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A thermodynamical approach towards group multi-criteria decision making (GMCDM) and its application to human resource selection

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ABSTRACT

In group multi-criteria decision making (GMCDM) problems, ratings are assigned to the alternatives on different criteria by an expert group. In this paper, we propose a thermodynamically consistent model for GMCDM using the analogies for thermodynamical indicators – energy, exergy and entropy. The most commonly used method for analysing GMCDM problem is technique for order of preference by similarity to ideal solution (TOPSIS). The conventional TOPSIS method uses a measure similar to energy for the ranking of alternatives. We demonstrate that the ranking of the alternatives is more meaningful if we use exergy in place of energy. The use of exergy is superior due to the inclusion of a factor accounting for the quality of the ratings by the expert group. The unevenness in the ratings by the experts is measured by entropy. The procedure for the calculation of the thermodynamical indicators is explained in both crisp and fuzzy environments. Finally, the effectiveness of the proposed method is demonstrated by applying to human resource selection problem.

1. Introduction

Group multi-criteria decision making (GMCDM) is a process used for ranking of alternatives based on different criteria. The applications of GMCDM are numerous and it has been applied to human resource management [1,2], transportation [3], portfolio optimization [4], product design [5], vendor selection [6,7], energy efficient network selection [8], robot selection [9] and visual inspection [10].

Different methods have been used in the past to solve the complex MCDM problems. Multi-attribute utility theory (MAUT) [11,12] was one among the first few methods developed for solving the MCDM problems. MAUT involves the representation of the preferences by means of a utility function to every possible consequence. MAUT takes into account the uncertainty and the preferences of the decision maker. However, its application requires extensive input at every step. On the similar lines of MAUT, analytic hierarchy process (AHP) [13] was developed. AHP uses pairwise comparison to estimate the criteria weights and also to compare the alternatives with respect to various criteria. Some of the advantages of AHP includes the ease of use, scalability and lesser data requirement compared to MAUT. AHP does not take into account the interdependence between the criteria and the alternatives. Pairwise comparisons in AHP lead to inconsistencies in judgement and rank reversal. AHP was later extended to analytic network process (ANP) [14] which is a nonlinear form of AHP, mostly suited for problems with network structure. ANP allows for the prioritization of the groups and handles interdependence of the alternatives better compared to AHP. However, ANP also ignores the interdependence among the groups. The ability of fuzzy set theory to deal with the imprecise and uncertain data has been utilized extensively by many researchers in MCDM [15–21]. Most of these methods were based on pairwise comparison. Fuzzy theory has also been used to develop consensus models in MCDM [22–27]. Various mathematical programming models have also been developed for MCDM in which the multiple and conflicting objective functions are optimized over a feasible set of decisions. An optimization problem is then solved to find the feasible alternative. Data envelopment analysis (DEA) uses linear programming technique to calculate the relative efficiencies of the alternatives [28–30]. The disadvantage of DEA is that it cannot handle imprecise data and it assumes that all inputs and outputs are exactly known. Among mathematical programming models, goal programming is considered as one of the most effective methods for solving MCDM problems as they are able to handle large scale problems [31]. Outranking methods is another class of methods which have been widely used for. Outranking methods calculate the degree of dominance of one alternative over the other.

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These methods use outranking relations for modelling the decision maker's preferences. Two most commonly used outranking methods are ELimination Et Choix Traduisant la REalit (ELECTRE) (ELimination and Choice Expressing REality) [32,33] and Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) [34]. The outranking methods have been popular for many decades because of their ease of use. Another method which has been quite popular for MCDM analysis is technique for order of preference by similarity to ideal solution (TOPSIS). The advantages of TOPSIS include [1] – scalar value accounting for both best and worst alternative; logical representation of human rationale and easy implementation. TOPSIS is based upon the concept that the chosen solution should be closest to positive ideal solution and farthest from negative ideal solution.

The motivation for the present study comes from the application of thermodynamics for bibliometric assessment by [35] who used the analogies of the energy, exergy and entropy associated with a bibliometric sequence to derive an indicator of scientist's performance. In the present study, we propose a model for GMCDM based on a novel approach within the paradigm of thermodynamics. We define analogies for thermodynamical indicators – energy, exergy and entropy with respect to GMCDM. It should be noted that the entropy defined in the present study is different from Shannon's entropy [36] which assumes a prior distribution. A natural definition of entropy derived from the first principles is used in the present study. It is observed that the conventional TOPSIS method uses a measure similar to what we define as energy indicator. We propose and also demonstrate, with the help of examples, that it is exergy indicator which makes better sense in the ranking of an alternative rather than energy indicator. The proposed model is quite simple to implement and is thermodynamically consistent. The proposed model is formulated for both crisp and fuzzy environments. The effectiveness of the proposed model is demonstrated by applying to human resource selection problem in both crisp and fuzzy environments.

The organization of the paper is as follows. The second section defines the preliminaries towards thermodynamics. In the third section, we define analogies for the energy, exergy and entropy in both crisp and fuzzy environments. The fourth section describes why using exergy indicator makes more sense than using an indicator based on energy. Fifth section lists out the procedure for GMCDM using thermodynamical indicators. In the sixth section, the method is applied to human resource selection. The results obtained from the proposed method are compared with those obtained from extended and fuzzy TOPSIS. The seventh section discusses the results obtained from the proposed method. The final section presents the conclusions drawn from the present study.

2. Preliminaries towards thermodynamics

Thermodynamics is viewed as the science of energy. In this section, we reproduce the definition of the terms like energy, exergy and entropy based on [37] for the sake of completeness. The section also describes the two basic laws which govern the science of thermodynamics.

Definition 2.1. Energy (U) of a system is defined as its ability to do work. It can neither be created nor be destroyed but can only be converted from one form to another. It depends on the parameters of the matter or energy flow only and is independent of environment parameters. It is a measure of quantity alone.

Definition 2.2. Exergy (X) of a system is the maximum useful work possible during a process that brings the system into equilibrium with the specified reference environment. Exergy is the potential of a system to cause a change as it achieves equilibrium with its environment. It depends upon parameters of matter or energy flow and environment. It is a measure of both quantity and quality.

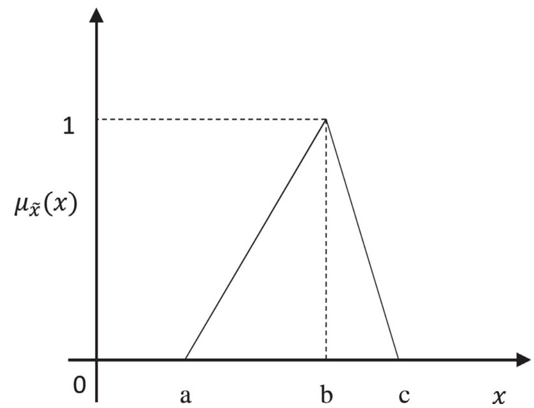

Fig. 1. Triangular fuzzy number.

Definition 2.3. Entropy (S) of a system is the measurement of the amount of disorder in the system. A system can generate entropy. The entropy of the system can be increased or decreased by energy transport across the system boundary. The direction of the change in the states of the system is from a state of low probability to the one with higher probability. Since, the disordered states are more probable than ordered states, the natural direction of the change in system states is from order to disorder.

First law of thermodynamics The energy is a thermodynamic property which can change from one form to another but the total amount of energy remains constant. It is based on the conservation of energy.

Second law of thermodynamics The energy has quality as well as quantity, and actual processes occur in the direction of decreasing quality of energy. Any process either increases the entropy of the universe – or leaves it unchanged.

The first law of thermodynamics gives no information about the direction of the energy conversion or the quality of energy. It is the second law of thermodynamics which establishes the difference in the quality of the various forms of energy. Based on second law of thermodynamics, entropy can be seen as the measure of energy which is unavailable for direct conversion to work. Two systems can have same energy but may not be capable of doing the same amount of useful work. A system which is capable of doing more useful work is said to have good quality of energy compared to others.

