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Creative Productivity and Age: A Mathematical Model Based on a Two-Step Cognitive Process

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It is argued that several empirical aspects of the relation between age and productivity can be explained by hypothesizing a simple two-step model of the creative process. Such a hypothesis permits a delayed single-peak function to result from an underlying process of constantly decelerating decay. The derived equation describes creative productivity as a function of individual age. The equation is not only shown to be consistent with empirical data on the relation between age and achievement, but additionally several important empirical predictions and theoretical consequences are inferred from the model. For instance, the model (a) maintains that the age curves may be largely the intrinsic outcome of cognitive processes rather than the extrinsic effect of developmental changes or sociological influences; (b) predicts the explanatory superiority of professional over chronological age; (c) explains the observed positive intercorrelation among creative precociousness, productivity, and longevity in terms of their mutual dependence upon individual differences in creative potential; and (d) provides a substantive basis for interpreting the variation in age peaks across disciplines by introducing the concepts of ideation rate, elaboration rate, and creative half-life. Tests to confirm or disconfirm the theoretical model are also proposed.

Lehman's (1953) book *Age and Achievement* probably constitutes the most comprehensive single attempt to empirically determine the agewise changes in creativity. By systematically measuring the longitudinal fluctuations in creative productivity for a wide array of fields within the arts and sciences, Lehman concluded that achievement tends to be a curvilinear function of age. From the onset of a creator's career, productivity tends to rapidly increase, then level off at some peak productive age, and thereafter slowly decline with increased aging. To be sure, the validity of Lehman's conclusions has been often questioned, especially his prognosis of a creative decline in the later years (e.g., Cole, 1979; Dennis, 1956a, 1956b, 1958, 1966; Riley, Johnson, & Foner, 1972, p. 437; Zuckerman & Merton, 1972; cf. Lehman, 1956, 1960). Nonetheless, recent research employing multivariate techniques has demonstrated the general truth of Lehman's basic conclusions, even though specific details often need to be qualified (e.g., Simonton, 1977a, 1980c, 1984, Chap. 7). The broad curvilinear trend holds up even after controlling for stress, physical illness, social rewards, competition, and such potential methodological artifacts as the compositional fallacy (cf. Riley et al., 1972, Chap. 2). Indeed, Lehman's work has even survived cross-cultural and transhis-

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torical replications (Simonton, 1975), thereby suggesting that the developmental trends displayed in Lehman's tables may reflect some universal law of human behavior.

The main deficiency in Lehman's research may not be its methodological failings but rather its conspicuous lack of even a minimal theoretical orientation. Lehman was more engaged in documenting the existence of the curvilinear relationship than in developing a theory of why achievement adopts a particular longitudinal form. This is not to say that Lehman does not offer some potential ad hoc explanations, for he does make just such an effort in the last chapter of his book, even if as a mere afterthought at the request of E. G. Boring (Lehman, 1953, p. xi). But neither he nor any of his successors has made any serious attempt to generate and test a formal model. The sole exception is the attempt of many sociologists to propose nonpsychological explanations of the trend curve (e.g., Cole, 1979). Yet even these sociological interpretations have not been translated into predictive models which can be subjected to direct empirical scrutiny. So the fact remains that we lack any theoretical model which can generate precise predictions and interesting substantive conclusions (cf. Bayer & Dutton, 1977).

Any model of the relationship between age and creativity must satisfy three fundamental requirements. First of all, the model must be based on a minimal set of theoretically reasonable assumptions. Second, the model must fit the facts and thereby provide substantive interpretations of the phenomena. Finally, any model must be testable; it must be capable of being proven wrong. It is my purpose in this paper to offer a model meeting these three specifications.

THE MODEL

In this section we begin by deriving a predictive equation from a minimal set of theoretical assumptions. Then the formal characteristics of the model will be developed in order to expand its empirical utility.

Derivation of the Predictive Equation

It is commonplace to refer to creators as "drying up" or as "running out of ideas," as if creators eventually must exhaust a nonrenewable supply of creative ideas. Although this notion is often given as an explanation for the postpeak phase of a creator's career, the prepeak phase can also be an immediate causal repercussion of just such a decline in creative resources. This apparent paradox can arise if the creative process consists of two or more consecutive stages. Declines in the final stages will then lag behind declines in earlier stages, and the last stages may even display peaks long after the earlier stages have entered upon an irreversible decline. To offer a simple physical analogy, the hottest time of the day usually falls a couple of hours after the amount of solar radiation has already begun to decline. That lag results from the fact that the ambient temperature is not directly dependent upon sunlight—the thermometer readings being taken "in the shade"—but rather the radiation must first heat the air which in turn heats the thermometer. Atmospheric heating thus constitutes an intervening step between solar radiation and the thermometer reading. Since several researchers have suggested that the creative act may indeed consist of two or more mental stages (e.g., Blatt, 1961; Wallas, 1926), we will not be departing from past work on creativity if we also postulate that creativity entails some sort of two-step process. In particular, let us propose the following two steps.

Step 1. Each creator begins with a supply of "creative potential" which, during the entire course of the creator's career, becomes actualized in the form of "creative ideations."

In the case of science we may define a scientist's creative potential as the total number of papers he or she is personally capable of producing due to the acquisition of a finite set of technical skills and theoretical issues. This scientific potential becomes realized in the form of actual ideas for research projects, such as those which accumulate in the laboratory journal or notebook. Likewise, an artist or composer with a given set of techniques and esthetic predilections collects ideations in the form of sketchbooks.

Step 2. The ideas produced in Step 1 are progressively translated into actual "creative contributions" through the established means of disciplinary communication or publication.

Thus in the case of a scientist, ideas for research projects are converted into concrete theoretical or empirical investigations which are eventually published in professional journals. Similarly, an artist or composer works over initial sketches and drafts into a final product suitable for exhibition or performance.

Needless to say, there is nothing especially profound in the two-step process we have just hypothesized. We must only posit a relatively noncontroversial view of the creative act which can serve as a plausible substantive basis for the derivation of a testable equation. In fact, the main purpose of this paper is to show that many important complexities concerning the relationship between age and productivity can result from a relatively simple underlying causal process. Only later will we devote time to discussing the substantive basis of the model.

Let us start the derivation by denoting the three variables of creative potential, ideation, and contribution by the letters x, y, and z, respectively. That is, x may be considered as the number of potential creative ideas which a given individual is still capable of conceiving but has yet to enter his or her awareness, y is the number of actual conscious ideas which have not yet undergone elaboration into a creative product, and z is the cumulative number of final creative contributions.[1] We next need to propose expressions for how each of these variables change with respect to time t. It is reasonable to assume that the rate at which creative potential is used up is proportional to the amount of creative potential still available. This assumption is analogous to the "law of mass action" in chemistry. In more concrete terms, as a creator's resources "dry up," inspirations become fewer and farther between. Using the nomenclature of calculus, the rate of diminution of x can then be expressed as

$$\frac{dx}{dt} = -ax, \tag{1}$$

where a is a proportionality constant greater than zero; a may be termed the "ideation rate."

At the other end of the two-step creative process we would anticipate that the rate at which contributions can be produced is directly proportional to the number of ideations. For instance, the more ideas for projects, the more projects can be going on simultaneously, and hence the higher the publication rate (cf. Hargens, 1978). Accordingly,

$$\frac{dz}{dt} = by, \tag{2}$$

where b is another nonnegative proportionality constant; b may be called the "elaboration rate."

Finally, we must acquire an expression for the rate at which ideation changes. Here we can employ differential Eq. (1) and (2) since the rate of ideation is equal to the difference between the rate of "drying up" and the contribution rate. That is,

$$\frac{dy}{dt} = ax - by. \tag{3}$$

Equations (1)–(3) make up a system of simultaneous first-order linear differential equations of the change rates of the three substantive variables. These equations can be solved if we set down two postulates. First, we will quite reasonably assume that both the number and the rate of contributions are zero at the outset of a creator's career (i.e., $z = dz/dt = 0$ at $t = 0$). Second, we must define the identity $x + y + z = m$ where m is a positive number representing the maximum number of contribu-

[1] We are providing this model with greater psychological realism by claiming that Step 1 entails an insight process where unconscious ideas become conscious. This claim is not essential for the model's acceptance. However, this idea connects the model both with the introspective reports of creative individuals (Ghiselin, 1952) and with theoretical models of the creative process which focus on subconscious thinking (e.g., Simonton, 1980a).

tions a creator is theoretically capable of producing in an infinite lifespan. Given these two assumptions, we can eliminate both x and y to yield a single second-order differential equation (Boyce & DiPrima, 1977, pp. 272–275). The solution to this new equation expresses z as a function of t alone, including estimates of the two integration constants. However, we are far more interested in obtaining the contribution *rate* for any given time t. The rate of contribution is the same as the productivity, so let us call the variable we are seeking "creative productivity" and symbolize it by p (i.e., $p = dz/dt$). Then the first derivative of the solution equation for the system of Eqs. (1)–(3) is

$$p(t) = c\,(e^{-at} - e^{-bt}), \tag{4}$$

where $c = abm/(b - a)$. Using two exponential functions ($e = 2.718 \ldots$), Eq. (4) expresses creative productivity totally as a function of the time t.

As is often the case, Eq. (4) is related to many other equations describing analogous physical or biological processes (Boyce & DiPrima, 1977, p. 141). For instance, this equation is identical to that describing the concentration of a drug in the bloodstream, where a, b, and c are constant for a given dosage of a drug injected into the body at time $t = 0$ (Burghes & Wood, 1980, pp. 73–74). Moreover, the integral of Eq. (4) (i.e., the solution equation to system 1–3) is identical to the equation for predicting the concentration of the end product of two consecutive unimolecular chemical reactions (Mellor, 1912/1955, pp. 434–436). Both phenomena concern two-step processes, just as we have assumed for in the current substantive problem.

