved.
- Second, financial time series contain many stylized facts that are not well understood.
- Third, financial markets provide a wealth of data volume that can be analyzed.

Use of ABM in macroeconomics is very recent, with a higher frequency in the last decade. In this sense, Le Baron [47], Hodgson [48] and Farmer and Foley [49] have stressed the need for agents to understand the micro and macroeconomic phenomena, because they can interact with each other and the environment, and agents are rationally limited. Due to these characteristics, Colander et al. [50] proposed the use of agents in Macroeconomics; in their words:

> The advantage of Computational Agent economy (Agent Based Models applied economics), in particular, in macroeconomics is that it removes the treated limitations of the analytical limit of macroeconomics. ABM allows researchers to choose the appropriate way to solve problems, including the types of agents, number of each type of agents and the hierarchy of their arrangements. They also allow the researchers consider the interactions between agents simultaneously with their decision, and study the dynamics of macro interaction between them.

Thus, Dosi et al. [51] built a model of evolutionary cycles and managed to reproduce some stylized facts in macroeconomics as the most volatile investment rather than consumption. The investment showed a procyclical growth and the model showed a multiplier equal to the Keynesian accelerator.

Because of the possibility that they may be incorporated into agent's variables such as learning, routines and evolution (somewhat restricted in traditional modeling firm theory), such qualities, according to Pykas and Fagiolo [38], can assist in the proposal of Nelson and Winter [52] to study the firm and Macroeconomics evolution. In this context, Hodgson [48] goes further and proposes ABM as an alternative model for the study of institutions in the twentieth century, since for him the institutions are a set of individuals who are constantly interacting and evolving and not a set of rational and atomized units.

In this way, the contribution of ABM on macroeconomics and/or firm theory is still difficult to measure because the research is fairly recent. However, ABM has been a challenging methodology and can greatly assist in understanding macroeconomic policies.

3.2. *Long term memory*

One way to characterize the long term memory of a time series is to associate it with a property called persistence. The determination of this property is generally related to a parameter, named H exponent or Hurst exponent. In the literature there are many methods for determining this exponent, but one of the most used is the classic R/S method. Specifically, R/S is a method with empirical bases. It has provided a solution for the problem of a finite reservoir subjected to a random input stream.

The question is based on determining the volume of a reservoir by knowing its water inflow $\xi(t)$, whereas its output flow is equal to the average $\xi(t)$, so that the tank never runs dry or overflows [53].

Considering a time interval τ, the mean inlet flow will be:

$$\langle\xi\rangle_\tau = \frac{1}{\tau}\sum_{i=1}^{\tau}\xi(\tau). \tag{3}$$

Let $X(t)$ be the cumulative difference between the input flow and its average

$$X(t,\tau) = \sum_{u=1}^{t}\left\{\xi(u) - \langle\xi\rangle_t\right\}, \quad (1 \le t \le \tau). \tag{4}$$

The maximum and minimum from (4) represents the minimum and maximum volume of water that passes through the shell in the period τ. Thus, the total volume of the reservoir, so that it never overflows or runs dry, should be the difference between the maximum and minimum from X,

$$R(\tau) = \max X(t,\tau)_{1\le t\le\tau} - \min X(t,\tau)_{1\le t\le\tau}. \tag{5}$$

Obviously $R(\tau)$ depends on the flow $\xi(t)$, which in turn depends on the period considered τ. Hurst investigated many natural phenomena and realized that R depended on the period τ as a power law,

$$R/S = \left(\frac{\tau}{2}\right)^H. \tag{6}$$

In (6) S is the standard deviation of the inflow, defined by:

$$S = \left(\frac{1}{\tau}\sum_{t=1}^{\tau}\left\{\xi(t) - \langle\xi\rangle_\tau\right\}^2\right)^{\frac{1}{2}}. \tag{7}$$

