Applied Soft Computing 60 (2017) 564–576

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A new extension to PROMETHEE under intuitionistic fuzzy environment for solving supplier selection problem with linguistic preferences

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ARTICLE INFO

Article history:
Received 6 January 2017
Received in revised form 10 July 2017
Accepted 14 July 2017
Available online 19 July 2017

Keywords:
Supplier selection
PROMETHEE
Linguistic based aggregation operator
Atanassov's intuitionistic fuzzy set
Decision making

ABSTRACT

This paper presents a new two-tier decision making framework with linguistic preferences for scientific decision making. The major reason for adopting linguistic preference is to ease the process of rating of alternatives by allowing decision makers (DMs) to strongly emphasize their opinion on each alternative. In the first tier, aggregation is done using a newly proposed operator called linguistic based aggregation (LBA), which aggregates linguistic terms directly without making any conversion. The main motivation for this proposal is driven by the previous studies on aggregation theory which reveals that conversion leads to loss of information and formation of virtual sets which are no longer sensible and rational for decision making process. Secondly, in the next tier, a new ranking method called IFSP (intuitionistic fuzzy set based PROMETHEE) is proposed which is an extension to PROMETHEE (preference ranking organization method for enrichment evaluation) under intuitionistic fuzzy set (IFS) context. Unlike previous ranking methods, this ranking method follows a new formulation by considering personal choice of the DMs over each alternative. The main motivation for such formulation is derived from the notion of not just obtaining a suitable alternative but also coherently satisfying the DMs' viewpoint during decision process. Finally, the practicality of the framework is tested by using supplier selection (SS) problem for an automobile factory. The strength and weakness of the proposed LBA-IFSP framework are verified by comparing with other methods under the realm of theoretical and numerical analysis. The results from the analysis infer that proposed LBA-IFSP framework is rationally coherent to DMs' viewpoint, moderately consistent with other methods and highly stable and robust against rank reversal issue.

1. Introduction

The process of decision making is often prone to ambiguity and imprecision as it involves human intervention and cognitive thinking. Human beings generally tend to make preferences over a set of items pertaining to a specific task based on the comparative analysis. This scheme of preference assignment generally invokes vagueness in the process as the choices are linguistically rated and compared. Liao and Xu [1] gave a mathematical definition for the multi-criteria decision making (MCDM) problems which is given by (1).

$$\max/\min\left(\xi_1(\lambda_i), \xi_2(\lambda_i), \ldots, \xi_n(\lambda_i)\right) \forall \lambda_i \in L \quad (1)$$

where $\xi_n(\lambda_i)$ is the rating value of an alternative λ_i over any criterion ξ_n, and $L = (\lambda_1, \lambda_2, \ldots, \lambda_i)$.

From (1) we observe that there is always a trade-off between criteria with respect to each alternative in the set L. Generally, criteria like profit, service, quality etc. are to be maximized and criteria like risk, resource utilization, cost etc. are to be minimized. Achieving such tradeoffs is often difficult and impossible in real time. So, researchers introduced the idea of a compromise solution in which alternatives satisfying majority of those competing criteria are chosen to be better candidates for the task [2]. For instance, we consider the car selection process in which attributes like safety, cost, fuel consumption, comfort etc. are popular. Among these, cost and fuel usage must be low; safety and comfort must be high. Such trade-off is termed as a *dominant trade-off* and is rarely achieved. So DMs adopt MCDM methods for choosing near dominant solutions. These MCDM methods are broadly classified as utility based and outranking based methods.

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http://dx.doi.org/10.1016/j.asoc.2017.07.028
1568-4946/

In the utility based method, alternatives are chosen based on utility functions. Popular L_p metric is used for selecting a compromise solution from the set of relevant solutions. The main idea of utility based method is to fuse all criteria values of an alternative into single value of that alternative and then rank the alternatives based on these values. Different aggregation operators are adopted by researchers for fusing values of the alternatives [3]. Liao and Xu [1] claimed that, utility based ranking is weak and provided some limitations of the scheme which are, (i) aggregation of different competing criteria for each alternative is logically wrong and (ii) also such aggregation cannot work fine with correlated set of criteria. To ***circumvent these issues***, scholars developed the idea of outranking based methods. In outranking based methods, ordinal data are used directly for ranking without converting them into abstract forms. This retains the natural preference by the DMs which resembles closely to the human thinking and decision making process. Also, outranking methods are better in handling fuzziness by using indifference and preference parameters [4]. The two popular outranking schemes are ELECTRE (elimination and choice expressing reality) and PROMETHEE. Out of these two methods, ELECTRE is based on concordance and discordance concept while, PROMETHEE is based on the concept of partial and net outranking. Several variants of both these schemes are proposed to address different scenarios. A detailed discussion on these variants are given in [4,5]. Both these schemes follow the concept of outranking relation and are affected by rank reversal issue. Upon investigation we infer that, these outranking variants vary only in their nature of preference representation. The popular forms of rating are interval numbers [6], triangular numbers [7], trapezoidal numbers [8], intuitionistic fuzzy numbers [9] etc.