Formal Properties of the Model

To obtain a more concrete idea of the general substantive nature of Eq. (4), it is advisable to work out some of its special mathematical features. From inspection it is clear that $p = 0$ when $t = 0$; that is, creative productivity is zero at the very beginning of a career (this attribute by assumption). It is equally evident that as t becomes very large, p approaches zero in the limit. So as a creator becomes very old his or her productivity declines asymptotically to zero. Thus, the equation satisfies a mandatory requirement of any function describing the relation between age and productivity: The predicted productivity value is never negative. There is maximum value between the beginning and the end of the creator's career. Using a little calculus, this single peak can be shown to occur at

$$t_1 = \frac{1}{b - a} \ln \frac{b}{a}, \tag{5}$$

where "ln" means the natural logarithm (to base e). Hence, the maximum point is a function of the two rate constants a and b, but not the integration constant c. The latter constant does not determine the general shape of the curve but rather serves only as a scaling constant along the p axis. That is, c determines the average level of productivity across a creator's career. This productivity constant can therefore be used to estimate the maximum contribution rate at t_1. Substituting Eq. (5) into Eq. (4) results in

$$p(t_1) = c\left(\frac{b - a}{a}\right)\left(\frac{a}{b}\right)^{b/(b-a)} \tag{6}$$

Thus the peak productivity value is totally determined by the two rate constants and the integration constant, as we would expect.

Examination of the second derivative of Eq. (4) indicates that the age curve will only display a peak at t_1 if $a < b$ (see, e.g., Burghes & Wood, 1980, p. 74). In substantive terms, the rate at which creative potential is converted into creative ideations must be less than the rate at which these ideas are elaborated into published contributions. In other words, it is the ideation rate and not the elaboration rate which limits the flow of contributions, a reasonable enough statement. The second derivative reveals another important attribute of Eq. (4), namely, that at a certain point after the peak the age curve changes from being concave downward to concave upwards. This zero-acceleration point occurs at

$$t_2 = \frac{1}{b - a}\ln\frac{b^2}{a^2}\,. \tag{7}$$

From t_2 on (where $t_2 > t_1$ if $a < b$), the decline in creative productivity begins to decay toward the asymptote of a zero contribution rate. This inflection point is certainly more substantively realistic than a simple second-order polynomial function with a constantly accelerating decline (cf. Simonton, 1977a). The latter would eventually yield *negative* productivity scores with increased time, a meaningless result to be sure. In fact, the model's inflection point and asymptotic decline render its functional curve superior to any finite polynomial age function.

Equation (4) has one final advantageous property: It can be used to calculate the quantity of contributions that can be expected between any two points in a creator's career. In particular, the number of contributions made between times $t = r$ and $t = s$ is given by the definite integral

$$\int_r^s p(t)dt = c\left[\frac{1}{b}\left(e^{-bs} - e^{-br}\right) - \frac{1}{a}\left(e^{-as} - e^{-ar}\right)\right]. \tag{8}$$

Equation (8) is probably the most useful for empirical purposes. Usually we are most interested in predicting the number of contributions in any given period of a creator's career. This equation will therefore be exploited when the model is weighed against actual empirical observations.

CONCRETE APPLICATIONS

After arriving at plausible estimates of the critical parameters, we will compare the predictions of the model with research data on the relationship between age and productivity. We will also specify three predictions which should be born out by future empirical research if the underlying theoretical model is to have any explanatory value whatsoever.

Parameter Estimates

To get a better idea of the shape of the curve as defined by Eq. (4), it will help to graph the function for a "typical" highly productive creator. This task requires that we first obtain estimates of the constants a, b, and c. Such estimates can be founded on four assumptions which place constraints on the values of these parameters:

1. Let us assume that creative ideation begins around the 20th year of life, and hence, that $t = 0$ at this age. This assumption is in approximate concordance with past research on creative longevity (e.g., Albert, 1975; Dennis, 1966; Lehman, 1953; Roe, 1952). Observe that t is operationalized not in terms of absolute chronological age but rather more in terms of relative "professional age" (Lyons, 1968) or "career age" (Bayer & Dutton, 1977).

2. In a manner also in keeping with prior empirical work (e.g., Dennis, 1966; Lehman, 1953; Simonton, 1975), let us assume that for our hypothetical prolific contributor, creative productivity peaks around the 40th year of life. Hence given that t is measured in years, the peak occurs around $t_1 = 20$ (i.e., about 2 decades after the first creative idea).

3. Let us set the maximum annual contribution rate for this productive individual at around five items (i.e., publications, paintings, poems, papers, etc.) (cf. Albert, 1975; Cole, 1979; Zuckerman, 1977, p. 302).[2]

4. Finally, because we have no a priori knowledge about the relative size of the two rates a and b, let us simply assume that they are very nearly equal (with the theoretically mandated provision that $a < b$).

Because of the extremely approximate nature of the above constraints, it is meaningless to look for constant values with several significant figures. Therefore, the estimates $a = .04$, $b = .05$, and $c = 61$ suffice for

[2] This rate is deliberately taken at a somewhat high figure to represent an upper bound for the most productive creators in a typical field. For less-productive creators in certain disciplines this estimate would be far too high.

our purposes. Using Eqs. (5) and (6), these values yield a peak productive age of about 42 years old (i.e., $t = 22.3$) and a maximum contribution rate of five items per year. Using these constants in Eqs. (4) and (8), we can calculate the descriptive data shown in Table 1 and plot the corresponding curve shown in Fig. 1.

Looking at Fig. 1 we should observe that the derived function does take the expected form: A rapid rise in creative productivity is followed by a gently sloping decline. Moreover, the curve is not concave downward throughout the entire range of the independent variable t. Since the second derivative is zero at $t = 44.6$, the curve becomes concave upward after around age 65. Accordingly, the rate of decline after this age is always *decreasing*. That is, creative productivity at that age levels off so as to approach a zero contribution rate at an ever decelerating pace.

Empirical Comparisons

We need not prove that Eq. (4) fits the data perfectly. Creative productivity over an individual's career is no doubt an extremely complex phenomenon with multiple determinants (see e.g., Roe, 1972; Simonton, 1977a, 1980b). But we must demonstrate that the model offers a sufficiently good first approximation that it is indeed worthwhile to deduce some further theoretical statements. This demonstration is facilitated by empirical data which satisfy the following four qualifications:

1. The data must be truly longitudinal rather than cross-sectional. That is, the information must follow individual creators throughout the course of their careers. Attempts to infer age effects from cross-sectional data of creators at different points in their respective careers are methodologically dangerous (e.g., Bayer & Dutton, 1977; Jaquish & Ripple, 1981). It becomes virtually impossible, for example, to divorce age effects from cohort effects (Riley et al., 1972).

2. The data should span the years 20 to 80 of an individual's life, and subdivide this span of time into decades. This specification assures maximum comparability across data sets.

3. The data should either give the productivity per decade as a percentage of total lifetime productivity or else present enough information so that such percentages can be easily calculated. This ipsatization also facilitates comparison across different data sets. It must be emphasized that the predicted proportions per decade are totally determined by the parameters a and b. Accordingly, we need not concern ourselves with estimating different values of c for the various data sets.

4. The data must make some provisions for a special type of compositional fallacy which can introduce artifacts into the agewise distributions of creative productivity (Riley et al., 1972, Chap. 2). Even though strictly speaking Eq. (4) applies only to the prediction of individual productivity,

TABLE 1

PROPERTIES OF THE CURVE $p = 61\,(e^{-.04\,t} - e^{-.05\,t})$ PREDICTING CREATIVITY AS A FUNCTION OF AGE

t Range	Age period	Productivity at end of decade	Total contributions of decade	Cumulative contributions of decade	Percentage total contributions for decade[a]	Cumulative percentage for decade[a]
0–10	20–30	3.89	22.7	22.7	10.0	10.0
10–20	30–40	4.97	45.8	68.5	20.2	30.2
20–30	40–50	4.76	49.3	117.8	21.7	51.9
30–40	50–60	4.06	44.3	162.1	19.5	71.4
40–50	60–70	3.25	36.5	198.6	16.1	87.5
50–60	70–80	2.50	28.6	227.2	12.6	100.1

[a] Assuming that the creator lives to age 80 ($t = 60$).

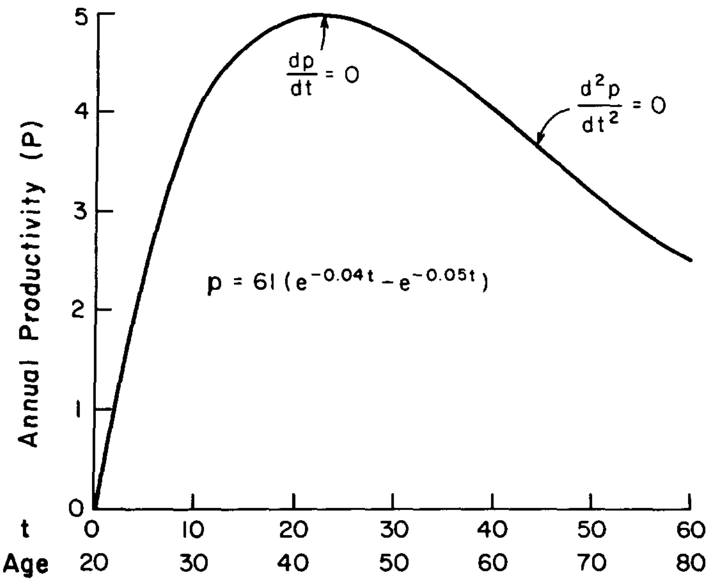

FIG. 1. Predicted productivity (i.e., contribution rate) as a function of age.

we are obliged to test the equation against data which has been aggregated across many individual careers. This necessity has a certain virtue in the highly ideographic factors in the career of any single individual (e.g., deaths in the family, job changes) that should cancel out when the results are averaged across many persons. Furthermore, if the individuals come from several different cohorts, period (or historical) effects are smoothed out in the aggregation process. So such aggregate data can actually offer more reliable indicators of agewise changes in productivity. Yet a problem remains: It is perfectly possible for a curve descriptive of the aggregate to poorly describe every single person composing the aggregate. This discrepancy can happen if the individuals are highly heterogeneous with respect to some crucial variable. In this case the critical variable is lifespan. Because no person can be very productive after death, to aggregate across individuals with very heterogeneous lifespans can greatly exaggerate the decline seen in the aggregate curve. The aggregate decline may not hold for any individual who lives into ripe old age. This compositional fallacy can be handled a number of ways. Dennis (1966), for example, only included creators who lived to be octagenarians. In contrast, one of the acute faults in Lehman's (1953) data is that this fundamental artifact is almost invariably ignored (Riley et al., 1972, p. 437).