Introduced only for the variable R/S is a dimensionless number making it easier to compare with other phenomena [54]. Although initially for hydrogeological reasons, this method has considerable application in economics. For example, Cajueiro and Tabak [55] proposed testing the assertion found in the literature that emerging financial markets were becoming more efficient over time and checked whether this was true or not. The calculation of the Hurst exponent was proposed from the R/S method over time using a time window of 4 years of data. These data were the stock exchange rates of some emerging countries and Japan and the United States. It was observed that for the latter two $H \approx 0.5$ that is, this market efficiency was related to the value of this Hurst exponent and that emerging countries including Brazil, the Philippines and Thailand had values this H exponent great then 0.5 which showed that these three countries were not following the literature by becoming more efficient over time.

Another application of this method was in Souza et al. [56] where they used the Hurst exponent by R/S method to investigate the long term memory in the transition between the fixed exchange rate regimes and floating exchange rate in Brazil.

Rejichi et al. [57] tested the efficiency of the MENA stock market using a certain Hurst Exponent with the R/S method. Mensi et al. [58] determined the Hurst exponent using the R/S method to measure the degree of long range dependence displayed by crude indexes for measuring the degree of long range dependence exhibited by West Texas Intermediate (WTI) and European Brent.

These and other applications of the classical method R/S are the physical point of view and basically work in autocorrelation time series. The application of these methods to the economy has been growing in the econophysics literature, promising a major evolution of this new area in the future.

This revolution continues when we look at another aspect of time series in econophysics associated with cross-correlations, which is one way to observe the behavior of grip a time series with another. As we are working with econophysics these series include the financial market such as: the foreign exchange market with the benchmark interest rate, country risk with international reserves, etc.

3.3. Cross-correlation

The study of cross-correlation is based on statistics that relates one time series with another. In the literature, a range of theories comprise this analysis. In this article we will focus on the theory [59] measuring the cross-correlation between time series, based on the autocorrelation of them [60] and the study of cross-correlation between time series by power law [61]. In order to improve the understanding of this theory we explore this in more detail below.

The theory of Peng et al. [60] is focused on determining the cross-correlation coefficient without trend, ρ_{DCCA} coefficient, which succinctly forms the 5 steps of his determination. Step I. Considering two time series, $\{x_t\}$ and $\{y_t\}$, with $t = 1, 2, \ldots, N$, (N is the total number of elements of the time series). Then we integrate time series obtaining two new series:

$$xx_k = \sum_{t=1}^{k} x_t \quad \text{and} \quad yy_k = \sum_{t=1}^{k} y_t, \quad k = 1, 2, \ldots, N. \tag{8}$$

Step II: We have divided these two integrated time series, $\{xx_k\}$ and $\{yy_k\}$, in $(N - s)$ overlapping boxes of equal length s, with $4 \leq s \leq N/4$.

Step III: We calculate the local trend of each box by the least squares fit of each series, $xP_i\,(k)$ and $yP_i\,(k)$. Now we calculate the covariance of waste in each box by:

$$f_{xy}^2\,(s,i) = \frac{1}{s+1} \sum_{k=i}^{i+s} (xx_k - xP_i\,(k))\,(yy_k - yP_i(k))\,. \tag{9}$$

Step IV: Now, the average over all overlapping boxes is calculated for the new covariance function:

$$F_{xy}^2\,(s) = \frac{1}{(N-s)} \sum_{i=1}^{N-s} f_{xy}^2\,(s,i)\,. \tag{10}$$

Step V: Finally we calculate the cross correlation coefficient ρ_{DCCA} by:

$$\rho_{DCCA}(s) = \frac{F_{xy}^2\,(s)}{F_{xx}\,(s)\,F_{yy}(s)}, \tag{11}$$

where $F_{xy}^2(s)$ is the correlation function determined by the method from Podobnik and Stanley [61] and $F_{xx}(s)$ and $F_{yy}(s)$ are the autocorrelation functions determined by the method from Peng et al. [60]. This cross-correlation coefficient depends on the size of each box s (time scale). One advantage of this cross-correlation coefficient is measuring the correlation between two time series with different time scales. The ρ_{DCCA} varies in the range $-1 \leq \rho_{DCCA} \leq 1$, where 1 means perfect cross-correlation, 1 perfect anti cross-correlation and 0 there is no correlation.