Before getting into the further discussion, it is worth to review some literature relating to *'supplier selection'* and *'IFS based MCDM methods'*. Scholars have widely used IFS based decision framework for making rational decisions at uncertain situations. The power of IFS to represent uncertainty and vagueness in three (preference, non-preference and indeterminacy) perspectives motivated much attractive research under IFS context. To realize the usefulness of IFS concept, some literature studies are reviewed. Xu and Liao [10] presented an intensive and extensive survey on intuitionistic fuzzy preference relation (IFPR) which dealt with the concept of IFPR, consistency measure in IFPR, consensus model and prioritization frameworks in IFPR. They claimed that IFPR is a powerful tool for uncertain decision making which needed some standard methods in each domain for making sensible and rational decision. Following this, Liao et al. [11] put forward a new IFPR based consensus reaching framework for suitable selection of outstanding Ph.D., student for the China Scholarship Council. They also proposed new consistency measure and automated procedure for repairing inconsistent IFPRs under the multiplicative consistency context. Further, Yu and Liao [12] presented an attractive scientometric review on IFS which quantified results related to IFS studies from various perspectives viz., influential articles in IFS, influential authors in IFS and most cited article in IFS. With a view of circumventing the weakness of [11], Liao et al. [13] presented an enhanced consensus reaching method under IFPR context that overcomes the drawback of previous consensus reaching method, by removing some opinions of the DM, rather than removing the entire DM from the group. The results clearly bring out the power of IFPR in managing uncertainty in the decision making process. Recently, Liao et al. [14] made a comparative study on newly proposed distinct consensus measure for a suitable selection of products in an organization under IFPR context. Two consensus measures viz., ordinal consensus measures and outranking flow based consensus measures were proposed and comparison results infer that, outranking flow based consensus measure produce a much reasonable decision in uncertain situations. Motivated by the work of Yu and Liao [12], Liu and Liao [15] conducted a deep bibliometric survey on fuzzy based decision making process from 1970 to 2015 which not only concentrates on IFS based decision making process but also focuses on other variants of fuzzy in decision making process. They present an overall picture of fuzzy decision making process and it's importance in the research ground based on quantitative analysis of factors like, influential journals, authors, application domains, countries, institutions and articles.

Based on the review process conducted above, the usefulness of IFS can be clearly realized. The extension of preference relation under IFS context is a powerful tool for sensible management of uncertainty and vagueness. Also, extension of outranking methods under IFS context is an attractive research topic which provides much reasonable consensus reaching process. Motivated by these inferences, we set our proposal in this direction.

From Table 1, we identify the following ***challenges***:

(1) Supplier selection is an attractive area for research which lacks significant scientific contribution from the linguistic based preference perspective. From the study, we infer that DMs often give numeric preferences which limit their cognitive thought process within a specific bound and are more prone to inaccuracies in rating.
(2) The vagueness and imprecision are argued to be an implicit factor in any process involving humans. Addressing such vagueness and imprecision in the decision process is an interesting challenge.
(3) Aggregation of group opinions in group decision making is another attractive challenge which not only seeks a proper fusion of viewpoints but also expects much sensible and rational aggregation of DMs' opinions that yield no virtual sets.
(4) Finally, we observe that sensible and rational ranking of alternatives under the linguistic context is also a complex challenge and the use of PROMETHEE based outranking method is rare in case of SS process, while the use of TOPSIS and AHP are most common in this field.

With the view of ***circumventing these challenges***, we ***gain better motivation*** and set our proposal for better decision making process:

(1) The challenge (1) ***motivated*** our proposal towards solving supplier selection problem under the roof of linguistic preferences. The ***motivation for choosing linguistic preferences as the source for rating is in two-fold:*** (a) from the DMs' point of view, metaphorical rating is often easy and well distinguishable and hence, DMs prefer this as a convenient source for rating alternatives. (b) from the methods' point of view, linguistic ratings are used for the better representation of vagueness and imprecision which are properly handled by using popular fuzzy schemes (triangular, trapezoidal, intuitionistic etc.).
(2) To better handle challenge (2), IFS based ranking is employed which is computationally powerful and attractive tool for handling vagueness and hence, ***this motivated us to*** set our proposal to convert linguistic terms to intuitionistic fuzzy values for proper and sensible ranking.
(3) Since group decision making and aggregation are an interesting process in MCDM (as per challenge (3)), we ***gain motivation and consider*** these two aspects in our proposal as well. We propose new aggregation operator called LBA for directly aggregating linguistic preferences without making any changes to the setup, with a view of preventing loss of information and maintaining rational decision process. We further use intuitionistic fuzzy hybrid aggregation operator for aggregating intuitionistic fuzzy values.