Three data sets were found to meet the foregoing specifications (viz. Dennis, 1966, Table 1, p. 2; Lehman, 1953, Table 58, p. 317; Zuckerman, 1977, Table E-1, p. 302). These are shown in Table 2 along with the

TABLE 2

CORRELATIONS BETWEEN PREDICTED AND OBSERVED PRODUCTIVITY PERCENTAGES ($a = .04$; $b = .05$)

			Observed percentages					
			Lehman (1953)	Dennis (1966)			Zuckerman (1977)[b]	
t Range	Age period	Predicted percentage	General	Scholarship[a]	Sciences	Arts	Laureates	Controls
0–10	20–30	10	10	4	6	9	6	8
10–20	30–40	20	30	14	17	23	17	22
20–30	40–50	22	27	19	22	26	21	18
30–40	50–60	20	19	20	21	21	17	23
40–50	60–70	16	9	21	20	14		
		} 29					39	29
50–60	70–80	13	3	20	15	6		
Correlation (r)			.82[c]	.55[d]	.86	.82[e]	.96	.93

Note. Calculations assume that the creator begins producing at 20 and ends at 80.

[a] I corrected an error in his Table 1 using his Table 2.

[b] These percentages were calculated from her productivity rates. The rate for "60+" was assumed to cover the 2 decades from 60 to 80 years.

[c] If $a = .05$ and $b = .06$, then the fit improves considerably ($r = .99$).

[d] If $a = .02$ and $b = .03$, then the fit improves considerably ($r = .999$).

[e] If $a = .05$ and $b = .06$, then the fit improves considerably ($r = .97$).

predicted values and the Pearson product–moment correlations between predicted and observed percentages.[3] Notice that the correlations between predicted and observed percentages tend to be respectably high, normally in the .80s and .90s. Thus anywhere between two-thirds and over 90% of the variance is shared. The one exception is the data on scholarly productivity where the shared variance is only 30%. Actually, these comparisons should be considered conservative given that they are based on a single set of values for a and b. If creators in different disciplines peak at different ages, this must be reflected in the parameter estimates (cf. Bayer & Dutton, 1977). As an example, relative to scientists, artists often tend to peak earlier whereas scholars tend to peak later (Dennis, 1966). These tendencies will not only affect the comparisons for the Dennis (1966) data, but also the Lehman (1953) data if some disciplines are more heavily weighted than others. If we assume that $a = .05$ and $b = .06$ for the Lehman general and the Dennis arts data, then the correlations between predicted and observed percentages are in the high .90s; if $a = .02$ and $b = .03$ for the Dennis scholarship data, the correlation becomes almost perfect (see Table 2). With these changes, the correlations range from .86 to .999, with an average correlation of around .95. In different terms, around 74 to almost 100% of the variance in observed scores is "explained" by the model. Such percentages are far superior to those found in previous research (e.g., Bayer & Dutton, 1977). A second-order polynomial age function, for instance, explains only about one-third of the variance (Simonton, 1977a).

We can make one final empirical comparison, this time with a data set which does not completely comply with the initial specifications but which may prove useful nonetheless. Cole (1979, Table 2, p. 962) presents data on scientific productivity for the age periods "Under 35," 35–39, 40–44, 45–49, 50–59, and "60+." Let "Under 35" be taken to mean 20–34 and "60+" to signify 60–80. Then converting his productivity rates for these six time periods (across six fields) into percentages yields 24, 9, 10, 8, 16, and 32. Employing Eq. (8) we obtain the corresponding predicted percentages 20, 11, 11, 11, 19, and 29. The correlation between these two proportions is .97, again an honorable degree of concordance. Some 94% of the variance is shared.

Besides illustrating the accuracy of the model, the above comparisons

[3] Here the correlation coefficient is used solely as a descriptive, not as an inferential, statistic. Of course, the correlation is not always the most suitable measures for a "goodness-of-fit" test. But that is not the real purpose of the present section. Theoretically, we would expect significant departures from the model's predictions. It should also be pointed out that the arcsine transformations so frequently used for calculating correlations between proportions are unnecessary here because that procedure made no difference in the outcome (Cohen & Cohen, 1975).

have instructed us about the most likely estimates of the key parameters a and b. In particular, we can fairly safely propose the interval estimates of $.02 < a < .05$ for the ideation rate and $.03 < b < .06$ for the elaboration rate, where always $a < b$. At one extreme, if the peak productive age comes early in the career, then estimates in upper ranges are appropriate. With an ideation rate at 5% and an elaboration rate at 6%, for instance, creative productivity tends to rise very rapidly to an early peak around 38 years and thereafter declines relatively rapidly.[4] At the other extreme, if the peak productive age arrives late in the creator's career, it is more reasonable to select the estimates in the lower range. If we take $a = .02$ and $b = .03$, for example, creative productivity increases very gradually to a peak age of 60 years and henceforth declines very slowly.

All of this makes sense when we underline the substantive meaning of these two parameters; a is the rate at which creative potential is transformed into ideations and b is the rate at which these ideations are elaborated into published contributions. The higher the numbers, the faster the rates, and hence the more quickly the creative potential is worked into contributions via the hypothesized two-step process. By comparison, low estimates of these parameters signify slow rates, and consequently a much bigger time scale is required for the same creative transformations to occur. These parameters should vary across disciplines accordingly. Certainly conceiving and polishing a sonnet or a mathematical proof is a small-scale task in comparison to the large-scale enterprise of outlining and elaborating a novel or biological taxonomy. Therefore, any future investigations attempting to check Eq. (4) should take care to choose the parameter estimates most suitable for the disciplines studied. Further, these coefficients can be usefully compared for the light they shed on the different rates at which the creative process operates in different disciplines. To illustrate, from the above empirical tests we have learned that the ideation rate is over twice as fast for artists as for scholars (i.e., 5 vs 2%, respectively).

Critical Predictions

Although Eq. (4) seems to be in good agreement with empirical data, especially once provision is made for interdisciplinary variation in the parameters, I should specify the three areas where the model is most vulnerable to empirical disconfirmation. It is one thing to demonstrate that the longitudinal movement in creativity follows the broad outline of Eq. (4), and quite another to show that the curve adequately describes

[4] If we want the peak to appear even earlier, the estimates $a = .09$ and $b = .1$ yield a peak productive age of 30 years, a youthful peak which might be more appropriate for physics, mathematics, poetry, and song compositions (cf. Lehman, 1953).

specific details concerning how a career begins, how it ends, and what happens in between.

Decelerating career onset. The first prediction has to do with the onset of creative productivity. According to the model, the agewise growth in creativity decelerates into the peak productive phase from the very beginning of the career at $t = 0$. Such a curve is similar to the usual "learning curve" so familiar to psychologists. This curve also contrasts markedly with the logistic curves which characterize growth processes bounded by some fixed growth-limiting constraint (e.g., population growth restrained by food production). A logistic function begins with an accelerating curve which only later decelerates into the maximum. Thus, the precise shape of the initial curve has definite and diverging theoretical implications. For the current model to be acceptable, empirical research must indicate that agewise productivity does not usually accelerate prior to deceleration. Regrettably, past research cannot be employed to verify this prediction. Although many investigators have found curves consistent with the model (e.g., Davis, 1954; Lehman, 1953, with some exceptions), these studies all dealt with aggregated data. As such, the compositional fallacy may lead to summary curves which are not congruent with those on the individual level. The impact of this incongruency can be devastating, biasing the results against the model. To be specific, if the age at which creative persons begin their productive careers is normally distributed about some mean value, then the average productivity scores computed across persons per age period will exhibit a positively accelerated initial curve even if the individuals uniformly display curves perfectly compatible with the model. Thus the first prediction should be tested using individual rather than aggregate data, preferably using a cross-sectional time-series design (e.g., Simonton, 1977a).

An alternative to employing individual data is to go ahead and aggregate the productivity tabulations across all sampled individuals, but then to define the units in terms of career age. This procedure is far more convenient, and enjoys the additional asset of smoothing over the transient disturbances in output for each individual that may deflect the productivity curves from the idealized form. To illustrate this approach, while concomitantly providing some empirical support for the first critical prediction, I took the 74 most eminent psychologists listed in Watson's (1974) *Eminent Contributors to Psychology,* a compendium that offers a bibliography of primary references for distinguished and deceased psychologists. The rationale for selecting the most eminent (namely, those with eminence scores of 24 or above) was to take advantage of the correlation between eminence and productivity, thereby obtaining the most reliable tabulation with the fewest subjects. In any case, I looked at the number of contributions made during the first, second, and third 3-year periods after the year of first publication for each psychologist. The aggregated tabulation of output was therefore defined in terms of individual career age, where the $t = 0$ was marked (albeit approximately) by the appearance of a first publication. The count in the first 3-year age period was 119, in the second 164, and in the third 179. If the age function were strictly linear, the tally for the second period should have been 149, and, more critically, that count should be less than 149 if some kind of growth process were operative. The count is thus far more than 10% higher than we would anticipate from an accelerating function. Hence, in accord with the model, the curve is decidedly concave downward in a decelerating positive monotonic function at the onset of a productive career. The rival logistic curve is thereby ruled out by the data.