In conclusion, the determination of the ρ_{DCCA} coefficient has some applications in econophysics. In M. F. da Silva et al. [62] this coefficient was used to measure the correlation between the prices of soybeans and corn in Barreiras/Bahia/Brazil with the Brazilian foreign exchange market. In Wang et al. [63] the ρ_{DCCA} was used to measure the cross-correlation between the larger coins 44 countries on a global network and in Reboredo et al. [64] the ρ_{DCCA} coefficient was used to verify the cross-correlation between the exchange rate and the price of oil in different countries before and after the 2008 economic crisis.

The theory of cross-correlation time series is significantly robust and well applicable in econophysics. There is great potential still to be explored in this area of econophysics and therefore a good opportunity for researchers who want to explore new avenues.

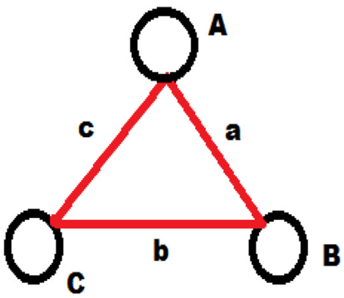

Fig. 1. Network consists of three components A, B and C which represent the vertices (nodes) of the network and a, b, c the edges.
Source: Self elaboration, 2015.

3.4. Complex networks

Complex networks are nodes (vertices) connected by edges, and they can be exemplified by: transport networks (network of airlines in Brazil, networks of roads), social interactions (knowledge networks, scientific collaboration networks), biological networks (regulatory networks of genes and protein interaction networks) and networks in economics (banking networks and networks of exporting countries), etc.

The study of networks began with the resolution of the Königsberg Problem, city of Prussia, currently Kaliningrad, in Russia. After the statement by Leonhard Euler in 1736 that the problem had no solution, mathematics revolutionized not the answer, but the way it was developed. Euler created basic categories like nodes and edges. Two centuries later, mathematicians Paul Erdös and Alfred Rényi, introduced a new concept that allows the study of these networks: the theory of random graphs. The idea was to combine the theory of graphs with the tools of probability theory. Two other fundamental contributions were: the discovery of the small world effect by Milgron [65] and Watts and Strogatz [66] and networks free of scale [67].

The structure of a complex network is represented in the same way as a graph by which a set R, for networks that do not have weights in the links is defined by $R\ (\upsilon, \varepsilon)$, where $\upsilon = \{\upsilon_1, \upsilon_2, \upsilon_3 \ldots \upsilon_N\}$ are the nodes (or vertices) and $\varepsilon = \{e1, e2, e3, \ldots, eM\}$ are the edges or connections that connect pairs of nodes, considering the number of elements υ and ε, N and M, respectively. To illustrate this complex structure, a simple schematic representation of a network with three edges and three nodes is shown in Fig. 1.

3.4.1. Complex network and financial markets

The use of complex networks in the financial markets has enabled a new view, mainly to measure the financial interaction between the stock exchanges, assets, banks or companies, in this case, the nodes are usually the assets, banks or countries. For financial markets, the greatest contribution of complex networks to date has been to show the financial markets as interdependent and subject to financial fragility. Until the 2008 crisis, the economy predominated the idea of efficient markets proposal by Eugene [11] where the assets reflect all available information. However, after the 2008 crisis, markets are not efficient and the financial interconnection between different assets or banks varies greatly (see [68]). This means that in times of financial crisis, stock markets tend to become more interconnected and can completely change the economic and monetary policy of a country.