Table 1
Survey on supplier selection from 2013 to 2016.

Ref #	Year	Fuzzy based MCDM methods	Aggregation	Application	Discussion
[16]	2016	Interval IFS (IVIFS) based ranking	Yes	Automobile supplier selection	Linguistic variables are converted to IVIFVs and were fused to form a single decision matrix. TOPSIS scheme was used to assign weights and linear programming model was proposed for ranking suppliers.
[17]	2016	IFS and IVIFS based ranking	No	Supplier selection	Modified Delphi is adopted to choose criteria for SS process. Choquet integral and fuzzy measures were proposed for selecting feasible suppliers. Expert opinion was also considered for evaluation purpose.
[18]	2016	IFS based ranking	Yes	Supplier selection	Associated IF probabilistic averaging and geometric operators were proposed and their properties were realized. Practicality was verified by using SS example.
[19]	2016	Triangular IFS (TrnIFS) based ranking	Yes	Green supplier selection	New aggregation operators were proposed with TrnIFS and their properties are realized.
[20]	2015	IFS based ranking	No	Natural gas supplier selection	IFS based TOPSIS was proposed for ranking 5 alternatives with respect to 30 criteria. The rating is made linguistically and criteria weights are calculated using entropy based measure.
[21]	2015	IFS based ranking	Yes	Green supplier selection	New index evaluation was modeled using IF-TOPSIS. Aggregated matrix was used to rank the alternatives with the help of IF-TOPSIS. Criteria were assigned weights using information entropy.
[22]	2015	IFS based ranking	No	Supplier selection	MOORA was formulated for IFS environment and practically tested for optimal supplier selection
[23]	2015	IFS based ranking	Yes	Supplier selection	New aggregation method was proposed using Einstein operators and its properties were realized. Ranking efficacy was tested using supplier selection for logistic service.
[24]	2015	IFS based ranking	No	Supplier selection	Criteria were assigned weights using optimization model. Ranking of supplier is done using IF-TOPSIS. Sensitivity analysis was done to verify stability of the proposed scheme.
[25]	2015	IVIFS based ranking	No	Green supplier selection	A new linear programming model was developed under IFS domain for ranking of suppliers. Weights of criteria were also calculated using optimization model.
[26]	2015	IVIFS based ranking	No	Supplier selection	Criteria were assigned weights based on maximization model. New ranking scheme was modeled which maximizes divergence and minimizes similarity.
[27]	2015	IFS based ranking	No	Vendor selection	Different ranking methods were applied under IFS domain to select better vendor for the company. Final ranking was done using lexicographic procedure.
[28]	2014	IFS based ranking	No	Supplier selection	Criteria were assigned weights using AHP method. Ranking was done using IF-TOPSIS method.
[29]	2014	IFS based ranking	No	Coal supplier selection	VIKOR ranking method was extended to IFS domain for selecting feasible coal supplier. Linguistic data was used by DMs for rating suppliers. Criteria weights were estimated using normalized procedure.
[30]	2014	IVIFS based ranking	No	Supplier selection	Criteria weights are estimated using entropy measure. IVIF-ELECTRE was proposed for ranking which uses graphical representation of indices. Practicality was tested using supplier selection problem.
[31]	2013	IFS based ranking	No	Supplier selection	New IFS based programming model is proposed for determining weights and ideal solution. The model processes heterogeneous data. Finally, ranking was done using distance measure.
[32]	2013	IFS based ranking	No	Supplier selection	Two ranking schemes were proposed under IFS domain namely, choquet integral and TOPSIS. Comparison was made between the methods in selecting a suitable supplier.
[33]	2013	IFS based ranking	No	Sustainable supplier selection	Criteria were initially identified based on literature investigation. A new MCDM algorithm under IFS domain was proposed for optimal selection of sustainable supplier.
[34]	2013	IFS based ranking	No	Supplier selection	IF based TODIM approach was proposed and its practicality was tested using SS problem.
[35]	2013	IFS based ranking	Yes	Manufacturer selection	New concordance and discordance measure was proposed using distance metric. IFWA was used to integrate decision matrices. ELECTRE under IFS domain was proposed and integrated with TOPSIS for solving SS problem.

(4) Finally, to tackle challenge (4), PROMETHEE ranking method is extended to IFS context (linguistic information converted to IFS information) which is a powerful outranking method that mitigates the issue raised by utility ranking methods. Also, such outranking method is rarely explored in the field of SS. Thus, ***motivated by these claims***, we set our proposal in this direction and a detailed discussion on the LBA-IFSP decision making framework is made in Section 4.