3. Thermodynamical analogies

In this section, we define analogies for the thermodynamical terms in both crisp and fuzzy environment. These analogies lay down the basis for the thermodynamically consistent model for GMCDM. Let an alternative (A) be rated by a decision maker (E), for a criterion (C). The weight assigned to the criterion by the expert is w. The rating and the weights are normalized between 0 and 1. The rating and the weights are expressed as fixed numbers (r, w) in case of crisp and triangular fuzzy number ($\tilde{r}$, $\tilde{w}$) in case of fuzzy environment. A triangular fuzzy number ($\tilde{x}$) is determined by a triplet (a, b, c) (Fig. 1) whose membership function is given by:

$$\mu_{\tilde{x}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{x-c}{b-c} & b \le x \le c \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

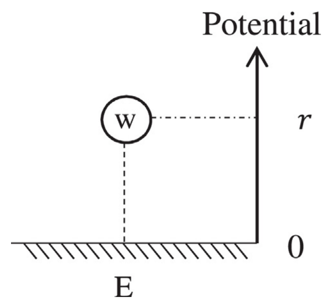

Fig. 2. Energy equivalence of an alternative (A) in GMCDM.

We assume that the fuzzy number associated with the rating $\tilde{r}$ is (r_a, r_b, r_c) and with the weight $\tilde{w}$ is (w_a, w_b, w_c).

Definition 3.1. The **force** due to gravity or weight of an alternative in GMCDM is defined as the weight assigned to it by an expert. In Fig. 2, the force/weight associated with alternative A is w.

Definition 3.2. The **potential** of an alternative in GMCDM is defined as the rating assigned to it by an expert. In Fig. 2, the potential associated with alternative A is r.

Definition 3.3. The **potential difference** between two states r_1 and r_2 of the system is given by:

$$d = |r_1 - r_2| \tag{2a}$$

$$\tilde{d} = (|r_{1a} - r_{2a}|, |r_{1b} - r_{2b}|, |r_{1c} - r_{2c}|) \tag{2b}$$

Definition 3.4. **Work** (W) done by the system during the change in state from r_1 to r_2 is equal to the change in its potential energy. The work done is given by:

$$W = w \cdot d(r_1, r_2) \tag{3a}$$

$$\tilde{W} = \tilde{w} \cdot \tilde{d}(\tilde{r_1}, \tilde{r_2}) \tag{3b}$$

Definition 3.5. **Energy indicator** (U) of an alternative is defined as the energy possessed by virtue of its rating in the system. The energy associated with alternative A in crisp and fuzzy environment is given by:

$$U = w \cdot r \tag{4a}$$

$$\tilde{U} = \tilde{w} \cdot \tilde{r} \tag{4b}$$

$$\begin{aligned} &= (w_a, w_b, w_c) \cdot (r_a, r_b, r_c) \\ &= (w_a r_a, w_b r_b, w_c r_c) \end{aligned} \tag{4b}$$

Definition 3.6. The **quality** of a rating is the measure of its degree of excellence compared to other rating. If all the experts have a consensus in the rating, then quality is equal to one. It is measured as one minus the relative distance of a rating from the mean rating. Let an alternative A be rated $(r_1, r_2, \ldots, r_n)$ by n experts and the mean rating is $\bar{r}$. The quality of ith rating is given by:

$$q = \left(1 - \frac{d(r_i, \bar{r})}{\bar{r}}\right) \tag{5a}$$

$$\tilde{q} = \left((1, 1, 1) - \frac{\tilde{d}(\tilde{r}_i, \bar{\tilde{r}})}{\bar{\tilde{r}}}\right) \tag{5b}$$

$$= \left(1 - \frac{|r_{ia} - \bar{r}_a|}{\bar{r}_a}, 1 - \frac{|r_{ib} - \bar{r}_b|}{\bar{r}_b}, 1 - \frac{|r_{ic} - \bar{r}_c|}{\bar{r}_c}\right) \tag{5b}$$

Definition 3.7. **Exergy indicator** of a rating is the measure of the quality energy that a rating carries. Mathematically, it is given by:

$$X = q \cdot U \tag{6a}$$

$$\tilde{X} = \tilde{q} \cdot \tilde{U} \tag{6b}$$

Definition 3.8. **Entropy indicator** is a measure of the unevenness in the ratings of an alternative. If an alternative is assigned exactly same rating by all the experts, then the entropy is equal to zero. The entropy in the present study is defined as the difference between energy and exergy [35] which is different from the Shanon's definition [36]. Shanon's entropy assumes a prior distribution for the ratings while no such assumption is made in the present study. Mathematically, the entropy of a rating can be written as: [35]:

$$S = U - X \tag{7a}$$

$$\tilde{S} = \tilde{U} - \tilde{X} \tag{7b}$$

4. Energy vs. exergy

Let us assume that there are K decision makers, rating m alternatives based on n criteria. In the classical TOPSIS method, the ratings and the weights are first aggregated using arithmetic mean or any other suitable method. The aggregated ratings and weights are then assembled to form decision (D) and weight (W) matrix as given below:

$$D = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \qquad W = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix} \tag{8}$$

where x_{ij} denotes the aggregate rating of ith alternative for jth criterion and w_j represents the weight for jth criterion.

In order to bring various criteria on a comparable scale, vector or linear normalization is carried out. Normalized decision matrix (R) given by:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{9}$$

where r_{ij} denotes the normalized rating of ith alternative for jth criterion. The weighted normalized decision matrix is constructed by multiplying the weights with the normalized rating and is given by:

$$\begin{aligned} V \quad &= \left[v_{ij} \right], \\ \text{where} \quad v_{ij} &= w_j(\cdot) r_{ij} \\ i &= 1, 2, \ldots, m \\ j &= 1, 2, \ldots, n \end{aligned} \tag{10}$$

On careful observation, it is found that the term v_{ij} is similar to what we have defined as energy in the previous section. The energy indicator associated with a rating gives the information only about the quantity and not the quality. In the process of aggregation of rating, the information on its quality is lost. There is a need for an indicator which accounts not only for the quantity but for the quality as well. Exergy indicator defined in the previous section includes both the quantity and the quality of the energy. This motivates the use of exergy indicator in place of energy. In this section, we highlight how the use of exergy indicator instead of energy makes more sense using two different examples covering both crisp and fuzzy environments.

Example 4.1. Consider a case where two alternatives (A_1 and A_2) are rated by 10 decision makers on a particular criterion. The aggregated weight (w) for the criterion is 0.7. The normalized ratings (r)

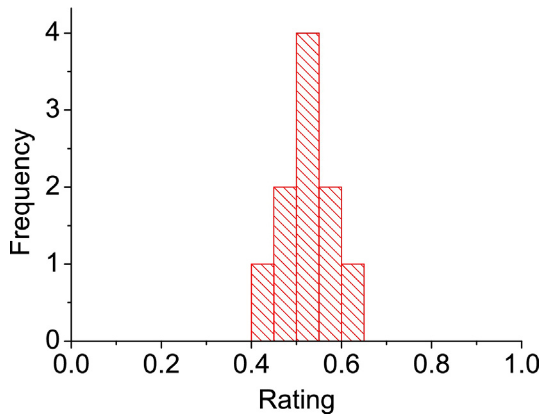

Fig. 3. Histogram of normalized rating assigned to A_1 for Example 4.1.