Decelerating career conclusion. The second critical point on the curve appears during the final decades of life. The model predicts that the decline in creative productivity decelerates in the later years, approaching the zero productivity level only asymptotically. Again, while data gathered to date appear largely consistent with this prediction (e.g., Lehman, 1953), the methodological precautions expressed regarding the first prediction deserve consideration here as well. In addition, the sample must include creators who lived long enough to exhibit the inflection point (e.g., octagenarians such as those selected by Dennis (1966)). According to the model, the decline may not begin to level off until after 65 years of age. To once more demonstrate the preferred route to verification, I sampled all 196 psychologists in the Watson (1974) book who lived to be at least 70 years of age. Such a large sample size was dictated by the lower productivity rates in the last years of a career. Three consecutive 2-year age periods were examined from ages 65 to 70 (viz. 65–66, 67–68, and 69–70). The three scores were 182, 146, and 132. If the function were linear, the middle score should have been 157, and consequently the function appears to be concave upward, or conspicuously decelerating, after age 65, the predicted inflection point for those fields, such as psychology, where the peak productive age occurs around the 40th year. Of course, these data cannot be adopted as proof, but they are collaborative.

Single major peak. The third and final prediction concerns the middle section of the age curve for productivity. The equation derived from the model yields an unequivocal single-peak function. Yet occasionally researchers have spotted two peaks in the relationship between age and productivity (e.g., Bayer & Dutton, 1977; Davis, 1954; Haefele, 1962, pp. 235–236, 295; Pelz & Andrews, 1966; cf. Simonton, 1977a). The primary peak is the highest and occurs usually at the age of around 40 years, whereas the smaller, secondary peak occurs somewhat closer to retirement age. A double-peaked function is particularly prominent in mathematical creativity (Cole, 1979; Dennis, 1966). Once more, these findings are all founded on aggregated data and therefore the results cannot be accepted with complete confidence. For example, it is possible that the double-peaked function found in mathematical output is a compositional fallacy due to the aggregation of two very different types of mathematicians, namely, those in pure mathematics who tend to peak relatively early and those in applied mathematics (especially statistics) who tend to peak relatively late. It is also not unlikely that double-peak age curves may reflect the distortion of the natural creative process by external forces. The fact that the secondary peak may occur toward retirement especially suggests the influence of extraneous factors. Nonetheless, if a double-peak function can be shown to be a stable attribute on the individual level, even after controlling for heterogenous populations and extrinsic influences, then the present model must be considered invalid (or at least for those disciplines which consistently feature such functions).

The most efficient procedure for testing this prediction is to determine whether in a given set of longitudinal data productivity fluctuations can be described by a fourth-order polynomial age function. That is, a double-peak function requires an equation with linear, quadratic, cubic, and quartic age terms. Accordingly, to render the model empirically implausible we need only show that the higher-order terms beyond the quadratic, and especially the quartic (or fourth-power) term, do not explain a significant amount of agewise variance over and above what can be already accounted for by a second-order polynomial with just linear and quadratic terms present. For example, a recent cross-sectional time-series analysis of eminent composers in classical music found that cubic and quartic age terms did not noticeably enhance the R^2 over what was already handled by a second-order age function descriptive of a single-peak curve (Simonton, 1977a). Such tests for the predictive utility of a fourth-order polynomial are mandatory before we accept the existence of a double-peak function: Due to the idiosyncracies of measurement errors, publication delays, extrinsic distortions, and other extraneous factors, longitudinal tabulations will seldom display a smooth curve, and, therefore, a modest departure from the expected form may be misinterpreted quite easily as a definite secondary peak. So before such an inference is drawn it is incumbent upon the researcher to demonstrate that the apparent resurgence is statistically reliable. Take, as a case in point, the putative double-peak function that Dennis (1966) found for mathematics (his Table 1). Even if we ignore the fact that the tabulation apparently combined pure and applied mathematics into a single composite, a trend analysis reveals that this distribution can be essentially considered a single-peak function anyway. The linear component accounts for 12% of the variance and the quadratic component for an additional 72%, signifying that a second-order age polynomial explains 84% of the variance.

A cubic term adds only 7%, an addition that does no more than to refine the tail in the later years of the age distribution. Most significantly, the fourth-power term increases the predictive power by only a 3% increment. Thus, even in this likely biased data the agewise curve can be said to be described by a quadratic (or cubic) single-peak function.

It should be stressed that I know of no other model having any substantive basis which makes predictions with as much elegance, precision, and comprehensiveness concerning the course of creative productivity throughout a career. For example, we have repeatedly compared Eq. (4) with a higher-order polynomial age function, especially the quadratic. Yet for two primary reasons such polynomials cannot be viewed as serious rivals. First, a theoretical foundation for this mathematical form has yet to be devised. When we say that a projectile, such as a bullet, follows the path of a second-order polynomial function (once we remove such disturbances as air resistance and turbulence), there is a firm theoretical rationale for doing so: A parabolic path is a logical consequence of two vectors, namely, the initial velocity and the downward acceleration due to gravitation. But a similar justification is wanting for a polynomial model of productivity. Until a theoretical underpinning is provided, a polynomial can only serve a descriptive role at best. Second, as we have observed earlier, a polynomial age function has the disadvantage of not offering realistic predicted values throughout the domain of the independant variable. Thus for those individuals who live long enough, a second-order polynomial will yield negative productivity scores, an absurd result to be sure. Equation (4), in comparison, always provides plausible predicted values even if a creator should happen to live as long as the Biblical patriarchs. Naturally, it is an established principle of mathematics that a polynomial can closely approximate any function if it is of sufficiently high degree. Even so, the insertion of higher powers of age very soon becomes a fruitless curve-fitting exercise that aggravates all the more any quest for a theoretical understanding. The key parameters of the current model (viz. a, b, and c) all enjoy specific substantive interpretations founded on how information is processed in a given discipline and how individuals vary in creative potential (two points that will become more obvious in the section that follows). If the three parameters of even a second-order polynomial (the intercept and the coefficients for the linear and quadratic terms) have yet to be assigned theoretical significance, how much more problematic it would be to give concrete interpretations for a higher-order polynomial! Therefore, polynomials simply cannot be considered competitive on either logical or empirical grounds.

Other rival models suffer from different drawbacks. For the most part, even though there do exist plausible mathematical functions with theoretically interpretable parameters, these cannot account for creative productivity over the lifespan. On the one hand, even if the onset of productivity were defined by an accelerating growth function, the logistic curve has nothing to tell us about either the shape of the peak or the form of the decline in later years. The same incompleteness attends, of course, the learning curve. On the other hand, even if we can easily resort to a simple exponential decay curve to predict the very last years, such a function is totally silent about the onset and peak portions of the agewise productivity distribution. In other words, in order to find a true competitor for the present model we must really piece together utterly separate theoretical processes and corresponding mathematical expressions. For instance, we could argue that the initial portion of the career is characterized by a learning process, the latter portion by a decay process (i.e., what we so arduously learn gradually decays). If so, we would have to define a third process to connect these two so as to smooth over the transition from a concave downward to a concave upward curve. Thus, in lieu of a single cognitive process with but three parameters we would end up with a patched-together collection of three separate processes each with its own set of parameters. Needless to say, such a mixture of incomplete models is both mathematically clumsy and theoretically cumbersome. Besides having to make arbitrary adjustments so as to assure that the first derivative exhibits no unrealistic discontinuities, we must somehow explain how or when one process takes over from the preceding one. This inelegance contrasts sharply with the present model in which a single cognitive process is presumed operative throughout, yielding an automatically unified curve. To the best of my knowledge, no other mathematical function having concrete theoretical meaning can account for the entire career course with as few parameters. Therefore, if the foregoing three critical predictions continue to be endorsed by the data, the current model would necessarily enjoy the greatest comprehensiveness with the most elegance. Moreover, as the following section will show, the model leads us to explanations and predictions beyond the mere shape of the curve—implications that also do not ensue from any other competing model.

THEORETICAL IMPLICATIONS

At this point we will work out the substantive consequences of the model in order to show that it can help explain some special aspects of creative productivity as well as provide a conceptual framework and nomenclature for discussing developmental changes in creativity. This discussion should indicate that the model is not merely a mathematical formalism but rather a proposition having substantive merit.

Precociousness, Productivity, and Longevity

Several investigators have pointed out that the most prolific creators in any given discipline tend to begin their productive careers very early and to end their productive careers very late in life (e.g., Albert, 1975; Dennis, 1954a, 1954b; Simonton, 1977b). This finding contradicts the common folklore view than an early start is associated with early "burn out." In any event, this empirical result cannot be considered a tautology. Even though high lifetime productivity would result from a long productive career, productive longevity may be the consequence of either an early start or a late termination, not necessarily both at once. Moreover, cross-sectional variations in lifetime productivity do not presuppose variations in career length; highly productive individuals may have careers equally long as their less prolific colleagues but still excel in lifetime total output due to superior productivity rates per annum. Thus taken altogether there are three independent sources of overall lifetime productivity, namely, how early the career begins, how late it ends, and the general level of the productivity rate throughout the career. Although these three factors are mathematically distinct, the present model predicts that all three influences should strongly intercorrelate with each other. The basis for this intercorrelation is the fact that all three factors are ultimately dependent on the single underlying variable of initial creative potential. All that we need to assume is that there are substantial individual differences in the amount of initial creative potential. Then, if we take two persons from the same discipline (and hence operating under identical a and b values), that individual with the higher initial creative potential will automatically exhibit earlier creative precociousness, higher productivity rates, and greater creative longevity in comparison with that individual with the lower initial creative potential.