Further, some ***key contributions*** of the proposed decision framework are also presented in a nutshell below for the ease of understanding:

(1) The LBA-IFSP framework is the first framework involving outranking relations and linguistic aggregation, whose adequacy is investigated to supplier selection problem. An analysis of this sort will help DMs to properly validate a framework for making rational decisions.
(2) This proposal complements the work done by Liao and Xu [1] by taking full advantage of the IFS environment which sensibly manages fuzziness and uncertainty. Unlike Liao and Xu framework [1], in this proposal, we gain motivation from the idea of Geldermann et al. [36] and formulate the rating information, preference and indifference parameters and criteria weights as

intuitionistic fuzzy values and retain the IFPR property throughout the ranking process.

(3) In this proposal, the strength of the framework is tested from both theoretical and numerical point of view. Such investigations clearly bring out the power of any decision framework. From the analysis made in Section 5, we infer that proposed framework is strong in both theoretic and numeric sense compared to its counterpart, IF-PROMETHEE method [1].

(4) The parameters used for theoretic analysis are gained by intuition, while the parameters used for numeric analysis are inspired from [37] and to the best of our knowledge, this is the first time, these parameters are used in a decision framework involving linguistic aggregation, outranking relations and supplier selection.

(5) Also, from the analysis, we infer that, unlike Liao and Xu [1] framework, the proposed LBA-IFSP framework (i) manages imprecision and vagueness better by retaining IFS property throughout evaluation, (ii) robust against rank reversal issue, (iii) remains moderately consistent with other methods and (iv) remains rationally coherent to DMs' viewpoint.

Following this, the rest of the paper is organized as Section 2 for preliminaries where the basic concepts of IFS and classical PROMETHEE are discussed, Section 3 for proposed methodology where the proposed decision framework is presented, followed by new aggregation operator for aggregating DMs' preferences and new ranking method for sensible ranking of alternatives. Section 4 demonstrates an illustrative example for realizing the practicality of the proposal. Section 5 further presents comparative analysis from both theoretic and numeric perspectives. Finally, Section 6 presents the conclusion and future scope of research.

2. Preliminaries

2.1. Basic concepts of IFS

Definition 1 *([9])*. Let Y be a fixed crisp set such that $S \subset Y$ is also fixed. Now IFS $\bar{S}$ in Y is an object which is of the form:

$$\bar{S} = (y, \mu_{\bar{s}}(y), \upsilon_{\bar{s}}(y) | y \in Y) \tag{2}$$

where μ is the membership term, ν is the non-membership term, $\mu + \upsilon \leq 1$, $\mu \in [0, 1]$ and $\upsilon \in [0, 1]$.

Definition 2 *([38])*. The intuitionistic fuzzy preference relation (IFPR) R on a set $Y = (y_1, y_2, \ldots, y_n)$ is a matrix of order $(n \times n)$ where, each instance $r^*_{kl} = ((y_k, y_l), \mu(y_k, y_l), \upsilon(y_k, y_l)) \forall k, l = 1, 2, \ldots, n$ follows the constraints as given by:

$$\mathrm{R} = \begin{cases} \mu_{kk} = \upsilon_{kk} = (0.5, 0.5) & \textit{at diagonal} \\ \mu_{kl} + \upsilon_{kl} \leq 1 & \forall \mu \in [0, 1], \nu \in [0, 1] \\ \mu_{kl} = \upsilon_{lk} & \textit{instances excluding diagonal} \end{cases} \tag{3}$$

where k is the k$^{\text{th}}$ row and l is the l$^{\text{th}}$ column of the preference matrix.

Definition 3 *([38])*. The IFS obeys the following operational laws as given by:

Consider any two instances $r^*_{mn} = (\mu_{mn}, \upsilon_{mn})$ and $r^*_{pq} = (\mu_{pq}, \upsilon_{pq})$ with $\mu \in [0, 1]$ and $\upsilon \in [0, 1]$.

$$r^*_{mn} \otimes r^*_{pq} = \left(\mu_{mn} + \mu_{pq} - \mu_{mn}\mu_{pq}, \upsilon_{mn}\upsilon_{pq}\right) \tag{4}$$

$$r^*_{mn} \otimes r^*_{pq} = \left(\mu_{mn}\mu_{pq}, \upsilon_{mn} + \upsilon_{pq} - \upsilon_{mn}\upsilon_{pq}\right) \tag{5}$$

$$\lambda r^*_{mn} = \left(1 - (1 - \mu_{mn})^{\lambda}, \upsilon^{\lambda}_{mn}\right) \tag{6}$$

$$r^{*\lambda}_{mn} = \left(\mu^{\lambda}_{mn}, 1 - (1 - \upsilon_{mn})^{\lambda}\right) \tag{7}$$

where m, n, p and q are some instances in the range 1, 2,. . .,n and $\lambda > 0$. Also all the operations described in ((4)–(7)) yield an AIFS whose range is given by [0, 1].