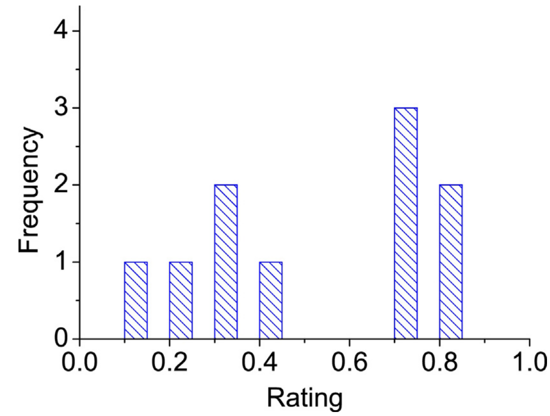

Fig. 4. Histogram of normalized rating assigned to A_2 for Example 4.1.

for A_1 are clustered and varies from 0.4 to 0.6 with a mean of 0.5. In case of A_2, the normalized ratings are more dispersed (varies from 0.1 to 0.8) but has the same mean as A_1 (that is 0.5). The histogram of the normalized ratings is plotted in Fig. 3 for A_1 and in Fig. 4 for A_2. The normalized ratings assigned to an alternative and the calculated thermodynamical indicators are given in Table 1 for A_1 and Table 2 for A_2. The variation of the thermodynamical indicators for A_1 and A_2 are shown in Fig. 5.

It is observed from Tables 1 and 2 that the weighted normalized decision (which is equivalent to energy indicator) for both the alternatives will lead to same value of 0.35, if we use classical TOPSIS method. Figs. 3 and 4 clearly suggests that the ratings for A_1 are more reliable than for A_2. This fact is also evident from the variation of thermodynamical indicator for A_1 and A_2 (Fig. 5). This information is lost if we use energy indicator. On the other hand, the exergy indicator clearly suggests that the A_1 is better rated than A_2. Mean entropy values of A_1 and A_2 indicates that the unevenness in the ratings assigned to an alternative by the decision makers is more in case of A_2 compared to A_1.

Example 4.2. In this example, the ratings ($\tilde{r}$) are assigned to the alternatives A_1 and A_2 in the form of triangular fuzzy number by 5 decision makers on a particular criterion. The weight ($\tilde{w}$) assigned to the criteria is a triangular fuzzy number (0.7, 0.8, 0.9). The normalized fuzzy rating assigned to A_1 and A_2 are shown in Figs. 6 and 7. The numbers in the bracket in Figs. 6 and 7 represent the decision maker corresponding to that rating. The normalized fuzzy rating assigned to an alternative and the calculated thermodynamical fuzzy indicators are given in Table 3 for A_1 and in Table 4 for A_2. The variation of the thermodynamical indicators for A_1 and A_2 are shown in Fig. 8. The mean normalized fuzzy rating and the mean fuzzy energy is same for A_1 and A_2. In this case also, the classical TOPSIS method will result in same weighted normalized fuzzy decision even though there is a large difference in the quality of ratings

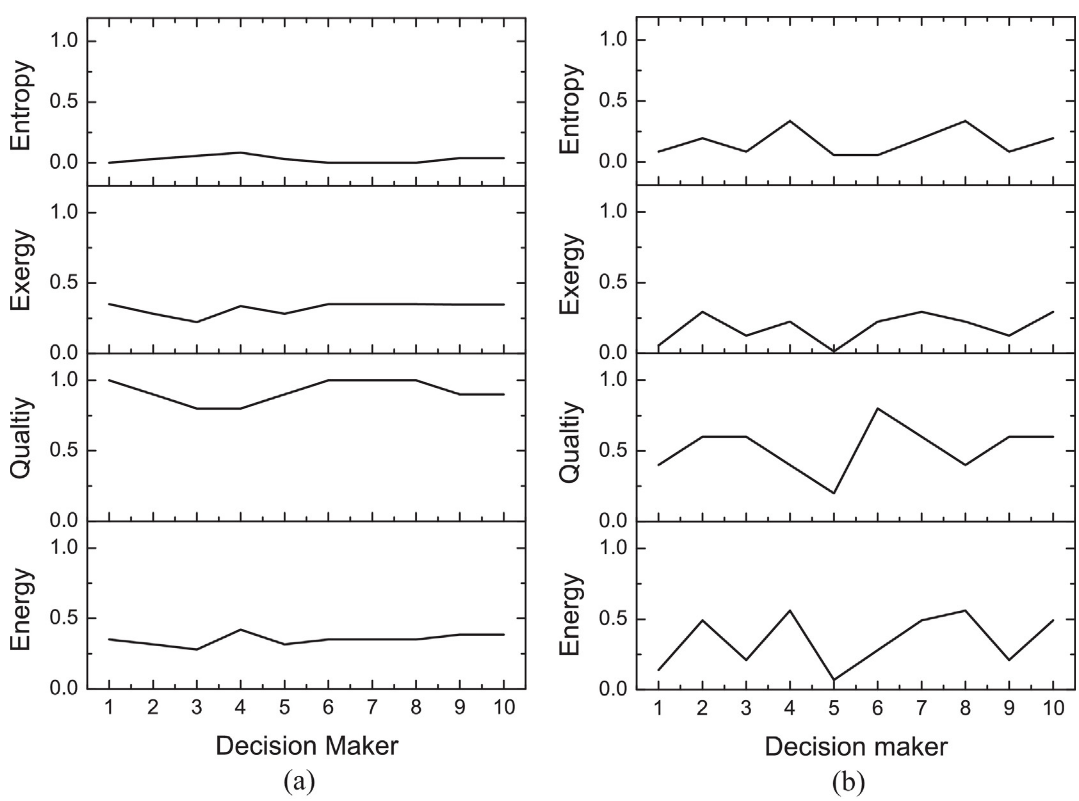

Fig. 5. Variation of thermodynamical indicators for Example 4.1 – (a) A_1, (b) A_2.

Table 1
Normalized ratings and thermodynamical indicators of A_1 for Example 4.1.

Decision maker	Rating r	Energy $U = w \cdot r$	Quality q	Exergy $X = q \cdot U$	Entropy $S = U - X$
1	0.500	0.350	1.000	0.350	0.000
2	0.450	0.315	0.900	0.284	0.032
3	0.400	0.280	0.800	0.224	0.056
4	0.600	0.420	0.800	0.336	0.084
5	0.450	0.315	0.900	0.284	0.032
6	0.500	0.350	1.000	0.350	0.000
7	0.500	0.350	1.000	0.350	0.000
8	0.500	0.350	1.000	0.350	0.000
9	0.550	0.385	0.900	0.347	0.039
10	0.550	0.385	0.900	0.347	0.039
Mean	0.500	0.350	0.920	0.322	0.028

Table 2
Normalized ratings and thermodynamical indicators of A_2 for Example 4.1.

Decision maker	Rating r	Energy $U = w \cdot r$	Quality q	Exergy $X = q \cdot U$	Entropy $S = U - X$
1	0.200	0.140	0.400	0.056	0.084
2	0.700	0.490	0.600	0.294	0.196
3	0.300	0.210	0.600	0.126	0.084
4	0.800	0.560	0.400	0.224	0.336
5	0.100	0.070	0.200	0.014	0.056
6	0.400	0.280	0.800	0.224	0.056
7	0.700	0.490	0.600	0.294	0.196
8	0.800	0.560	0.400	0.224	0.336
9	0.300	0.210	0.600	0.126	0.084
10	0.700	0.490	0.600	0.294	0.196
Mean	0.500	0.350	0.520	0.188	0.162

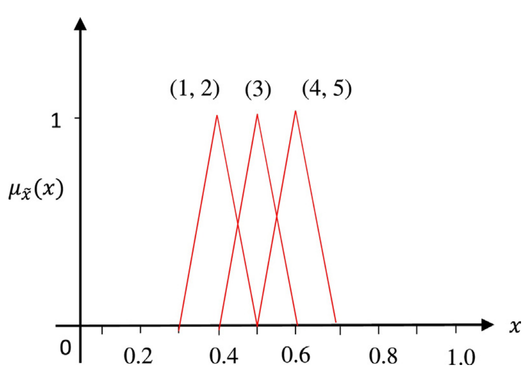

Fig. 6. Normalized fuzzy ratings assigned to A_1 for Example 4.2.