3.4.2. Complex networks and production

Currently, to measure the degree of connection between the various export sectors and importers of the economy, noting that in Hildalgo et al. [69], a new idea about the level of economic growth was formulated. For them an important element is connectivity between some sectors, for example, to produce a computer the company must rely on others that will produce the battery, liquid crystal display, chips, mouse, software, etc. In other words, the more complex and interconnected the relations are between the various sectors, the greater the chances of a country to grow economically, because the greater the knowledge produced by the country in question.

This type of research may represent an advance in the theories of economic growth with an analysis of various disaggregated sectors. Usually, economic models tend to consider only two sectors such as assets and capital. In the case of complex networks there is a paradigm shift in that you can see how each sector interfered in another or the probability of developing a new activity or new sector.

4. Prospects for econophysics

The growth of econophysics has been considerable in the last two decades with several indications including the creation of Ph.D. courses in econophysics around the world such as Houston University or creating econophysics as a discipline in various postgraduate courses both in economics and in physics. Moreover, many conferences and meetings have directly and indirectly encompassed econophysics both in Brazil and around the world, with the most well known being the Econophysics Colloquium, which in 2016 will be its twelfth year and has been held in various countries around the world.

The institutionalization of economics is documented in Gingras and Schinckus [2], which uses bibliometric methods to identify scientific publications with the econophysics theme. Have been identified 242 articles on the subject published in the period from 1980 to 2008; 147 Published in scientific journals, 90 of which are published in *Physica A*, 27 in *European Physical Journal B*, and the others in *Physical Review E*, *Quantitative Finance*, and *Journal of Economic Behavior & Organization*.

Some of the most cited economists in the world, according to various indices that measure the number of citations such as *repec*, *Google Scholar* or *webometric* use directly or indirectly the theoretical tools of complex systems. For example, Joseph Stiglitz, Nobel laureate in economics, who has the highest H[3] index of the world according to *webometric* [70] with an $H = 177$ and one of the most cited according to *repec* [71], has several works using complex systems and with physical partnerships (see [72–75]). According to *repec*, two of the fifteen most cited economists in the world in the last ten years use complex systems in their recent works Daron Acemoglu and Xavier Gabaix. In the case of Acemoglu, which is the most cited economist in the world in the last ten years, according to *repec* [71], uses heavy tails to explain the large macroeconomic fluctuations including works with coauthor Eugene Stanley, one of econophysics' founders. And the Xavier Gabaix's relationship is even more directly involved with econophysics, as the winner of several awards in economics and finance including the top award *Fischer Black* of finance, given by the *American Finance Association*. He uses econophysics in several works mainly using power laws [29,32,76,77] and writing several articles with physicists and with Eugene Stanley.

Another point is the creation of the *Institute for New Economic Thinking*, by mega investor George Soros, with the intention of helping the economic sciences to rebuild and put together strategies to prevent possible financial crises. The institute has the participation of physicists such as Doyne Farmer from *Santa Fe Institute* and Dirk Helbing from *ETH Zurich (Swiss Federal Institute of Technology in Zurich)*.

In this context, the financial market has been absorbing physicists to work as *traders* or in providing advice, where they can apply the concepts of physics to complex systems in finance. For that to happen, a great beacon was undoubtedly the 2008 crisis, because while models of physics indicated a likely drop in the North American stock exchange [77], economists were still rooted in the efficient markets hypothesis and that, in addition to physics providing an easier way to deal with complex systems such as the financial markets, it is crucial for greater visibility and application of the concepts or methods of econophysics on stock exchanges [78].

Therefore, economists and physicists are discovering that econophysics can be of great help in modeling various economic problems, leading to a natural process of science that is the consolidation of econophysics, just like mathematical economics, behavioral economics and evolutionary economics.

5. Final considerations

This article explored the contribution physics has had on economics beginning with Adam Smith being influenced by Newtonian mechanics and the concept of utility or the Black–Scholes formula. Additionally, this paper was shown as a one more discussion's alternative about this topic.