Definition 4 *([39])*. Consider the decision matrix $D = \left(r^*_{ij}\right)^p_{m \times n}$ with $r^*_{ij} = \left(x_{ij}, \mu\left(x_{ij}\right), \nu\left(x_{ij}\right)\right)$ for m alternatives and n criteria over p DMs. The aggregation operator follows a mapping $\Psi^n \to \Psi$ that is used for fusing the matrices and is given by:

$$\text{IFHA} = \left(1 - \prod_{k=1}^{p}\left(1 - \mu_{ij}\right)^{\gamma^k}, 1 - \prod_{k=1}^{p}\left(1 - \upsilon_{ij}\right)^{\gamma^k}\right) \tag{8}$$

where IFHA is the intuitionistic fuzzy hybrid aggregation and γ is the relative weight/strategy of the DM. For integrating IFPRs, we use the operator with a constraint such as $i < j$. By this, we calculate the upper triangular values and hence we can easily form the whole IFPR using the property given in Definition 2.

Remark 1. We can also divide the result from IFHA operator by a factor n. Here n represents the number of matrices being considered for aggregation. The reason for performing this operation is to maintain the IFS property of $\mu + \nu \leq 1$ which gets relaxed when $\sum$ weight $\neq 1$ (in IFHA operator this refers to $\sum \gamma$).

Definition 5 *([40,41])*. Let $a = (\mu_1, \nu_1)$ and $b = (\mu_2, \nu_2)$ be two IFVs with μ_1, μ_2, ν_1 and ν_2 all belonging to the range [0, 1]. Now, these two IFVs can be ranked using the following 3 schemes viz.,

Scheme a:

- If $S(a) < S(b)$ then $a < b$
- If $S(a) = S(b)$ then, apply the following
  - ○ If $H(a) < H(b)$ then $a < b$
  - ○ If $H(a) = H(b)$ than $a = b$

Here, score $S = (\mu - \upsilon)$ and accuracy $H = (\mu + \upsilon)$.

Scheme b:

$$\rho(a) = 0.5\left(1 + \pi^a\right)\left(1 - \mu^a\right)$$

In Scheme b, the highest value is preferred more.

Scheme c:

- If $L(a) < L(b)$ then $a < b$
- If $L(a) = L(b)$ then, apply the following;
  - ○ If $H(a) > H(b)$ then $a > b$
  - ○ If $H(a) = H(b)$ then $a = b$

Here, similarity function $L(a) = {}^{(1 - \upsilon_a)}\!/\!(1 + \pi_a)$ which is defined in the range [0, 1].

Remark 2. Xu and Liao [41] argue that, Scheme c is effective in ranking alternatives compared to all three schemes. So, in this paper, we adopt scheme c for ranking.

2.2. Procedure for classical PROMETHEE method

Let us review the steps involved in classical PROMETHEE method. We consider PROMETHEE II for the analysis in this paper, as the method deals with the complete ranking of alternatives. For better understanding the general idea of PROMETHEE and its family, readers are encouraged to refer [42].

Step 1: For a given set of alternatives, $A = (a_1, a_2 \ldots, a_m)$ over the criteria, $C = (c_1, c_2 \ldots, c_m)$, we estimate the deviation of preferences using (9).

$$\textit{Deviation}\ d = (c(a) - c(b)) \tag{9}$$

where a and b are any two alternatives, c is any criterion.

Step 2: Determine the preference values between any two alternatives using any one of the six functions. Mathematically, it is represented by (10).

$$\textit{Preference } p = f(a,b) \tag{10}$$

where $f(a,b)$ is the assessment function that translates preferences between alternatives over each criterion in the range [0, 1]. There are six popular functions developed by researchers for preference analysis viz., usual, U-shape, V-shape, level, V-shaped with indifference and Guassian [43].

Step 3: Estimate the total preference index using (11).

$$\textit{Total Preference Index } T = \sum_{k=1}^{n} \omega_k p_k \tag{11}$$

where ω is the weight of the criteria with $\sum \omega = 1$.

Step 4: Estimate the positive and negative outranking values using ((12) and (13)).

$$\phi_+(a) = \frac{1}{m-1}\sum_{x\in A} T(a,x) \tag{12}$$

$$\phi_-(a) = \frac{1}{m-1}\sum_{x\in A} T(x,a) \tag{13}$$

where A is the set of m alternatives, $\phi_+(a)$ is the positive outrank and $\phi_-(a)$ is the negative outrank. We say an alternative a outranks b if $\phi_+(a) \geq \phi_+(b)$and $\phi_-(a) \leq \phi_-(b)$. If we have strictly less than (or) greater than condition then, the estimate becomes incomparable.