To illustrate this point, let us return to Table 1 where we see a hypothetical individual with a creative potential of 305 contributions and a cumulative contribution score to age 80 of 227, and who is governed by the parameters $a = .04$, $b = .05$, and $c = 61$. Now suppose we examine a second hypothetical person whose initial creative potential is only 10% of the first person's. Since the ideation and elaboration rates are fixed under the assumption that both creators work in the same field, the difference must be made up in the integration constant c which is only 10% of the standard size for the second individual (i.e., 6.1). Because c determines the maximum productivity rate, the second individual will necessarily exhibit an average productivity rate much smaller than the first, one-tenth as small in fact. Furthermore, let us closely study what happens at the beginning and end of their respective careers. According to Table

1, the more productive creator is expected to produce about 23 contributions in the first decade of the career, whereas the less productive creator would be expected to yield about 2 contributions over the same time period. Since the productivity function is increasing over this time interval, the consequence of this differential expected output is that contributions will be more likely to be made earlier in the interval for the more productive creator. Thus the first creator has a higher probability of making a first contribution in the early 20s while the second creator is more likely to be near 30 before productivity begins.

This same probabilistic logic applies just as well to the last decades of life, only this time the creative productivity function is decreasing over the final intervals. Thus in the last decade from 70 to 80 years, the first individual is expected to make around 29 contributions and the second around 3 contributions. By the laws of probability (i.e., by converting the productivity function into a probability density function), we would anticipate that the more prolific individual would necessarily have a longer career. Hence, even though creative precociousness, productivity, and longevity are mathematically distinct, all three factors become mutually related under the model by their ultimate dependence upon creative potential. If a prolific creator tends to begin early, end late, and produce at exceptional rates, that creator presumably began with a superior supply of creative potential.

The preceding explanation was founded on the fact that the hypothesized age curve is increasing at the beginning and decreasing toward the close of the productive career. It is also significant that the rate of increase at the beginning is much larger in absolute value than the rate of decrease at the end. That is, the average slope prior to the peak productive age is steeper than the average slope after that maximum point. This contrast between initial and terminal slopes provides the basis for an empirically verifiable prediction. If individual differences in creative potential are necessarily represented as individual differences in precociousness and longevity, as just demonstrated, then that variation will be differentially represented in these two variables to the extent that the slopes differ. Since the slope at which productivity begins is so steep, variation in initial creative potential will translate into less variance in precociousness than is the case for longevity, a variable based on the much more gentle slope of the later years. To be more precise, the model unambiguously predicts that the variance in the age of first contribution should be smaller than the variance in the age of last contribution across any sample heterogeneous as to creative potential. Naturally, to empirically test this prediction other sources of differential variance must be controlled. Most critically, lifespan is far more likely to affect when a productive career ends that when it begins (Simonton, 1977b), and therefore this variable must be partialled out of the computed variances (also see Simonton, 1975). Hence, even though there exists some evidence suggesting that the two variances differ as predicted (e.g., Allison & Stewart, 1974; Simonton, 1977b), no conclusive support can be summoned at this point. No study to date has controlled for lifespan. It should also be pointed out that the current model is not the only one which would predict this contrast in variances. The theory of "accumulative advantage" makes the same prediction (see Allison & Stewart, 1974). Nonetheless, if differential social rewards are partialled out as well (cf. Simonton, 1977a), then the two alternative models would yield divergent predictions. If both lifespan and social rewards are controlled, only the current model would still predict that the variance for age of productivity onset will be smaller than that for age of productivity termination.

But the most significant point to remember is that the model predicts the observed relationship among the three individual-difference variables of creative precociousness, productivity, and longevity. So far as I have been able to determine, it is the only theoretical model which can account for this significant empirical finding. Moreover, this explanation is predicated upon a most minimal assumption: All we have to assume is that there exists individual differences in creative potential. Those differences do not even have to be described by a specific probability distribution, whether normal, beta, negative binomial, or otherwise (cf. Allison & Stewart, 1974; Lotka, 1926; Price, 1976; Simon, 1955). The existence of such individual differences is not only prima facie plausible, but additionally there is ample empirical reason for believing that human beings are not homogeneous in creative potential (see, e.g., Allison & Stewart, 1974).

Productive Decline

As previously noted, one of the recurrent criticisms of Lehman's (1953) research involved his suggestion that creativity is more a manifestation of youth than of mature old age (e.g., Cole, 1979; Dennis, 1956; Zuckerman & Merton, 1972). In the absence of a theoretical model, this controversy has had no choice but to revolve around methodological issues. But the current model does indeed provide a theoretical framework for dealing with this debate. On the one hand, the curve defined by Eq. (4) is usually asymmetrical, so that the peak productive age falls somewhere in the middle of the first half of a creator's productive career (an outcome quite in keeping with the "harmonic mean" model of Zusne (1976)).[5] So

[5] According to Zusne's (1976) model, the age at which a creative psychologist produces his or her most significant work is equal to the harmonic mean of the age at the first and last publication. Though Zusne provides no theoretical justification for this peculiar rela-

the supposed creative fertility of youth appears endorsed by the present model. The sole exceptions are long-maturing disciplines such as scholarship. On the other hand, the "center of the mass," if you will, for a creator's productive career is situated in the more mature years of life. Returning to Table 1, we note that if a creator lives to age 80, the cumulative percentage to age 50 is around 52%. The proportion signifies that almost half of a creator's career remains after the half-century point. In fact, around *two-thirds* of the creator's lifetime output comes after the supposed "decline" sets in (thus endorsing the argument of Zuckerman & Merton, 1972, pp. 306–307). Now compare the productivity rates of the first and last decades in the same table. Although 10% of the lifetime contributions can be attributed to the first decade, almost 13% of the total contributions come from the last decade. In more concrete terms, creators in their 70s are slightly superior to creator's in their 20s. Indeed, it is in the middle of the next to the last decade that the decline in productivity tends to level off. As pointed out earlier, the second-derivative of Eq. (4) equals zero at around 65 years. From that point on the rate of decline continually decreases, approaching the $p = 0$ asymptote ever more slowly with age. Consequently, the creator can never really be said to "burn out" or to "dry up" or to "run out of ideas." In fact, using Eq. (8) we can obtain $m = 305$ as the theoretical upper limit to the number of lifetime contributions. Accordingly, by age 80 a creative person has produced only around 74% of his or her theoretical potential. Therefore, the present model maintains that creative productivity can continue virtually indefinitely despite the existence of a productive decline after the early peak.

Even though the decline in productivity cannot be said to be highly salient, there is another interpretation of the model which does indicate a pronounced decline in creative powers with age. The peak productive

tionship, he did show that the predicted value correlated .52 with the actual value for a sample of 213 eminent psychologists. To now offer an explanation, let us assume that, according to the constant-probability-of-success model, the most significant contribution tends to appear at the age in which the creator is most productive, a relationship that has been shown to be true for psychologists as well as for creators in other disciplines (Simonton, 1983). Let us further postulate that taking the age of first and last publication is approximated by taking the ages at the beginning and at the end of a career in which Eq. (4) yields the same predicted productivity scores. If $a = .04$, $b = .05$, and $c = 61$, for example, the productivity will be three papers per annum at both $t = 7$ and $t = 53$, or at chronological ages 27 and 73, respectively. Similarly, at $t = 10$ and $t = 42$, or at ages 30 and 62, the predicted annual rate is just shy of four papers. The harmonic mean of 27 and 73 is 39, that of 30 and 62 is 40, figures that fall comfortably close to the actual productivity peak of 42 years (or $t = 22$) for these given parameters. Nonetheless, the present model predicts that Zusne's model will not work for those disciplines such as scholarship, in which the peak age is much later in the career.

age may appear in the early 40s or later, but the peak age for creative ideation occurs much earlier. In fact, the onset of the individual's creative career marks the age at which creative ideas are emerging at the most rapid rate. To establish this point, let me merely recall the fact that according to Eq. (2) the number of creative ideas is directly proportional to creative productivity (viz. $y = p/b$). Using Eq. (4) it is easy to show, therefore, that the rate of change in ideation is given by the formula

$$\frac{dy}{dt} = \frac{c}{b}\left(be^{-bt} - ae^{-at}\right) . \tag{9}$$

The above function describes how fast a creative individual is adding or subtracting from the store of ideas for scientific or artistic projects. When this function is positive, the creator is arriving at ideas faster than they can be elaborated into contributions. But when the function is negative, the withdrawals exceed deposits in this bank account of creative ideas. Thus it is significant that this function assumes its highest positive value at $t = 0$. At the onset of the career creative ideations are accumulated faster than they are being exploited. For example, if $a = .04$, $b = .05$, and $c = 61$, then the ideation rate at the onset of the career is a little over 12 ideas per year. Yet this ideation rate declines very rapidly as the reservoir of ideas becomes quickly transformed into creative productivity. By age 30 (i.e., $t = 10$) the ideation rate would be only around four ideas per annum and by age 40 this rate is almost zero ($dy/dt = 0$ at $t = 22.3$). In other words, the peak productive age marks the point at which there is a perfect balance of payments, ideas being placed into the cache of potential contributions at the same rate that some of these projects become finished products. Thereafter, the change rate is negative and thus the store of planned projects begins to decline. Fortunately, this depletion rate reaches a minimum at around 65 years (i.e., $t = 44.6$), and from then on approaches zero asymptotically from below. This minimum point in the inspiration depletion rate is precisely where the curve for creative productivity becomes concave upward. At that inflection point the difference between deposits to and withdrawals from the stockpile is -1.64. To illustrate, the maximum depletion rate of, say, a creative scientist would be slightly less than two papers per year. After this low point the annual drain from the stored resources is progressively reduced. In loose terms we can conclude that the picture actually becomes slightly more optimistic once the creator passes the traditional "retirement age" of 65 years old.