Some instruments used by econophysics were presented such as: power laws, ABM, long term memory, cross-correlation and complex networks in order to provide future researchers in this area information about early concepts in some of this field's areas. This, of course, does not exhaust all areas of econophysics that, due to its dynamic behavior, also already have good contributions on literature. Therefore, these areas are considered a sample in a universe of possibilities areas to investigate about this topic.

In summary, the emergence of the expression econophysics opened a new horizon of research and scientific contribution that has grown considerably in the last twenty years. Initially, the research was almost exclusively on financial markets, but with a greater number of researchers and the emergence of new techniques, it has expanded and today, there are applications in macroeconomics, theory of the firm, income distribution, microeconomics, etc. This rapid growth of econophysics has brought many perspectives becoming a new discipline, not to be subsumed by other traditional disciplines within the economy.

References

[1] C. Schinkus, Is Econophysics a new discipline? Physica A 389 (18) (2010) 3814–3821.
[2] Y. Gingras, C. Schinckus, The institutionalization of econophysics in the shadow of physics, J. Hist. Econ. 34 (2012) 109–130.
[3] S. Hetherington, Isaac Newton's influence on Adam Smith's natural laws in economics, J. Hist. Ideas 44 (3) (1983) 497–505.
[4] J.A. Paula, Walras no 'Journal des Economistes': 186065, Rev.Bras. Econ. 56 (1) (2002) 121–146.
[5] A. Carbonne, G. Kaniadakis, A.M. Scarfone, Where do we stand on econophysics? Physica A 382 (1) (2007) 11–14.
[6] JeanMichel Courtault, Youri Kabanov, Bernard Bru, Pierre Crepel, Isabelle Lebon, Arnaud Le Marchand, Louis Bachelier On the centenary of Théorie de la Spéculation, Math. Finance 10 (3) (2000) 339–353.

[3] The H Index was created in 2005 by the physicist J. E. Hirsch of the University of California. It is a proposal to quantify the productivity and impact of scientists based on papers most cited. In other words, the H index is the number of papers having greater than or equal to this number of quotes. Examples certainly help illustrate the concept: a researcher with $H = 5$ has 5 papers receiving 5 or more citations; a researcher with H index equal to 30 has published 30 scientific papers, and each received at least 30 citations in other papers. Papers with fewer citations would not be considered.