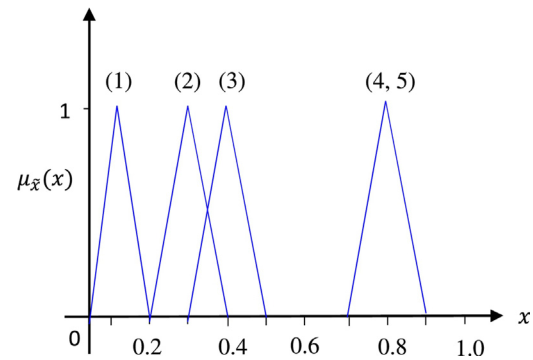

Fig. 7. Normalized fuzzy ratings assigned to A_2 for Example 4.2.

of A_1 and A_2 (Fig. 8). This difference is reflected in the mean fuzzy exergy indicator.

Based on the examples studied, we conclude that the use of exergy indicator in place of indicator based on energy will bring more rationality to the decision making process. The use of exergy indicator will enable to account for the quality of the ratings which is neglected in the classical TOPSIS method.

5. Evaluation of thermodynamical indicators

A systematic approach is presented in this section for the ranking of alternatives in GMCDM based on exergy indicator in both crisp and fuzzy environments. The GMCDM problem consists of K decision makers rating m alternatives based on n criteria. The detailed step-by-step procedure is described below:

5.1. Crisp environment

Step 1: Formulate decision matrices $(D^1, \ldots, D^K)$ for each of the decision maker.

$$D^k = \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \quad (11)$$

Table 3
Normalized fuzzy ratings and thermodynamical fuzzy indicators of A_1 for Example 4.2.

Decision maker	Rating $\tilde{r}$	Energy $\tilde{U} = \tilde{w} \cdot \tilde{r}$	Quality $\tilde{q}$	Exergy $\tilde{X} = \tilde{q} \cdot \tilde{U}$	Entropy $\tilde{S} = \tilde{U} - \tilde{X}$
1	(0.30,0.40,0.50)	(0.21,0.32,0.45)	(0.75,0.80,0.83)	(0.16,0.26,0.38)	(0.05,0.06,0.08)
2	(0.30,0.40,0.50)	(0.21,0.32,0.45)	(0.75,0.80,0.83)	(0.16,0.26,0.38)	(0.05,0.06,0.08)
3	(0.40,0.50,0.60)	(0.28,0.40,0.54)	(1.00,1.00,1.00)	(0.28,0.40,0.54)	(0.00,0.00,0.00)
4	(0.50,0.60,0.70)	(0.35,0.48,0.63)	(0.75,0.80,0.83)	(0.26,0.38,0.53)	(0.09,0.10,0.11)
5	(0.50,0.60,0.70)	(0.35,0.48,0.63)	(0.75,0.80,0.83)	(0.26,0.38,0.53)	(0.09,0.10,0.11)
Mean	(0.40,0.50,0.60)	(0.28,0.40,0.54)	(0.80,0.84,0.87)	(0.22,0.34,0.47)	(0.06,0.06,0.07)

Table 4
Normalized fuzzy ratings and thermodynamical fuzzy indicators of A_2 for Example 4.2.

Decision maker	Rating $\tilde{r}$	Energy $\tilde{U} = \tilde{w} \cdot \tilde{r}$	Quality $\tilde{q}$	Exergy $\tilde{X} = \tilde{q} \cdot \tilde{U}$	Entropy $\tilde{S} = \tilde{U} - \tilde{X}$
1	(0.10,0.20,0.30)	(0.07,0.16,0.27)	(0.25,0.40,0.50)	(0.02,0.06,0.14)	(0.05,0.10,0.14)
2	(0.20,0.30,0.40)	(0.14,0.24,0.36)	(0.50,0.60,0.67)	(0.07,0.14,0.24)	(0.07,0.10,0.12)
3	(0.30,0.40,0.50)	(0.21,0.32,0.45)	(0.75,0.80,0.83)	(0.16,0.26,0.38)	(0.05,0.06,0.08)
4	(0.70,0.80,0.90)	(0.49,0.64,0.81)	(0.25,0.40,0.50)	(0.12,0.26,0.41)	(0.37,0.38,0.41)
5	(0.70,0.80,0.90)	(0.49,0.64,0.81)	(0.25,0.40,0.50)	(0.12,0.26,0.41)	(0.37,0.38,0.41)
Mean	(0.40,0.50,0.60)	(0.28,0.40,0.54)	(0.40,0.52,0.60)	(0.10,0.20,0.31)	(0.18,0.20,0.23)

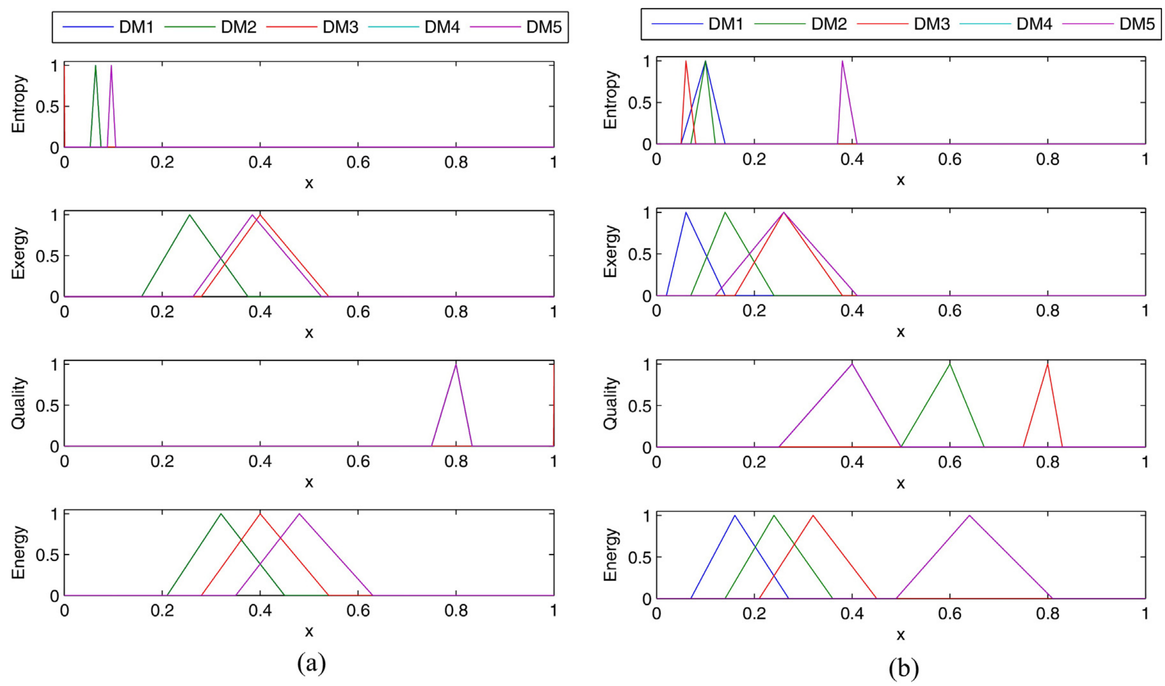

Fig. 8. Variation of thermodynamical indicators for Example 4.2 – (a) A_1, (b) A_2.

where $k = 1, \ldots, K$ and x_{ij}^k denotes the rating assigned by kth decision maker to ith alternative for jth criterion.

Step 2: Construct the normalized decision matrix $(R^1, \ldots, R^K)$ for each of the decision maker.

$$R^k = [r_{ij}^k]_{mxn},\ r_{ij}^k = \begin{cases} \dfrac{x_{ij}^k}{x_j^{k+}} & \text{for benefit criterion } j \\ \dfrac{x_j^{k-}}{x_{ij}^k} & \text{for cost criterion } j \end{cases} \tag{12}$$

where $x_j^{k+} = \max_i(x_{ij}^k)$ and $x_j^{k-} = \min_i(x_{ij}^k)$ for $i = 1, \ldots, m$, and $j = 1, \ldots, n$.