Isaac Newton claimed that his most creative ideas came to him when he was in his 20s the rest of his life more or less being devoted to the mere elaboration of these thoughts; the modern Nobel prize-winning sci entist Dirac has asserted that theoretical physicists are past their prime after 30 (Zuckerman, 1977, p. 164). In any event, it has been long maintained that originality is a prerogative of youth (see, e.g., Beard, 1874). At the same time, there is no doubt that creative productivity can continue very late into life, up to the final days (see Lehman, 1953, Chap. 14). By comparing the contribution rate described by Eq. (4) with the ideation rate described by Eq. (9), the model demonstrates that these two views are not necessarily contradictory. The highest ideation rate indeed occurs in the early 20s and precipitiously declines thereafter. Yet the peak age for productivity does not occur until over 20 years later, and the decline in the contribution rate decreases relatively slowly. Moreover, there is nothing in the current model to suggest that there should appear any decline in quality with increased age. If fewer "great" or "major" works are produced in the last years of a creator's career, it is only because the number of works contributed is too small for the odds of success to be favorable (see the "constant–probability-of-success" model developed by Simonton (1977a, 1980c, 1983) from Dennis (1966)).

The foregoing discussion does lead to some potentially useful questions for further research. Will longitudinal studies of the work habits of productive individuals reveal that the agewise curve representing the accumulation of ideas for projects is parallel to the productivity curve? Do ideas for projects accumulate faster in earlier portions of the individual's career? Will the age difference between the peak ideation rate and the peak contribution rate affect both the creator's self-view and the perceptions of others? Is the early portion of the career viewed as a phase of more conspicuous originality, inspiration, and enthusiasm, the later portion as a phase of more routine working through the previously acquired ideas? Does the fact that the ideation rate becomes negative by the mid-forties have anything to do with the so-called "mid-career crisis" which often occurs about the same time? These questions are deliberately suggestive. But research addressing these issues may help evaluate the theoretical plausibility of the model.

Chronological versus Professional Age

The model presumes that the longitudinal ups and downs in productivity are inherent in the creative process and not in age-related developments in intellectual or motivational factors. Thus no assumptions have to be made about longitudinal changes in IQ, creativity, achievement motive, or other psychological attributes. Nor do we have to make any assumptions about changes in the reward structure during the course of a creator's career (cf. Cole, 1979). On the contrary, once creativity begins with the first ideation, productivity takes its natural course over time (except where affected by drastic personal or social events). This is not to say that the onset of the career could take place any time within an individual's lifespan, but only that the agewise productivity curve is more determined by career age than by chronological age once the creative process begins. Taking two persons absolutely equal in initial levels of creative potential, the one who starts producing earlier will tend to peak earlier and to end productivity earlier. This aspect of the model can be cast as a critical test. Equation (4) should explain more variance in individual productivity if t is defined in terms of professional age than in terms of chronological age. In other words, t must be allowed to vary across individuals so that $t = 0$ at the beginning of each person's career. If subsequent research indicates otherwise—that a nomothetic definition does as well or better than an ideographic one—then there would be ample cause for rejecting the hypothesis that the age curves are an intrinsic outcome of a creative process. If chronological age explains more variance, then the age curves are more likely to be the extrinsic manifestation of some developmental changes in intellectual or motivational characteristics.

Naturally, few developmental psychologists would argue that chronological age is necessarily more central than professional age in the determination of longitudinal trends. Even the course of biological changes varies across individuals. Nevertheless, the current model at least provides one possible interpretation for the superiority of professional age as a predictor. Moreover, the model can handle other aspects of the agewise changes in creativity which are not so readily accommodated otherwise. For instance, by viewing the age curve as a manifestation of an information-processing activity, the model is able to explain differences in developmental trends across various disciplines. Different fields probably require contrary ideation and elaboration rates, these in turn directly affecting the course of the creator's career. An explanation which strove to attribute the career course to extrinsic developmental influences would encounter greater difficulty in treating such interdisciplinary contrasts, whatever the success in anticipating the superiority of an ideographic definition of age. Hence, the present model probably stands on firmer ground than any of its rival interpretations. And certainly the model is perforce disconfirmed if chronological age is not a more inferior predictor of productivity than is professional age. A developmental analysis can always fall back upon strict chronological age in a post hoc fashion, a luxury not enjoyed by an information-processing viewpoint.

The Half-Life of Creative Potential

If we solve differential Eq. (1) (using integration by separation of variables) we obtain

$$x = ke^{-at}, \tag{10}$$

where k is yet another integration constant ($k > 0$). This equation has the typical form of most natural decay processes, such as radioactive decay. Like those phenomena, creative potential is first depleted at a very rapid rate, yet the rate of decrease itself continually declines. The curve then levels off, approaching the asymptote $x = 0$ at an extremely sluggish pace. Consequently, creative potential is never really exhausted, but rather it is merely reduced to very small levels.

To acquire a better view of the magnitude of the decline, suppose we plot Eq. (10). For illustrative purposes, let us again assume that $a = .04$ and that the theoretical upper limit to the total number of contributions under the estimated constants is 305. If x measures the potential number of creative contributions, the x must equal 305 at $t = 0$. Hence $k = m = 305$, and we obtain the graph in Fig. 2.

Since the function represented by Eq. (10) is a typical decay curve, we can meaningfully speak of the half-life of creative potential just as we can speak of the half-life for a given radioactive element. The half-life is solely dependent upon the ideation rate represented by constant a, and hence that value has an additional substantive importance for the model. Specifically, when $a = .04$, the half-life of creative potential is 17.3 years. This number means that at $t = 17.3$, half of the initial creative potential

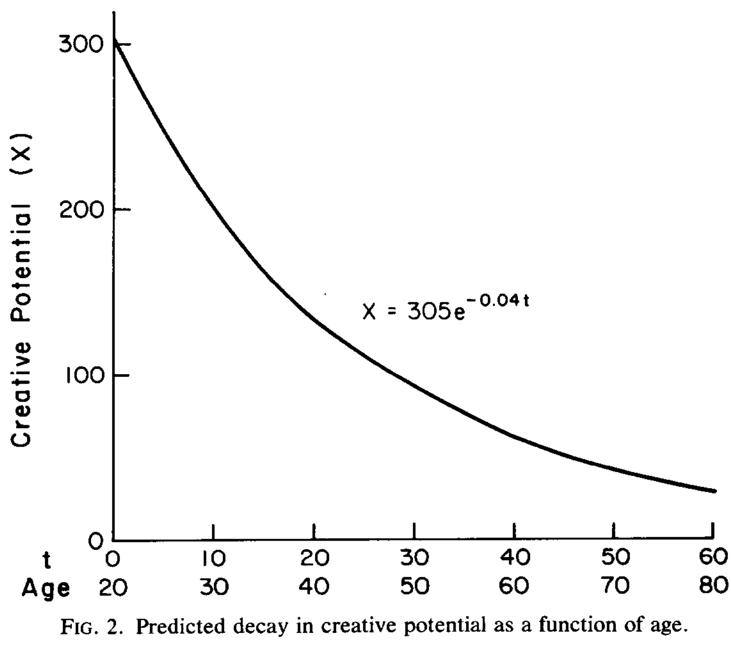

FIG. 2. Predicted decay in creative potential as a function of age.

has been used up in the form of ideations. Continuing the decay process, some 75% of the original potential has been actualized by $t = 34.6$ and almost 88% has been so consumed by $t = 51.9$. Translated into terms of chronological age, 50% of a creator's original potential remains at age 37, 25% at age 55, and 12% at age 72. Yet as with other decay phenomena, the supply of creative potential never really vanishes altogether. Even at age 90 the creator will still have the potential for around 18 additional inspirations for projects, and this number does not include the creative ideas already accumulated which have yet to be elaborated into published contributions. Hence we have again seen that even a model predicated on some kind of irreversible "drying up" process nevertheless predicts that creativity endures well into ripe old age.

This concept of half-life may be of value in comparing different fields. Since the half-life always equals the natural logarithm of 2 divided by the decay constant a, it is easily calculated from the estimated value of a. Returning to Table 2, we found that although a is .04 for the sciences, a is .05 for the arts and .02 for scholarship. Consequently, the half-life of creative potential is 13.9 for the arts and 34.6 for scholarship, a substantial difference of over 20 years. Some creative activities, such as mathematics or poetry, may have a's as large as .09, yielding a half-life of 7.7 years, a very short time span indeed (cf. Footnote 4). Whatever the details, the estimate of half-life values for the various disciplines can provide insights into how fast creative potential is exploited in various disciplines. This knowledge may then have implications for the nature of the creative process in different endeavors. To illustrate, the ideation rate of scholars is so relatively slow that scholars are almost 55 years old before they use up even half of their creative potential. By comparison, ideas for poems may emerge so fast that the half-way point for the consumption of the poet's creative potential may be reached before the 30th birthday. It is no wonder, then, that poets can actually die younger than scholars without suffering any appreciable damage to their overall reputation (Simonton, 1975). And it may not be too glib to suggest that this striking contrast between scholarship and poetry may be partially responsible for the very divergent stereotypes most of us have of slow and steady scholars versus rash and impulsive poets.

One particular asset of the concept of creative half-life is that it is not dependent upon the determination of the constant k. Two persons can have the same ideation rate even though their k values are totally unequal. On the one hand, k is presumed to be an individual-difference factor which gauges the initial creative potential of each person. As such, it must be determined ideographically. On the other hand, the ideation rate a is presumed to be an attribute of a given creative discipline. Defined by the analytical and synthetic strategies demanded in a particular endeavor, this value can be estimated nomothetically across individuals.