[7] Digital library USP, The Black–Scholes equation with impulse action. http://www.teses.usp.br/teses/disponiveis/55/55135/tde02072008101527/ptbr.php, 2008 (acessed 01.08.14).
[8] L. Bachelier, Théorie de la speculation. http://archive.numdam.org/ARCHIVE/ASENS/ASENS1900317/ASENS1900317210/ASENS1900317210.pdf, 1900 (accessed 01.08.14).
[9] A. Einstein, Ann. Phys. 17 (1905) 549.
[10] P.A. Samuelson, Proof that properly anticipated prices fluctuate randomly, Ind. Manag. Rev. 6 (1965) 41–49.
[11] F. Eugene, Efficient capital markets: a review of theory and empirical work, J. Finance 25 (2) (1970) 383–417.
[12] Fama E F, Efficient capital markets: II, J. Finance 46 (5) (1991) 1575–1618.
[13] P.L. Bernestein, Against the Gods: The Remarkable Story of Risk. http://hostel.ufabc.edu.br/~nelson.faustino/Ensino/IPE2016/Livros/Peter%20L.%20BernsteinAgainst%20the%20Gods%20The%20Remarkable%20Story%20of%20RiskWiley%20(1998)%20(1).pdf, 1998 (accessed 01.08.14).
[14] A. Sheifler, Inefficient Markets: A Introduction to Behavioral Finance, Oxford University Press, Oxford, 2000, p. 216.
[15] R.E. Lucas, Asset prices in an exchange economy, Econometrica 46 (6) (1978) 1429–1445.
[16] S. Grossman, J. Stiglitz, On the impossibility of informationally efficient markets, Amer. Econ. Rev. 70 (1980) 393–408.
[17] R. Mategna, E. Stanley, An Introduction to Econophysics: Correlation and Complexity in Finance, Cambridge University Press, Cambridge, 1999, p. 137.
[18] M. Ausloos, F. Jovanovic, C. Schinckus, On the ''usual'' misunderstandings between econophysics and finance: Some clarifications on modelling approaches and efficient market hypothesis, Int. Rev. Financ. Anal. 47 (2016) 7–14.
[19] F. Jovanovic, C. Schinckus, Econophysics: A New Challenge for Financial Economics? (with Franck Jovanovic), J. Hist. Econ. Thought 35 (3) (2013) 319–352.
[20] F. Jovanovic, C. Schinckus, Econophysics and Financial Economics: An Emerging Dialogue, Oxford University Press, 2017, (forthcoming).
[21] B.V. Gnedenko, A.N. Kolmogorov, Limit Distributions for Stums of Independent Random Variables, AddisonWesley, Cambridge, 1954.
[22] B. Mandelbrot, Very longtailed probability distributions and the empirical distribution of city sizes, in: F. Massarik, P. Ratoosh (Eds.), Mathematical Explanations in Behavioral Science, Homewood Editions, New York, 1965, pp. 322–332.
[23] B. Mandelbrot, The variation of certain speculative, J. Bus. 36 (4) (1963) 392–417.
[24] I. Gleria, R. Matsushita, S. Silva, Sistemas complexos, criticalidade e leis de potência, Rev. Bras. Ensino Fís. 26 (2) (2004) 98–108.
[25] H.E. Stanley, V. Afanasyev, L. Amaral, S. Buldyrev, A.L. Goldberger, S. Havlin, H. Leschhorn, P. Maass, R. Mantegna, C.K. Peng, A. Prince, M.A. Salinger, M. Stanley, G.M. Viswanathan, Anomalous fluctuations in the dynamics of complex systems: from DNA and physiology to econophysics, Physica A 224 (1996) 302–321.
[26] R. Mantegna, E. Stanley, Scaling behaviour in the dynamics of an economic index, Nature 376 (1995) 46–49.
[27] J. Bouchoud, R. Cont, A. Langevin, Approach to stock market fluctuations and crashes, Eur. Phys. J. B 6 (1998) 543–550.
[28] X. Gabaix, P. Gopikrishnam, V. Plerou, H.E. Stanley, A theory of power law distributions in financial market fluctuations, Nature 423 (2003) 267–270.
[29] X. Gabaix, Zipf's law for cities: An explanation, Quart. J. Econ. 114 (3) (1999) 739–767.
[30] X. Gabaix, A. Landier, Why has CEO pay increased so much? Quart. J. Econ. 123 (1) (2008) 49–100.
[31] X. Gaibaix, Power laws in economics and finance, Annu. Rev. Econ. 1 (2009) 255–293.
[32] The Economist, AGENT of change: Conventional economics models failed to foresee the financial crisis. Could agentbased modeling do better? http://www.economist.com/node/16636121, 2010 (acessed 20.07.11).
[33] S.L. Ferreira, R. Girardi, Arquitetura de software baseadas em agentes: do nível global ao detalhado, Rev. Eletrôn. Iniciação Cient. 2 (2) (2002) 1–17.
[34] M. Wooldridge, J. Nicholas, Intelligent Agents Theories, Architectures, and Languages, in: Lecture Notes in Artificial Intelligence, Springer Verlag, Berlin, 1995, p. 407.
[35] S. Russel, G. Norvig, Artificial Intelligence: A Modern Approach –The Intelligent Agent Book, Prentice Hall, 1995, p. 1152.
[36] L.A.M. Garcia, Analisando flutuações de um mercado financeiro artificial baseado na expectativa de riqueza dos agentes. https://www.lume.ufrgs.br/bitstream/handle/10183/15532/000685927.pdf?sequence=1, 2008 (acessed 01.08.14).