Step 3: Construct weight matrix $(W^1, \ldots, W^K)$ for each of the decision maker.

$$W^k = [w_1^k, \ldots, w_n^k] \tag{13}$$

where w_j^k is the weight assigned to jth criterion by kth decision maker.

Step 4: Construct energy matrix $(U^1, \ldots, U^K)$ for each of the decision maker.

$$U^k = \begin{bmatrix} w_1^k \cdot r_{11}^k & w_2^k \cdot r_{12}^k & \cdots & w_n^k \cdot r_{1n}^k \\ w_1^k \cdot r_{21}^k & w_2^k \cdot r_{22}^k & \cdots & w_n^k \cdot r_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ w_1^k \cdot r_{m1}^k & w_2^k \cdot r_{m2}^k & \cdots & w_n^k \cdot r_{mn}^k \end{bmatrix} \tag{14}$$

Step 5: Construct quality matrix $(q^1, \ldots, q^K)$ for each of the decision maker.

$$q^k = \begin{bmatrix} \left(1 - \dfrac{d(r_{11}^k, \bar{r}_1^k)}{\bar{r}_1^k}\right) & \left(1 - \dfrac{d(r_{12}^k, \bar{r}_2^k)}{\bar{r}_2^k}\right) & \cdots & \left(1 - \dfrac{d(r_{1n}^k, \bar{r}_n^k)}{\bar{r}_n^k}\right) \\ \left(1 - \dfrac{d(r_{21}^k, \bar{r}_1^k)}{\bar{r}_1^k}\right) & \left(1 - \dfrac{d(r_{22}^k, \bar{r}_2^k)}{\bar{r}_2^k}\right) & \cdots & \left(1 - \dfrac{d(r_{2n}^k, \bar{r}_n^k)}{\bar{r}_n^k}\right) \\ \vdots & \vdots & \cdots & \vdots \\ \left(1 - \dfrac{d(r_{m1}^k, \bar{r}_1^k)}{\bar{r}_1^k}\right) & \left(1 - \dfrac{d(r_{m2}^k, \bar{r}_2^k)}{\bar{r}_2^k}\right) & \cdots & \left(1 - \dfrac{d(r_{mn}^k, \bar{r}_n^k)}{\bar{r}_n^k}\right) \end{bmatrix} \tag{15}$$

where $\bar{r}_j^k = \frac{1}{m} \cdot (r_{1j}^k + \cdots + r_{mj}^k)$.

Step 6: Construct exergy matrix $(X^1, \ldots, X^K)$ for each of the decision maker.

$$X^k = [q_{ij}^k \cdot U_{ij}^k]_{mxn} \tag{16}$$

Step 7: Calculate the average energy and exergy of ith alternative with respect to kth decision maker.

$$\begin{aligned} U_i^k &= (U_{i1}^k + U_{i2}^k + \cdots + U_{in}^k)/n \\ X_i^k &= (X_{i1}^k + X_{i2}^k + \cdots + X_{in}^k)/n \end{aligned} \tag{17}$$

Step 8: Calculate the energy (U_i) and exergy (X_i) indicators associated with an alternative i.

$$\begin{aligned} U_i &= (U_i^1 + U_i^2 + \cdots + U_i)^K/K \\ X_i &= (X_i^1 + X_i^2 + \cdots + X_i)^K/K \end{aligned} \tag{18}$$

Step 9: Calculate the entropy (S_i) indicator of an alternative.

$$S_i = U_i - X_i \tag{19}$$

Step 10: Rank the alternatives in the order of their exergy indicator.

5.2. Fuzzy environment

The ratings and the weights are assigned in terms of linguistic variables which are then converted to triangular fuzzy numbers.

Step 1: Formulate fuzzy decision matrices ($\tilde{D}_1, \ldots, \tilde{D}_K$) for each of the decision maker.

$$\tilde{D}_k = \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \ldots & \vdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \cdots & \tilde{x}_{mn}^k \end{bmatrix} \quad (20)$$

where $k = 1, \ldots, K$ and $\tilde{x}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$ denotes the fuzzy rating assigned by kth decision maker to ith alternative for jth criterion.

Step 2: Construct the normalized fuzzy decision matrix ($\tilde{R}^1, \ldots, \tilde{R}^K$) for each of the decision maker.

$$\tilde{R}^k = [\tilde{r}_{ij}^k]_{mxn}, \; \tilde{r}_{ij}^k = \begin{cases} \left(\dfrac{a_{ij}^k}{c_j^{k+}}, \dfrac{b_{ij}^k}{c_j^{k+}}, \dfrac{c_{ij}^k}{c_j^{k+}}\right) & \text{for benefit criterion } j \\ \left(\dfrac{a_j^{k-}}{c_{ij}^k}, \dfrac{a_j^{k-}}{b_{ij}^k}, \dfrac{a_j^{k-}}{a_{ij}^k}\right) & \text{for cost criterion } j \end{cases} \quad (21)$$

where $c_j^{k+} = \max_i(c_{ij}^k)$ and $a_j^{k-} = \min_i(a_{ij}^k)$ for $i = 1, \ldots, m$, and $j = 1, \ldots, n$.

Step 3: Construct fuzzy weight matrix ($\tilde{W}^1, \ldots, \tilde{W}^K$) for each of the decision maker.

$$\tilde{W}^k = [\tilde{w}_1^k, \ldots, \tilde{w}_n^k] \quad (22)$$

where $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$ is the weight assigned to jth criterion by kth decision maker.

Step 4: Construct fuzzy energy matrix ($\tilde{U}^1, \ldots, \tilde{U}^K$) for each of the decision maker.

$$\tilde{U}^k = \begin{bmatrix} \tilde{w}_1^k \cdot \tilde{r}_{11}^k & \tilde{w}_2^k \cdot \tilde{r}_{12}^k & \cdots & \tilde{w}_n^k \cdot \tilde{r}_{1n}^k \\ \tilde{w}_1^k \cdot \tilde{r}_{21}^k & \tilde{w}_2^k \cdot \tilde{r}_{22}^k & \cdots & \tilde{w}_n^k \cdot \tilde{r}_{2n}^k \\ \vdots & \vdots & \ldots & \vdots \\ \tilde{w}_1^k \cdot \tilde{r}_{m1}^k & \tilde{w}_2^k \cdot \tilde{r}_{m2}^k & \cdots & \tilde{w}_n^k \cdot \tilde{r}_{mn}^k \end{bmatrix} \quad (23)$$

Step 5: Construct fuzzy quality matrix ($\tilde{q}^1, \ldots, \tilde{q}^K$) for each of the decision maker.

$$\tilde{q}^k = \begin{bmatrix} \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{11}^k, \bar{\tilde{r}}_1^k)}{\bar{\tilde{r}}_1^k}\right) & \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{12}^k, \bar{\tilde{r}}_2^k)}{\bar{\tilde{r}}_2^k}\right) & \cdots & \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{1n}^k, \bar{\tilde{r}}_n^k)}{\bar{\tilde{r}}_n^k}\right) \\ \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{21}^k, \bar{\tilde{r}}_1^k)}{\bar{\tilde{r}}_1^k}\right) & \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{22}^k, \bar{\tilde{r}}_2^k)}{\bar{\tilde{r}}_2^k}\right) & \cdots & \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{2n}^k, \bar{\tilde{r}}_n^k)}{\bar{\tilde{r}}_n^k}\right) \\ \vdots & \vdots & \cdots & \vdots \\ \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{m1}^k, \bar{\tilde{r}}_1^k)}{\bar{\tilde{r}}_1^k}\right) & \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{m2}^k, \bar{\tilde{r}}_2^k)}{\bar{\tilde{r}}_2^k}\right) & \cdots & \left((1,1,1) - \dfrac{\tilde{d}(\tilde{r}_{mn}^k, \bar{\tilde{r}}_n^k)}{\bar{\tilde{r}}_n^k}\right) \end{bmatrix} \quad (24)$$

where $\bar{\tilde{r}}_j^k = \frac{1}{m} \cdot (\tilde{r}_{1j}^k + \cdots + \tilde{r}_{mj}^k)$.