Step 5: Finally, calculate the net outranking to avoid confusions from partial outranking. The higher net value is preferred more. Mathematically, net outranking is given by (14).

$$\phi_{net}(a) = \phi_+(a) - \phi_-(a) \tag{14}$$

3. Proposed methodology

3.1. Proposed two-tier framework

The Fig. 1 depicts the proposed two-tier framework. This consists of aggregation and ranking mechanisms. The first tier does the aggregation of linguistic terms by the DMs using LBA operator, while the second tier does the ranking using PROMETHEE under IFS domain. The framework is self-contained and easy to follow. A detailed discussion on these two tiers is made in Section 3.2 and 3.3.

3.2. Proposed LBA operator

The LBA is an aggregation operator that uses linguistic terms directly for integration without making any quantitative conversion. The reason for proposing such mechanism is that LBA based fusion maintains the originality of the linguistic term set defined by the DM. On the other hand, classical aggregation operators convert linguistic terms into quantifiable units and their corresponding aggregation yields values that no longer matches with the term set. To better represent preferences, without considerable loss of originality, LBA based fusion mechanism is adopted.

Xu [44] conducted a deep survey on different linguistic aggregation operators and classified these operators into five zones. He claimed that the operators which directly worked with words are more effective than those which quantify terms. In the past decades, researchers have proposed many operators within this category. But, these operators tend to yield original as well as virtual term sets. To avoid such virtual terms during the fusion process, we propose a simple and straightforward LBA operator that yields results that are highly matching with the original term set.

Definition 6 *([45])*. Consider a term set S, defined by $\left[S_{\beta_i} | \forall \beta_i \in [-s, s]\right]$ with s being a non-negative integer. Every term S_{β_i} is a linguistic term with the following properties:

- If $\beta_i > \beta_j$ then, $S_{\beta_i} > S_{\beta_j}$.
- The $neg\left(S_{\beta_i}\right) = S_{-\beta_i}$.

Definition 7. The aggregation operator is a mapping function defined as U: $\Psi^n \to \Psi$ such that,

$$LBA = \begin{cases} \text{Scheme 1} & \textit{if all preference instances are unique} \\ \text{Scheme 2} & \textit{otherwise} \end{cases} \tag{15}$$

where,

Scheme 1: Find the zone where maximum preferences occur, then choose the moderate term of that particular zone as the aggregated value. If tie occurs in the count, then break the tie arbitrarily by choosing S_0 value.

Scheme 2: Calculate the maximum #(occurrences of each term) and choose that term which gives the maximum value.

Example 1. Consider three DMs A, B and C rating an alternative W with respect to a criterion P, with A = (S_2), B = (S_1) and C = (S_1). Now, since all terms are not unique, apply Scheme 2. We observe that, $\#(S_1) = 2$ and $\#(S_2) = 1$, so S_1 is chosen as the fused value for an alternative W over criterion P.

3.3. Procedure for IFSP method

The IFSP is an extension to classical PROMETHEE under IFS environment. The method adopts IFVs as input to the decision matrix which helps the procedure to take better advantage of the IFS theory. This idea is lacking in the previous version of IFS based PROMETHEE [1]. For a clear understanding of IFSP method, working procedure is given below:

Step 1: Determine the set of alternatives and the set of criteria for the problem under study. Use IFVs for rating each alternative with respect to each criterion.

Step 2: The relative importance of each alternative is collected from the DM and an integrated relative importance value for each alternative is obtained using LBA operator.

Step 3: Calculate the membership and non-membership deviation between alternatives for each criterion separately using ((16) and (17)). Apply popular V preference function to estimate the preference values of each alternative pair using ((18) and (19)).

$$d_a^{\mu}(m,n) = c_a^{\mu}(m) - c_a^{\mu}(n) \tag{16}$$

$d_a^{\upsilon}(m,n) = c_a^{\upsilon}(m) - c_a^{\upsilon}(n)$ (17)where m, n are any two alternatives and c_a is the a$^{\text{th}}$ criterion of the decision matrix.

$$\mu_{mn}^{a} = \begin{cases} 0 & d_a^{\mu}(m,n) \leq q^{\mu} \\ \left(d_a^{\mu}(m,n) - q^{\mu}\right) / \left(p^{\mu} - q^{\mu}\right) & q^{\mu} < d_a^{\mu}(m,n) \leq p^{\mu} \\ 1 & d_a^{\mu}(m,n) > p^{\mu} \end{cases} \tag{18}$$

$$\upsilon_{mn}^{a} = \begin{cases} 0 & d_a^{\upsilon}(m,n) \leq q^{\upsilon} \\ \left(d_a^{\upsilon}(m,n) - q^{\upsilon}\right) / \left(p^{\upsilon} - q^{\upsilon}\right) & q^{u} < d_a^{\upsilon}(m,n) \leq p^{\upsilon} \\ 1 & d_a^{\upsilon}(m,n) > p^{\upsilon} \end{cases} \tag{19}$$