[37] V. Grimm, U. Berger, F. Bastiansen, S. Eliassen, V. Ginot, J. Giske, J. GossCustard, T. Grand, S.K. Heinz, G. Huse, A. Huth, J.U. Jepsen, C. Jorgensen, W.M. Mooij, B. Müller, G. Peer, C. Piou, S.F. Railsback, A.M. Robbins, M.M. Robbins, E. Rossmanith, N. Rüger, E. Strand, S. Souissi, R.A. Stillman, R. Vabo, U. Visser, D.L. DeAngelis, A standard protocol for describing individualbased and agentbased models, Ecol. Modell. 198 (2006) 115–126.
[38] A. Pykas, G. Fagiolo, Agent based modeling: a methodology for NeoSchumpeterian economics. http://www.wiwi.uniaugsburg.de/vwl/institut/paper/272.pdf, 2005 (acessed 10.07.09).
[39] C. Schinckus, Econophysics, a new step in the evolution of physical sciences (on invitation), Contemp. Phys. 54 (1) (2013) 17–32.
[40] J. Holland, H. Miller, Artificial adaptive agents in economic theory, Amer. Econ. Rev. 81 (1991) 365–370.
[41] B. Heath, R. Hill, F. Ciarello, A survey of agentbased modeling practices, J. Artif. Soc. Soc. Simul. 12 (4) (2009) 135.
[42] R. Palmer, W.B. Arthur, J.H. Holland, B. Le Baron, P. Taylor, Artificial economic life: a simple model of stock market, Physica D 75 (1994) 264–275.
[43] T. Lux, M. Marchesi, Scaling and criticality in a stochastic multiagent model of a financial market, Nature 397 (1999) 498–500.
[44] K. Suzuki, T. Shimokawa, T. Misawa, An agentbased approach to option pricing anomalies, IEEE Trans. Evol. Comput. 13 (2009) 19–32.
[45] F. Black, M. Scholes, The pricing of options and corporate liabilities, J. Polit. Econ. 81 (3) (1973) 637–654.
[46] B. Le Baron, Agentcomputational finance, in: K. Judd, L. Tesfatsion (Eds.), Handbook of Computational Economic: Agent Based Computational Economics, Elsevier, NorthHolland, 2006, pp. 1180–1227.
[47] B. Le Baro, L. Tesfatsion, Modeling macroeconomies as openended dynamic systems of interacting agents, Amer. Econ. Rev. 98 (2) (2008) 246–250.
[48] G. Hodgson, The Great Crash of 2008 and the reform of economics, Camb. J. Econ. 33 (6) (2009) 1205–1221.
[49] J.D. Farmer, D. Foley, The economy needs agent based modeling, Nature 460 (2009) 685–686.
[50] D. Colander, P. Howitt, A. Kirman, A. Leijonhufvud, P. Mehrling, Beyond DSGE models: toward an empirically based macroeconomics, Amer. Econ. Rev. 98 (2) (2008) 236–240.
[51] G. Dosi, G. Fagiolo, A. Roventini, The microfoundations of business cycles: an evolutionary, multiagent model, J. Evol. Econ. 18 (2008) 413–432.
[52] R.R. Nelson, S.G. Winter, An Evolutionary Theory of Economic Change, Belknap Press, Cambridge, 1982.
[53] J. Feder, Fractals, Plenum Press, New York, 1988.
[54] J.G. Miranda, Análise fractal de reescalonamento temporal para chuvas (1997) https://blog.ufba.br/pgif/dissertacoes1990ate1999/ (acessed 20.07.07).
[55] D.O. Cajueiro, B.M. Tabak, The Hurst Exponent over time: testing the assertion that emerging markets are becoming more efficient, Physica A 336 (3) (2004) 521–537.
[56] S.R. Souza, B.M. Tabak, D.O. Cajueiro, Investigação da Memória de Longo Prazo na Taxa de câmbio no Brasil, Rev. Bras. Econ. 60 (2006) 193–209.
[57] I.Z. Rejichi, C. Aloui, Hurst Exponent behavior and assessment of MENA stock markets efficiency, Res. Int. Bus. Finance. 26 (2012) 353–370.
[58] W. Mensi, M. Beljid, S. Managi, Structural breaks and the time varying levels of weak form efficiency in crude oil markets: Evidence from Hurst exponent and Shannon entropy methods, Int. Econ. 140 (2014) 89–106.
[59] G.F. Zebende, DCCA cross-correlation coefficient: Quantifying level of cross-correlation, Physica A 390 (2011) 2438–2443.
[60] C.K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, Mosaic organization of DNA nucleotides, Phys. Re E. 49 (1994) 1685–1689.
[61] B. Podobnik, H.E. Stanley, Detrended cross-correlation analysis: a new method for analyzing two nonstationary time series, Phys. Rev. Lett. 100 (8) (2008) 14.
[62] M.F. da Silva, A.M. da Silva Filho, A.P.N. de Castro, Quantificando a Influência do Mercado de Câmbio nos Preços do Milho e da Soja no Município de Barreiras, Conjunt. Planej. 182 (2014) 45–51.
[63] G.J. Wang, C. Xie, Y.J. Che, S. Che, Statistical properties of the foreign exchange network at different time scales: evidence from detrended cross-correlation coefficient and minimum spanning tree, Entropy 15 (2013) 1643–1662.
[64] J.C. Reboredo, M.A. Rivera-Castro, G.F. Zebende, Oil and US dollar exchange rate dependence: a detrended cross-correlation approach, Energy Econ. 42 (2013) 132–139.