Step 6: Construct exergy matrix ($\tilde{X}^1, \ldots, \tilde{X}^K$) for each of the decision maker.

$$\tilde{X}^k = [\tilde{q}_{ij}^k \cdot \tilde{U}_{ij}^k]_{mxn} \quad (25)$$

Step 7: Calculate the average fuzzy energy and exergy of ith alternative with respect to kth decision maker.

$$\begin{aligned} \tilde{U}_i^k &= (\tilde{U}_{i1}^k + \tilde{U}_{i2}^k + \cdots + \tilde{U}_{in}^k)/n \\ \tilde{X}_i^k &= (\tilde{X}_{i1}^k + \tilde{X}_{i2}^k + \cdots + \tilde{X}_{in}^k)/n \end{aligned} \quad (26)$$

Step 8: Calculate the energy (U_i) and exergy (X_i) indicator associated with an alternative i.

$$\begin{aligned} U_i &= (s(\tilde{U}_i^1) + s(\tilde{U}_i^2) + \cdots + s(\tilde{U}_i)^K))/K \\ X_i &= (s(\tilde{X}_i^1) + s(\tilde{X}_i^2) + \cdots + s(\tilde{X}_i)^K))/K \end{aligned} \quad (27)$$

where $s(\tilde{x}) = \sqrt{\frac{1}{3}(a^2 + b^2 + c^2)}$.

Step 9: Calculate the entropy (S_i) indicator of an alternative.

$$S_i = U_i - X_i \quad (28)$$

Step 10: Rank the alternatives in the order of their exergy indicator.

6. Human resource selection

In this section, we take up two examples from the literature to demonstrate the effectiveness of the proposed methodology to the critical problem of human resource selection. The ranking based on the thermodynamical indicators is then compared with those obtained from extended and fuzzy TOPSIS.

Example 6.1. Crisp environment This problem is adopted from [1]. A company wants to recruit a manager. There are 17 eligible candidates to be evaluated by 4 decision makers (DM) on 7 benefit criteria out of which five are objective and two are subjective. The objective criteria include language test (C1), professional test (C2), safety rule test (C3), professional skills (C4) and computer skills (C5). The subjective criteria include panel interview (C6) and one-on-one interview (C7). The score of the candidates in objective and subjective criteria are given in Table 5. The weights assigned to the different criteria are given in Table 6.

The energy, exergy and entropy indicators are evaluated using the procedure described in the previous section. The alternatives are ranked in terms of energy and exergy indicators. The ranking obtained from the thermodynamical indicators is then compared with that reported in [1]. The values of the calculated thermodynamical indicators and ranking of the candidates are given in Table 7. The variation of the thermodynamical indicators is shown in Fig. 9. It is observed that the ranking obtained from the energy indicator is in close agreement with those obtained from extended TOPSIS. Major difference is observed between the rankings based on exergy indicator and extended TOPSIS. The reason for the observed difference is highlighted in the next section.

Example 6.2. Fuzzy environment This problem is adopted from [38]. A software company wants to hire system analysis engineer. There are three eligible candidates (A1, A2, A3) to be evaluated by three decision makers (DM1, DM2, DM3) on five benefit criteria – emotional steadiness (C1), oral communication skill (C2),

Table 5
Scores of the candidates for different criteria

Candidate	Objective criteria					Subjective criteria							
						DM1		DM2		DM3		DM4	
	C1	C2	C3	C4	C5	C6	C7	C6	C7	C6	C7	C6	C7
A1	80	70	87	77	76	80	75	85	80	75	70	90	85
A2	85	65	76	80	75	65	75	60	70	70	77	60	70
A3	78	90	72	80	85	90	85	80	85	80	90	90	95
A4	75	84	69	85	65	65	70	55	60	68	72	62	72
A5	84	67	60	75	85	75	80	75	80	50	55	70	75
A6	85	78	82	81	79	80	80	75	85	77	82	75	75
A7	77	83	74	70	71	65	70	70	60	65	72	67	75
A8	78	82	72	80	78	70	60	75	65	75	67	82	85
A9	85	90	80	88	90	80	85	95	85	90	85	90	92
A10	89	75	79	67	77	70	75	75	80	68	78	65	70
A11	65	55	68	62	70	50	60	62	65	60	65	65	70
A12	70	64	65	65	60	60	65	65	75	50	60	45	50
A13	95	80	70	75	70	75	75	80	80	65	75	70	75
A14	70	80	79	80	85	80	70	75	72	80	70	75	75
A15	60	78	87	70	66	70	65	75	70	65	70	60	65
A16	92	85	88	90	85	90	95	92	90	85	80	88	90
A17	86	87	80	70	72	80	85	70	75	75	80	70	75

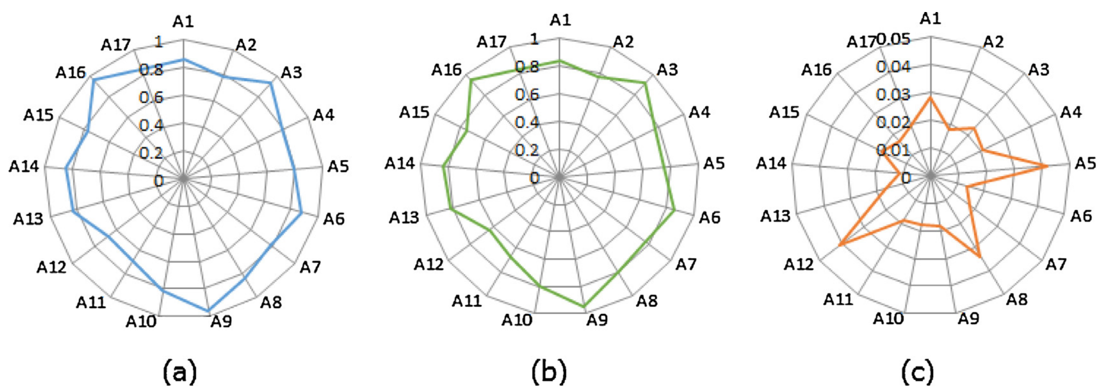

Fig. 9. Variation of thermodynamical indicator for Example 6.1 – (a) energy, (b) exergy, (c) entropy.

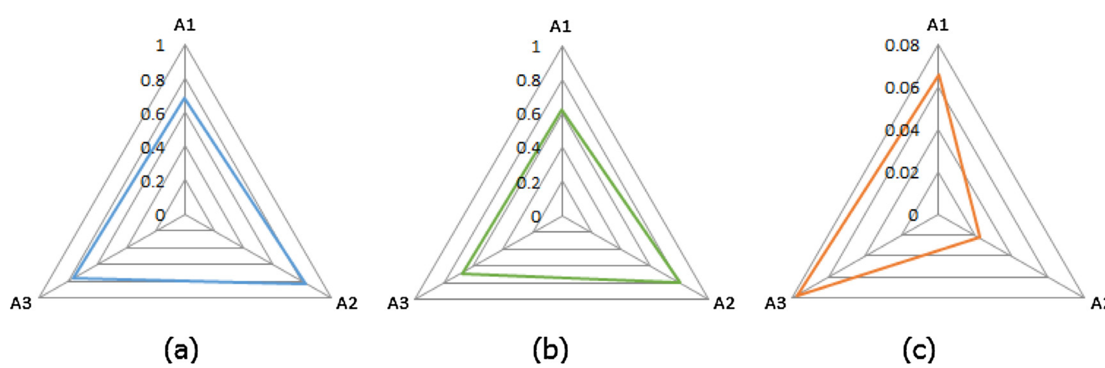

Fig. 10. Variation of thermodynamical indicator for Example 6.2 – (a) energy, (b) exergy, (c) entropy.