On applying the preference function, we obtain a separate set of square matrices of alternatives defined over each of the criterion

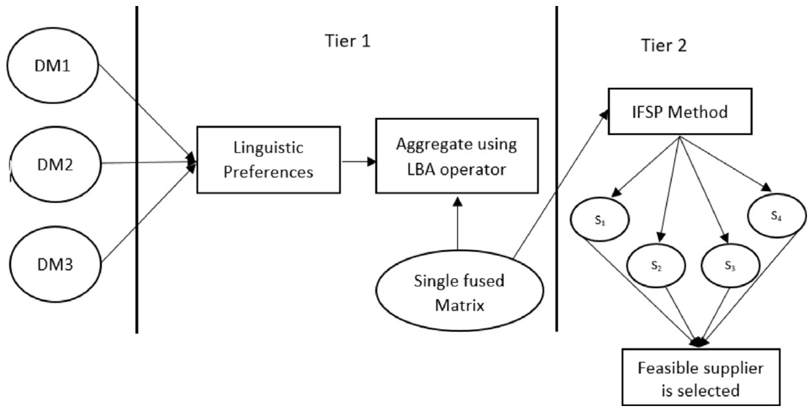

Fig. 1. Proposed two tier framework.

in both membership and non-membership domain. Here, the preference value P and indifference value Q are defined by the DM for each of the criterion.

Step 4: Form IFPR of each of these square matrices using the IFPR property, $\mu_{ij} = \nu_{ji}$ and vice versa. Aggregate these IFPRs with respect to their membership and non-membership pairs respectively using (8). Suppose there are 3 alternatives and 4 criteria, we form 4 square matrices of order (3×3) for membership and non-membership domains separately. We then form 8 IFPRs of order (3×3) where, first three IFPRs correspond to membership and next three corresponds to non-membership domains. Now, we integrate $((1, 4), (2, 5), (3, 6)$ *and* $(4, 8))$ square matrices to form 4 IFPRs.

Step 5: Further, aggregate these IFPRs to form a single IFPR using (8). This resultant matrix gives the preference relation between every alternative pair for all the criteria.

Step 6: Using the matrix from Step 5, calculate positive and negative outranking.

$$\phi^{+}(\alpha_i) = \frac{1}{(n-1)} \oplus_{l=1}^{m} r_{kl} \tag{20}$$

$$\phi^{-}(\alpha_i) = \frac{1}{(n-1)} \oplus_{k=1}^{m} r_{kl} \tag{21}$$

where α_i is the i^{th} alternative, r_{ij} is the instance of the IFPR matrix.

$$\text{Here, } \oplus_{l=1}^{m} r_{kl} = \left(1 - \left(\prod_{a=1}^{m} \left(1 - \mu_{kl}^{a} \mu_{\omega^k}\right)\right),\right.$$
$$\left.\left(\prod_{a=1}^{m} \left(\upsilon_{kl}^{a} + \upsilon_{\omega^k} - \upsilon_{kl}^{a} \upsilon_{\omega^k}\right)\right) \forall k \neq l\right) \tag{22}$$

where $\left(\mu_{\omega^k}, \upsilon_{\omega^k}\right)$ is the IFS weights of each alternative given by the DM and m is the total number of alternatives.

Similarly, $\oplus_{k=1}^{m} r_{kl}$ can also be estimated.

Step 7: Calculate the net outranking value for each alternative using (23).

$$\phi^{net} = \phi^{+} \oplus \phi^{-} \tag{23}$$

where $\oplus$ is given by (4), $\phi^{+}$and ϕ^{-} are obtained from ((20) and (21)).

Step 8: Now, rank the alternatives based on the net outranking value from Step 7. This value is estimated for each alternative separately and it is an IFV. Now, apply Scheme c of Definition 5. The alternative with highest L value is preferred. Preference order is thus formed based on the decreasing order of L value. Finally, construct rank value set by multiplying each L value by a factor 100. This determines the weight (relative importance) of each alternative in percentage and hence, DMs can make sensible decisions. This also encourages DMs to form backup sets for other tasks.