In fact, speaking in more general terms, we have assumed on several occasions that both the ideation rate and the elaboration rate are more characteristic of the information-processing requirements of a discipline than of individual creative potential. If we assume that the ultimate eminence of an individual within a given discipline is dependent upon the actualization of that potential in the guise of published contributions, then the placement of the peak within a career would not be influenced by the eminence that the person is able to attain on the basis of the initial potential. In other words, the shape of the agewise curve, including the location of the peak, is more a function of the discipline's particular attributes, whereas the height of that peak alone is partially decided by the individual-difference factors that define creative potential and hence eminence. Although more eminent persons may begin productivity earlier and end productivity later relative to their less-distinguished colleagues, the peak productive age should not differ. This specific prediction of the model has empirical support in the literature. Zusne (1976), for example, has reported that the estimated peak age is virtually identical for famous and less-famous psychologists (39.79 vs 39.48, respectively). Simonton (1975) found similar results for a sample of nonfiction authors. If future research continues to endorse a null relation between eminence and age of optimal output, we will have even more reason to believe that the peak productive age is determined mostly by the kind of intellectual operations required in a given creative endeavor.

CONCLUSION

The principal point of this paper is that many complex details concerning creative productivity can be explained using a simple model. Based on a conception of the creative act as a two-step process of ideation and elaboration, a single-peak function was derived which accurately predicts agewise changes in creative productivity. The ideation and elaboration rates determine both the age at which the peak productive age occurs and the steepness of the postpeak decline, thereby offering an information-processing interpretation of interdisciplinary differences in the functional relation between age and achievement. These interdisciplinary differences can be usefully characterized by the theoretical construct of creative half-life which was introduced to describe the decay in creative potential with age. The model also explains the observed empirical correlation among the three mathematically distinct individual-difference variables of creative precociousness, productivity, and longevity, the association appearing as the necessary repercussion of personological variation in creative potential. In addition to fitting the empirical facts gathered so far, the model makes predictions precise enough to be testable. Thus the model demands that creative productivity begin with a decelerated, concave-downward age curve which leads into a single peak and thereafter approaches the zero contribution baseline rate asymptotically. The model also requires that the independent variable in the predictive equation be operationalized in terms of professional rather than chronological age if the amount of explained variance is to be maximized. And the model predicts that the variance in the age of productivity onset must be smaller than the variance in the age of productivity termination across any heterogeneous sample of creators, even after making all due statistical provision for differential lifespan and social rewards.

It cannot be overemphasized that no other theoretical model can explain the same range of phenomena pertaining to longitudinal and personological variation in creative productivity. Even more critically, no other model offers predictions which are so precise that it can be disproven. The model can be empirically falsified. Yet these explanatory and predictive assets are obtained with a high degree of theoretical elegance. No assumptions are demanded about developmental changes in intellectual performance or motivation. Nor was it necessary to postulate alterations in social rewards or responsibilities. Creative productivity is seen to be a function of the creative process itself, independent of extrinsic developmental or sociological influences. Hence, the model suggests that the relationship between age and achievement is not automatically germane to questions about intellectual impairment (cf. Lehman, 1953) or about social reward structures (cf. Cole, 1979). To be sure, there is no theoretical reason for claiming that such extrinsic factors do not play some part in creative productivity. Indeed, such external influences may cause the age curves for any given individual to depart from the predicted values. Even so, Eq. (4) may be adopted as a convenient theoretical baseline. The corresponding function embodies the agewise flow of creativity in the ideal case of test-tube isolation. It describes the expected trend in productivity whenever the creative process is permitted to proceed sans environmental interference or facilitation.

No doubt some critics may challenge the theoretical utility of the model, whatever its empirical or explanatory value. Two major objections may be especially prominent. First of all, one of the primary postulates is that the creative process consists of the two steps of ideation and elaboration. Although it is absolutely essential to postulate more than one step, and even though such a postulate appears prima facie plausible, there is no particular reason why these steps have to specifically consist of ideation and elaboration. We might just as well assume that the two steps are analysis and synthesis (Blatt, 1961), diversive and directed thought (Berlyne, 1965), primary and secondary processing (Suler, 1980), divergent and convergent thinking (Guilford, 1967)—or any of a number of such multiple-stage processes discussed in the rich literature on creativity and problem solving. Since only the assumption of a multiple-step process is mandatory, however, it does not really matter if some other specification is preferred. Ideation and elaboration have been selected over the alternatives because these two paired processes make the fewest theoretical presumptions and are most intimately related to recordable behavior. There is certainly nothing to prevent a more detailed examination of actual creative individuals to assess the plausibility of these particular constructs. Thus, surveys of the work habits of productive persons can be used to verify whether the rate of change in ideas for projects is indeed described by Eq. (9) (cf. Hargens, 1978). So even if the predictive validity of the model is not dependent upon the acceptance of the ideation–elaboration operationalization, this particular choice seems both reasonable a priori and verifiable a posteriori.

It is essential to realize that the model is by no means committed to any particular substantive definition of the creative process, nor, significantly, does it even require that the creative process be confined to just two steps. This last point has to be stressed. The *only* prerequisite for the model to be theoretically justified is that the creative process consist of at least two steps. The model can easily accommodate more than two steps so long as we are only interested in the input variable of creative potential (x) and the output variable of creative contributions (z) with its corresponding first derivative called creative productivity (p). The intervening processes are then summarized by the intermediate variable y which we have called the ideation rate but which we could label otherwise should we ever have good substantive reason for doing so. Accordingly, the four-step description advanced by Wallas (1926)—preparation, incubation, illumination, and verification—could have been made the concrete basis of the formal model if we so chose. The first stage which the model has called "ideation" can be adopted as a schematic summary of the true underlying substeps of preparation, incubation, and illumination, while the second, so-called "elaboration" stage may be restyled "verification." Of course, if we do assume that the creative process consists of more than two steps, Eq. (9) becomes a rather less-meaningful description of the internal workings of the creative act. Nevertheless, Eq. (4) for creative productivity and Eq. (10) for creative potential remain totally unchanged no matter how many intervening steps are involved. Consequently, the concepts of creative half-life, the explanation of the relationships among precociousness, productivity, and longevity, the specific shape of the age curve for productivity, and the several testable predictions are all valid implications of the model. The sole essential postulate is that the creative process consists of two stages or more. The substantive foundation of the formal model is therefore extremely robust.[6]

The second vulnerable theoretical assumption is that each individual is assigned a finite initial amount of creative potential which is then used up during the course of the creator's productive career. This concept of creative potential was introduced with the least amount of operationalization. Unlike the number of creative ideas or the number of contributions, the quantity of creative potential is not immediately measurable. On the first glance it appears possible that this construct might be usefully operationalized in terms of standard creativity tests. There is some evidence that creativity scores may decline in a fashion not unlike the pattern seen in Fig. 2 (see, e.g., Bromley, 1956; Eisenman, 1970; cf. Horn, 1980; Jaquish & Ripple, 1981). Yet such an attempt at establishing empirical equivalence is misguided. Creativity tests assess broad individual differences in cognitive style, whereas the concept of creative potential is tied to a particular discipline. The potential of a young artist would have to be measured by a very different psychometric instrument than the potential of a young scientist (cf. Hudson, 1966). Nonetheless, this lack of an empirical definition can actually be made into something of an advantage. Despite the fact that a direct measure has not been specified, individual differences in creative potential can be calculated indirectly from available productivity data. That is, given the specific parameters descriptive of a particular discipline along with data on the number of con-

[6] This expressed robustness is still not the same thing as saying that *any* multiple-step (or even *any* two-step) process will do as well as the offered substantive basis. To qualify, first of all, a transformation must operate on some initial quantity of cognitions to alter it into a second quantity via some mediating transformation which has some "storage capacity" for an intermediate cognitive product. This transformational sequence, moreover, cannot operate at two separate phases of a career but must operate together throughout a career, albeit at changing relative rates. Finally, the precise manner in which these cognitive transformations take place must be consistent with the differential equations that establish Eq. (4) (e.g., the rate of transformation for the first quantity must be proportional to the amount of that quantity and the rate of emergence of the final product must be proportional to the quantity stored of the intermediate product). Thus in the current model a beginning supply of creative potential is transformed into a respository of ideations which are in turn transformed into outright contributions; this ideation elaboration process is assumed to occur throughout the career, that is, the two steps refer to the course of an idea and not to the course of a career; and the pace of ideation is proportional to the given creative potential remaining at a given point in the career, while the pace of elaboration is proportional to the amount of creative ideations, or works in progress, still awaiting refinement. In contrast, one reviewer suggested the following alternative two-step process: (1) social rewards reinforce novelty and creativity in the early stages of a career and (2) social rewards shift subsequently toward capitalizing on the results of the first step. But this will not do. Not only do these steps apply to careers rather than cognitions, but, additionally, some unrealistic assumptions must be made about how social rewards are dispensed proportionately to such intrapsychic events as creative potential.

tributions up to any given professional age, it is possible to work backward to obtain creative potential scores for sampled individuals. The model would also provide a theoretical justification for estimating creative potential from precociousness or productivity scores (cf. Simonton, 1977b). However calculated, these estimated values may prove useful in isolating the environmental factors most conducive to creative development (cf. Simonton, 1978).

Needless to say, many would probably object to the construct of creative potential not so much because it cannot be directly measured as because it posits that the agewise changes in creative productivity are governed by some sort of "drying up" process. And it obviously must be admitted that the current model takes this conventional notion as a fundamental foundation for explicating the relation between age and productivity. Still, in saying that creative potential is used up over time it is not necessary to assume that individuals learn nothing once their careers have begun. There is nothing to prevent the construct of creative potential from being flexible enough to include feedback from other professionals or from the world at large. The creative potential of a scientist, for example, can be said to entail the methodological skills and theoretical orientations obtained during graduate and postdoctoral training plus the advancements and revisions in those abilities and perspectives resulting from newly collected data or from critical reactions of colleagues. In this sense, the concept of creative potential which we are using to describe the individual is very similar to Kroeber's (1944) notion of cultural pattern exhaustion or Kuhn's (1970) idea of paradigm development, both within the larger realm of sociocultural systems (also see Sorokin's (1941) concept of "immanent change").[7] Thus, creative potential can involve a dynamic interaction with the environment over the course of a creator's career rather than a mere automatic and positive production of preconceived ideas. So the construct is genetic rather than preformationist. The postulate of an initial creative potential only excludes one situation for the theoretical justification of the model to be preserved: The creative individual cannot undergo any major "retraining" or "retooling" tantamount to starting professional life all over from scratch. In that case the progression of creative acts itself is reset back to the beginning, the ultimate upshot being another peak. Fortunately, the number of creative individuals who undergo such drastic revision of their scientific or esthetic directions is quite small. The vast majority of scientists and artists

[7] In fact, it is not unreasonable to suggest that the timewise form of Kroeber's cultural configurations and Kuhn's paradigm development may be dictated by a sociocultural analog of the current individualistic model. Such transhistorical movements tend to be characterized by a distribution of creative activity not unlike that seen in Fig. 1.

work within the same basic framework of viewpoint and technique throughout their careers (see, e.g., Crane, 1965; Hull, Tessner, & Diamond, 1978).