[65] S. Milgron, The smallworld problem, Psychol. Today 1 (1) (1967) 61–67.
[66] D. Watts, S.H. Strogatz, Collective dynamics of 'smallworld' networks, Nature 393 (1998) 440–442.
[67] A.L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (1999) 509–512.
[68] B.M. Tabak, J. Rocha, M. Takami, D.O. Cajueiro, S.R. Souza, Directed clustering coefficient as a measure of systemic risk in complex banking networks, Physica A 394 (2014) 211–216.
[69] C.A. Hidalgo, B. Klinger, A.L. Barabaśi, R. Hausmann, The product space conditions the development of nations, Science 317 (2007) 482–487.
[70] S. Battiston, G. Caldarelli, CoPierre Georg, R. May, J.E. Stiglitz, Complex derivatives, Nat. Phys. 9 (2013) 123–125.
[71] IDEAS, Top 10% Authors (Last 10 years publications), as of July 2016. https://ideas.repec.org/top/top.person.all10.html, 2016 (acessed 28.07.16).
[72] D. Delli Gatti, M. Gallegati, B. Greenwald, A. Russo, J.E. Stiglitz, Business fluctuations in a creditnetwork economy, Physica A 370 (2006) 68–74.
[73] S. Battiston, D. Delli Gatti, M. Gallegati, B. Greenwald, Default cascades: When does risk diversification increase stability? J. Financ. Stab. 8 (3) (2012) 138–149.
[74] D. Acemoglu, A. Ozdaglar, A. Tahbaz-Salehi, Microeconomic Origins of Macroeconomic Tail Risks, NBER Working Paper 20865, 2015, pp. 1–47.
[75] P. Gopikrishnan, V. Plerou, X. Gabaix, H.E. Stanley, Statistical properties of share volume traded in financial markets, Phys. Rev. E 62 (4) (2000) 4493–4496.
[76] X. Gabaix, P. Gopikrishnan, V. Plerou, H.E. Stanley, A theory of power law distributions in financial market fluctuations, Nature 423 (2003) 267–270.
[77] D. Sornette, Why Stock Markets Crash: Critical Events in Complex Financial Systems, Princeton University Press, Princeton, 2002.
[78] C. Shea, Econophysics, http://www.nytimes.com/2005/12/11/magazine/econophysics.html, 2005 (acessed 01.08.14).