Table 6
Weights for different criteria.

Decision maker	C1	C2	C3	C4	C5	C6	C7
DM1	0.066	0.196	0.066	0.130	0.130	0.216	0.196
DM2	0.042	0.112	0.082	0.176	0.118	0.215	0.255
DM3	0.060	0.134	0.051	0.167	0.100	0.203	0.285
DM4	0.047	0.109	0.037	0.133	0.081	0.267	0.326

personality (C3), past experience (C4) and self-confidence (C5). The ratings and the weights are assigned in terms of linguistic variables. The triangular fuzzy number corresponding to the linguistic variables for ratings and weights are given in Table 8. The weights assigned to the different criteria are given in Table 9. The ratings of the three candidates for each of the criteria are given in Table 10. The values of the thermodynamical indicators and ranking of the candidates are given in Table 11. The variation of the thermodynamical indicators is shown in Fig. 10. The ranking of the alternative

Table 7
Thermodynamical indicators and ranking of candidates.

Candidate	Energy	Exergy	Entropy	Rank based on		
	U	X	S	U	X	Extended TOPSIS [1]
A1	0.860	0.831	0.028	5	6	5
A2	0.789	0.771	0.018	13	11	14
A3	0.934	0.910	0.024	3	3	3
A4	0.790	0.768	0.021	12	13	12
A5	0.791	0.749	0.042	11	14	11
A6	0.873	0.860	0.014	4	4	4
A7	0.788	0.770	0.018	14	12	13
A8	0.836	0.802	0.034	8	9	8
A9	0.964	0.946	0.018	2	2	2
A10	0.811	0.793	0.018	10	10	10
A11	0.690	0.672	0.018	16	16	16
A12	0.673	0.632	0.041	17	17	17
A13	0.831	0.813	0.017	9	8	9
A14	0.849	0.838	0.011	6	5	6
A15	0.768	0.748	0.020	15	15	15
A16	0.966	0.950	0.017	1	1	1
A17	0.846	0.826	0.020	7	7	7

Table 8
Triangular fuzzy numbers assigned to ratings and weights.

Ratings		Weights	
Linguistic variable	Fuzzy number	Linguistic variable	Fuzzy number
Very poor (VP)	(0,0,1)	Very low (VL)	(0,0,0.1)
Poor (P)	(0,1,3)	Low (L)	(0,0.1,0.3)
Medium poor (MP)	(1,3,5)	Medium low (ML)	(0.1,0.3,0.5)
Fair (F)	(3,5,7)	Medium (M)	(0.3,0.5,0.7)
Medium good (MG)	(5,7,9)	Medium high (MH)	(0.5,0.7,0.9)
Good (G)	(7,9,10)	High (H)	(0.7,0.9,1.0)
Very good (VG)	(9,10,10)	Very high (VH)	(0.9,1.0,1.0)

Table 9
Weights assigned to different criteria.

Criteria	Decision maker		
	DM1	DM2	DM3
C1	H	VH	MH
C2	VH	VH	VH
C3	VH	H	H
C4	VH	VH	VH
C5	M	MH	MH

based on energy and exergy indicators is found to be same as that obtained from fuzzy TOPSIS [38].

The two examples demonstrates the effectiveness of the proposed methodology in both crisp and fuzzy environments.

Table 10
Rating of the candidates for different criteria.

Criteria	Candidate	Decision maker		
		DM1	DM2	DM3
C1	A1	MG	G	MG
	A2	G	G	MG
	A3	VG	G	F
C2	A1	G	MG	F
	A2	VG	VG	VG
	A3	MG	G	VG
C3	A1	F	G	G
	A2	VG	VG	G
	A3	G	MG	VG
C4	A1	VG	G	VG
	A2	VG	VG	VG
	A3	G	VG	MG
C5	A1	F	F	F
	A2	VG	MG	G
	A3	G	G	MG

Table 11
Thermodynamical indicators and ranking of candidates.

Candidate	Energy	Exergy	Entropy	Rank based on		
	U	X	S	U	X	Fuzzy TOPSIS [38]
A1	0.685	0.620	0.065	3	3	3
A2	0.825	0.803	0.023	1	1	1
A3	0.761	0.683	0.077	2	2	2

7. Discussion

In Example 6.1, the ratings which are different from those reported in the literature are highlighted in Table 7. It is observed that the energy indicator ranks the alternative almost similar to the ranking based on extended TOPSIS [1]. The reason being the terms in the decision matrix of extended TOPSIS are similar to what we defined as energy. The ranking based on exergy indicator is also close to that of extended TOPSIS except for A2 and A5. If we carefully look at the subjective rating of A2 and A5 (highlighted in Table 5), we observe that the variation in the ratings of A5 is more than A2. For A2, the rating ranges from 60 to 70 for C6 and 70 to 77 for C7. In case of A5, the rating ranges from 50 to 75 for C6 and 55 to 80 for C7. This is also evident from the entropy values of A2 and A5. The quality of the rating reduces with increase in variations. The confidence in the information that we have depends on its quality. This factor is accounted well if we use exergy indicator.

In Example 6.2, the rating of the alternatives are clustered, and therefore, the variations are less. This proves that when the variations in the ratings are small, same ranking order is obtained from

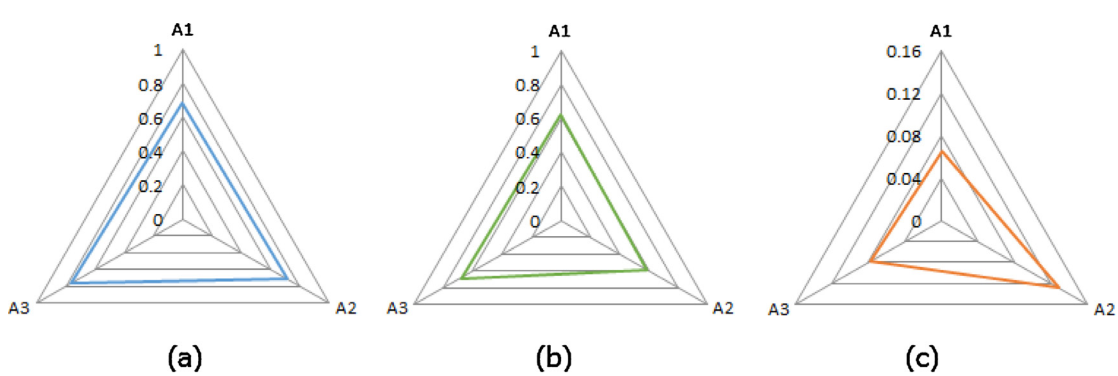

Fig. 11. Variation of revised thermodynamical indicator for Example 6.2 – (a) energy, (b) exergy, and (c) entropy.

Table 12
Revised thermodynamical indicators and ranking of candidates.

Candidate	Energy	Exergy	Entropy	Rank based on		
	U	*X*	*S*	*U*	*X*	Fuzzy TOPSIS
A1	0.685	0.62	0.065	3	2	3
A2	0.717	0.591	0.126	2	3	2
A3	0.761	0.683	0.077	1	1	1

TOPSIS, energy and exergy indicators. An exercise is taken up to highlight the effect of variations on the ranking of candidates based on different methods. The ratings of the candidate A2 by DM1 for criteria C1 and C2 are changed from (G, VG) to (VP, VP), respectively. The revised thermodynamical indicators and ranking of the candidates are given in Table 12. The variation of the revised thermodynamical indicators for the ratings is shown in Fig. 11. The variation in the rating of A2 has resulted in increase of its entropy indicator. The effect of this variation is accounted only in the ranking based on exergy indicator.