Perhaps the presumption of an initial nonrenewable supply of creative potential is objectionable because it seems on first blush to imply that even creative people "go downhill" with age. Since most of us will probably be lucky enough to get old, it is an easy issue to become sensitive about. Yet the theoretical model is very optimistic about the agewise course of creativity. After all, the formal equation predicts that by age 40 more than three-fourths of the creator's potential contributions have yet to appear, and by age 50 over half of that potential remains in the form of present or future ideas for projects. The number of contributions made in the sixth decade of a career is even predicted to be slightly larger than the number contributed in the first decade. And the postpeak decline is itself said to decline in such a way that a creator will never really "dry up" at no matter what age. Finally, the model neither assumes nor claims any developmental decrease in intellectual prowess, valuable knowledge, conceptual originality, motivational vigor, or any other presumably complimentary attribute. Seen from this perspective, the model cannot be very threatening in its implications. Even if one does not accept the idea of an irreversible decline in creative potential, the present model can be taken as a worse-case scenario which still features a rather positive view of the creator's last years. To the extent that creative potential can truly be replenished, furthermore, the picture regarding the relationship between age and achievement can only improve above this minimal comparative baseline.

REFERENCES

Albert, R. S. Toward a behavioral definition of genius. *American Psychologist,* 1975, **30,** 140–151.

Allison, P. D., & Stewart, J. A. Productivity differences among scientists: Evidence for accumulative advantage. *American Sociological Review,* 1974, **39,** 596–606.

Bayer, A. E., & Dutton, J. E. Career age and research—Professional activities of academic scientists: Tests of alternative non-linear models and some implications for higher education faculty policies. *Journal of Higher Education,* 1977, **48,** 259–282.

Beard, G. M. *Legal responsibility in old age.* New York: Russell, 1874.

Berlyne, D. E. *Structure and direction in thinking.* New York: Wiley, 1965.

Blatt, S. J. Patterns of cardiac arousal during complex mental activity. *Journal of Abnormal and Social Psychology,* 1961, **63,** 272–282.

Boyce, W. E., & DiPrima, R. C. *Elementary differential equations.* New York: Wiley, 1977. 3rd ed.

Bromley, D. B. Some experimental tests of the effect of age on creative intellectual output. *Journal of Gerontology,* 1956, **11,** 74–82.

Burghes, D. N., & Wood, A. D. *Mathematical models in the social, management and life sciences.* New York: Halsted, 1980.

Cohen, J., & Cohen, P. *Applied multiple regression/correlation analysis for the behavior sciences*. Hillsdale, N.J.: Erlbaum, 1975.
Cole, S. Age and scientific performance. *American Journal of Sociology*, 1979, **84**, 958–977.
Crane, D. Scientists at major and minor universities: A study of productivity and recognition. *American Sociological Review*, 1965, **30**, 699–714.
Davis, R. A. Note on age and productive scholarship of a university faculty. *Journal of Applied Psychology*, 1954, **38**, 318–319.
Dennis, W. Bibliographies of eminent scientists. *Scientific Monthly*, 1954, *79*, 180–183. (a)
Dennis, W. Predicting scientific productivity in later decades from records of earlier decades. *Journal of Gerontology*, 1954, **9**, 465–467. (b)
Dennis, W. Age and Achievement: A critique. *Journal of Geronotology*, 1956, **11**, 331–333. (a)
Dennis, W. Age and productivity among scientists. *Science*, 1956, **123**, 724. (b)
Dennis, W. The age decrement in outstanding scientific contributions: Fact or artifact? *American Psychologist*, 1958, **13**, 457–460.
Dennis, W. Creative productivity between the ages of 20 and 80 years. *Journal of Gerontology*, 1966, **21**, 1–8.
Eisenman, R. Creativity change in student nurses: A cross-sectional and longitudinal study. *Developmental Psychology*, 1970, **3**, 320–325.
Ghiselin, B. (Ed.) *The creative process*. Berkeley: Univ. of California Press, 1952.
Guilford, G. P. *The nature of human intelligence*. New York: McGraw–Hill, 1967.
Haefele, J. W. *Creativity and innovation*. New York: Reinhold, 1962.
Hargens, L. L. Relations between work habits, research technologies, and eminence in science. *Sociology of Work and Occupations*, 1978, **5**, 97–112.
Horn, J. L. Concepts of intellect in relation to learning and adult development. *Intelligence*, 1980, **4**, 285–317.
Hudson, L. *Contrary imaginations*. Baltimore: Penguin, 1966.
Hull, D. L., Tessner, P. D., & Diamond, A. M. Planck's principle: Do younger scientists accept new scientific ideas with greater alacrity than older scientists? *Science*, 1978, **202**, 717–723.
Jaquish, G. A., & Ripple, R. E. Cognitive creative celebitus and self-esteem across the adult life-span. *Human Development*, 1981, **24**, 110–119.
Kroeber, A. *Configurations of culture growth*. Berkeley: Univ. of California Press, 1944.
Kuhn, T. S. *The structure of scientific revolutions*. Chicago: Univ. of Chicago Press, 1970. 2nd ed.
Lehman, H. C. *Age and achievement*. Princeton, N.J.: Princeton Univ. Press, 1953.
Lehman, H. C. Reply to Dennis' critique of Age and Achievement. *Journal of Gerontology*, 1956, **911**, 333–337.
Lehman, H. C. The age decrement in outstanding scientific creativity. *American Psychologist*, 1960, **15**, 128–134.
Lotka, A. J. The frequency distribution of scientific productivity. *Journal of the Washington Acadamy of Sciences*, 1926, **16**, 317–323.
Lyons, J. Chronological age, professional age, and eminence in psychology. *American Psychologist*, 1968, **23**, 371–374.
Mellor, J. W. *Higher mathematics for students of chemistry and physics*. New York: Dover, 1955. 4th ed. (Originally published, 1912, Longmans, Green, New York.)
Pelz, D. C., & Andrews, F. M. *Scientists in organization*. New York: Wiley, 1966.
Price, D. S. A general theory of bibliometric and other cumulative advantage processes. *Journal of the American Society for Information Science*, 1976, **27**, 292–306.
Riley, M., Johnson, M., & Foner, A. (Eds.) *Aging and society*. New York: Russell Sage Foundation, 1972. Vol. 3.

Roe, A. *The making of a scientist*. New York: Dodd, Mead, 1952.

Roe, A. The maintenance of creative output through the years. In C. W. Taylor (Ed.), *Climate for creativity*. Elmsford, N.Y.: Pergamon, 1972.

Simon, H. A. On a class of skew distribution functions. *Biometrika*, 1955, **42**, 425–440.

Simonton, D. K. Age and literary creativity: A cross-cultural and transhistorical survey. *Journal of Cross-Cultural Psychology*, 1975, **6**, 259–277.

Simonton, D. K. Creative productivity, age and stress: A biographical time-series analysis of 10 classical composers. *Journal of Personality and Social Psychology*, 1977, **35**, 79–804. (a)

Simonton, D. K. Eminence, creativity, and geographic marginality: A recursive structural equation model. *Journal of Personality and Social Psychology*, 1977, **35**, 805–816. (b)

Simonton, D. K. The eminent genius in history: The critical role of creative development. *Gifted Child Quarterly*, 1978, **22**, 187–195.

Simonton, D. K. Intuition and analysis: A predictive and explanatory model. *Genetic Psychology Monographs*, 1980, **102**, 3–60. (a)

Simonton, D. K. Techno-scientific activity and war: A yearly time-series analysis, 1500–1903 A.D. *Scientometrics*, 1980, **2**, 251–255. (b)

Simonton, D. K. Thematic fame, melodic originality, and musical zeitgeist: A biographical and transhistorical content analysis. *Journal of Personality and Social Psychology*, 1980, **39**, 972–983. (c)

Simonton, D. K. Quality, quantity, and age: The careers of 10 distinguished psychologists. *International Journal of Aging and Human Development*, 1983, in press.

Simonton, D. K. *Genius, creativity, and leadership: Historiometric inquiries*. Cambridge, Mass.: Harvard Univ. Press, 1984.

Sorokin, P. A. *Social and cultural dynamics*. New York: Amer. Book, 1941. Vol. 4.

Suler, J. R. Primary process thinking and creativity. *Psychological Bulletin*, 1980, **88**, 144–165.

Terman, L. M. *Mental and physical traits of a thousand gifted children*. Stanford, Calif.: Stanford Univ. Press, 1926.

Wallas, G. *The art of thought*. New York: Harcourt, Brace, 1926.

Watson, R. I. (Ed.) *Eminent contributors to psychology*. New York: Springer, 1974. Vol 1.

Zuckerman, H. *Scientific elite*. New York: Free Press, 1977.

Zuckerman, H., & Merton, R. K. Age, aging, and age structure in science. In M. W. Riley, M. Johnson, & A. Foner (Eds.), *Aging and society*. New York: Russell Sage Foundation, 1972. Vol. 3.

Zusne, L. Age and achievement in psychology: The harmonic mean as a model. *American Psychologist*, 1976, **31**, 805–807.

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