8. Conclusions

A model is proposed for GMCDM based on thermodynamical analogies. The definition of thermodynamical indicators is derived from the first principles. The energy indicator associated with a rating gives an idea of the quantity of potential energy that a rating carries (based on the first law of thermodynamics). The expression of the exergy indicator is derived from the second law of thermodynamics. The exergy indicator gives information on the amount of energy which can be converted to useful work. The entropy indicator gives an idea about the unevenness among the ratings of an alternative. The confidence in the ratings reduces if there are large variations in the rating given by different decision makers. The classical TOPSIS method uses what we define as energy for the formulation of the decision matrix. The information on the quality of rating was neglected. We suggest the use of exergy indicator in place of energy to effectively account for the quality of the ratings in the decision making process. The effectiveness of the proposed model is demonstrated by applying it to human resource selection problem in both crisp and fuzzy environments. The new model is simple to implement and involves less computations compared to TOPSIS. In the proposed model, the alternatives can be ranked directly based on the value of exergy indicator eliminating the calculation of the separation measures from positive and negative ideal solutions which is required in TOPSIS.

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References

[1] H.-S. Shih, H.-J. Shyur, E.S. Lee, An extension of TOPSIS for group decision making, Math. Comput. Model. 45 (7) (2007) 801–813.
[2] X. Sang, X. Liu, J. Qin, An analytical solution to fuzzy TOPSIS and its application in personnel selection for knowledge-intensive enterprise, Appl. Soft Comput. 30 (2015) 190–204.
[3] S.-H. Tsaur, T.-Y. Chang, C.-H. Yen, The evaluation of airline service quality by fuzzy MCDM, Tour. Manag. 23 (2) (2002) 107–115.
[4] M. Ehrgott, K. Klamroth, C. Schwehm, An MCDM approach to portfolio optimization, Eur. J. Oper. Res. 155 (3) (2004) 752–770.
[5] H.-T. Liu, Product design and selection using fuzzy QFD and fuzzy MCDM approaches, Appl. Math. Model. 35 (1) (2011) 482–496.
[6] H.-J. Shyur, H.-S. Shih, A hybrid MCDM model for strategic vendor selection, Math. Comput. Model. 44 (7) (2006) 749–761.
[7] F.R.L. Junior, L. Osiro, L.C.R. Carpinetti, A comparison between fuzzy AHP and fuzzy TOPSIS methods to supplier selection, Appl. Soft Comput. 21 (2014) 194–209.
[8] I. Chamodrakas, D. Martakos, A utility-based fuzzy TOPSIS method for energy efficient network selection in heterogeneous wireless networks, Appl. Soft Comput. 12 (7) (2012) 1929–1938.
[9] T. Rashid, I. Beg, S.M. Husnine, Robot selection by using generalized interval-valued fuzzy numbers with TOPSIS, Appl. Soft Comput. 21 (2014) 462–468.
[10] M. Verma, J. Rajasankar, N. Anandavalli, A. Prakash, N.R. Iyer, Fuzzy similarity approach for ranking and health assessment of towers based on visual inspection, Adv. Struct. Eng. 18 (9) (2015) 1399–1414.
[11] P.C. Fishburn, Conjoint measurement in utility theory with incomplete product sets, J. Math. Psychol. 4 (1) (1967) 104–119.
[12] P.C. Fishburn, R.L. Keeney, Seven independence concepts and continuous multiattribute utility functions, J. Math. Psychol. 11 (3) (1974) 294–327.
[13] T.L. Saaty, What is the analytic hierarchy process? in: Mathematical Models for Decision Support, Springer, 1988, pp. 109–121.
[14] T.L. Saaty, Rank from comparisons and from ratings in the analytic hierarchy/network processes Eur. J. Oper. Res. 168 (2) (2006) 557–570.
[15] F. Herrera, E. Herrera-Viedma, J.L. Verdegay, A sequential selection process in group decision making with a linguistic assessment approach, Inform. Sci. 85 (4) (1995) 223–239.
[16] F. Herrera, E. Herrera-Viedma, et al., A model of consensus in group decision making under linguistic assessments, Fuzzy Sets Syst. 78 (1) (1996) 73–87.
[17] F. Chiclana, F. Herrera, E. Herrera-Viedma, Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations, Fuzzy Sets Syst. 97 (1) (1998) 33–48.
[18] F. Chiclana, F. Herrera, E. Herrera-Viedma, Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations, Fuzzy Sets Syst. 122 (2) (2001) 277–291.
[19] S.A. Orlovskii, Calculus of Decomposable Properties, Fuzzy Sets, and Decisions, Allerton Press, 1994.
[20] H.-J. Zimmermann, Multi criteria decision making in crisp and fuzzy environments, in: Fuzzy Sets Theory and Applications, Springer, 1986, pp. 233–256.
[21] K. Atanassov, G. Pasi, R. Yager, Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making, Int. J. Syst. Sci. 36 (14) (2005) 859–868.
[22] E. Herrera-Viedma, F. Herrera, F. Chiclana, A consensus model for multiperson decision making with different preference structures, IEEE Trans. Syst. Man Cybern. Part A: Syst. Hum. 32 (3) (2002) 394–402.
[23] E. Herrera-Viedma, L. Martinez, F. Mata, F. Chiclana, A consensus support system model for group decision-making problems with multigranular linguistic preference relations, IEEE Trans. Fuzzy Syst. 13 (5) (2005) 644–658.
[24] E. Herrera-Viedma, S. Alonso, F. Chiclana, F. Herrera, A consensus model for group decision making with incomplete fuzzy preference relations, IEEE Trans. Fuzzy Syst. 15 (5) (2007) 863–877.
[25] F.J. Cabrerizo, S. Alonso, E. Herrera-Viedma, A consensus model for group decision making problems with unbalanced fuzzy linguistic information, Int. J. Inform. Technol. Decis. Mak. 8 (1) (2009) 109–131.
[26] F.J. Cabrerizo, I.J. Pérez, E. Herrera-Viedma, Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information, Knowl. Based Syst. 23 (2) (2010) 169–181.
[27] J. Xu, Z. Wu, Y. Zhang, A consensus based method for multi-criteria group decision making under uncertain linguistic setting, Group Decis. Negot. 23 (1) (2014) 127–148.
[28] C.A. Weber, A data envelopment analysis approach to measuring vendor performance, Supply Chain Manag. Int. J. 1 (1) (1996) 28–39.
[29] R. Baker, S. Talluri, A closer look at the use of data envelopment analysis for technology selection, Comput. Ind. Eng. 32 (1) (1997) 101–108.
[30] R. Narasimhan, S. Talluri, D. Mendez, Supplier evaluation and rationalization via data envelopment analysis: an empirical examination, J. Supply Chain Manag. 37 (2) (2001) 28–37.
[31] B. Karpak, E. Kumcu, R.R. Kasuganti, Purchasing materials in the supply chain: managing a multi-objective task, Eur. J. Purch. Supply Manag. 7 (3) (2001) 209–216.
[32] B. Roy, Classement et choix en présence de points de vue multiples. Revue française d'automatique, d'informatique et de recherche opérationnelle, Recherche opérationnelle 2 (1) (1968) 57–75.
[33] B. Roy, The outranking approach and the foundations of ELECTRE methods, Theory Decis. 31 (1) (1991) 49–73.
[34] J.-P. Brans, P. Vincke, Note – a preference ranking organisation method: (the PROMETHEE method for multiple criteria decision-making), Manag. Sci. 31 (6) (1985) 647–656.
[35] G. Prathap, The energy-exergy-entropy (or EEE) sequences in bibliometric assessment, Scientometrics 87 (3) (2011) 515–524.
[36] C.E. Shannon, W. Weaver, The Mathematical Theory of Communication, University of Illinois Press, 1963.
[37] I. Dincer, Y.A. Cengel, Energy, entropy and exergy concepts and their roles in thermal engineering, Entropy 3 (3) (2001) 116–149.
[38] C.-T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets Syst. 114 (1) (2000) 1–9.