Before demonstrating the practicality of the proposed decision making framework (LBA-IFSP), it is worth to clarify certain details of the framework. They are as follows:

(1) The step 1 deals with the construction of decision matrix. The committee decides on the total number of alternatives and criteria for evaluation after the pre-screening process. As mentioned earlier, DMs initially rate alternatives linguistically and upon sensible aggregation of linguistic terms, a decision matrix is formed for evaluation.
(2) In step 2, DMs give their personal choices (relative importance value) about each alternative and those values are aggregated to form a vector of order $(m \times 1)$, where m is the total number of alternatives taken for evaluation.
(3) In step 3, deviation values are estimated separately for both membership and non-membership terms. This is done with a view of preventing loss of information and effective handling of vagueness. Though IFS lacks subtraction operator, we make a new formulation of splitting IFS terms as membership and non-membership and then apply the subtraction for each of those fragments. Some scholars suggest subtraction of score values, which eventually aggravates imprecision by the loss of information.
(4) Further, in step 3, we also estimate the preference values using preference functions. Based on the suggestion made by Liao and Xu [1], we use V-shape function for estimation. With a view of preventing loss of information, preference values are estimated for both degrees separately and then, IFPRs are constructed from these matrices using the IFPR property discussed in Definition 2.
(5) The IFPR matrices from step 4 (membership and non-membership) are further integrated using Definition 4. Suppose there are p criteria, there are 2p initial IFPR matrices that are formed. Upon integration, these 2p matrices reduces to p matrices of order $(m \times m)$, where m is the number of alternatives.
(6) In step 5, final aggregation of the p IFPRs is done using Definition 4 and this IFPR matrix is taken for rank estimation. Unlike IF-PROMETHEE method [1], in the proposed framework, the IFPR property is maintained till the end by allowing the operator to perform aggregation only to the upper part of the matrix. The lower is part is constructed simply by using Definition 2.
(7) In the last step, the aggregated matrix is taken for rank evaluation. Both partial and complete ranks are estimated for each alternative using ((20)–(23)). In order to retain the property of IFV in net outranking value, we apply (23). Since the positive and negative outranking are intuitionistic fuzzy values; we apply (4) to form net outranking value which is also intuitionistic fuzzy in nature. Finally, these values are taken and scheme

Table 2
Linguistic term set to corresponding IFVs [47].

Linguistic Rating (Decision matrix)	Corresponding IVFs	Linguistic Rating (Alternative)	Corresponding IVFs
Extremely Good (EG, L_3)	(0.95, 0.05)	Highly Preferred (HP, L_2)	(0.90, 0.10)
Moderately Good (VG, L_2)	(0.85, 0.10)	Moderately Preferred (MP, L_1)	(0.75, 0.20)
Good (G, L_1)	(0.70, 0.20)	Neutral (N, L_o)	(0.50, 0.40)
Medium (M, L_o)	(0.50, 0.35)	Less Preferred (LP, L_{-1})	(0.35, 0.60)
Bad (B, L_{-1})	(0.35, 0.55)	Highly Less Preferred (HLP, L_{-2})	(0.10, 0.90)
Moderately Bad (VB, L_{-2})	(0.25, 0.70)		
Extremely Bad (EB, L_{-3})	(0.10, 0.90)		

Table 3
Relative weights of decision maker(s).

Personal rating of alternatives by DMs	S_1	S_2	S_3	S_4
D_1	L_o	L_0	L_1	L_0
D_2	L_{-1}	L_0	L_1	L_{-1}
D_3	L_{-1}	L_1	L_2	L_1

Table 4
Decision matrix for supplier selection.

DMs	Alternatives	Criteria				
		C_1	C_2	C_3	C_4	C_5
DM1	S_1	L_1	L_0	L_1	L_1	L_{-1}
	S_2	L_0	L_{-1}	L_0	L_1	L_2
	S_3	L_2	L_2	L_1	L_0	L_1
	S_4	L_1	L_1	L_2	L_{-1}	L_0
DM2	S_1	L_0	L_{-1}	L_1	L_1	L_{-1}
	S_2	L_1	L_0	L_1	L_2	L_1
	S_3	L_1	L_2	L_0	L_0	L_1
	S_4	L_2	L_1	L_2	L_1	L_0
DM3	S_1	L_1	L_0	L_2	L_1	L_0
	S_2	L_2	L_2	L_1	L_1	L_1
	S_3	L_0	L_1	L_{-1}	L_2	L_2
	S_4	L_1	L_{-1}	L_1	L_0	L_{-1}

c of Definition 5 is applied. This yields a ranking order which is more sensible and reasonable for evaluation. Using this, rank value set is constructed and backup alternatives are selected based on this set. The usage of (23) not only helps IFSP method to retain IFS property better but also avoids the problem of negative values in the rank value set. Thus, the new formulation for net outranking, unlike the differencing of positive and negative outranks yields a more sensible inference (see Section 5.2 for further clarification).

4. An illustrative example

In this section, we test the applicability of the proposed LBA-IFSP framework by applying it to the popular SS problem [46]. In this problem, we consider a manufacturing firm that wants to buy a critical component for the assembly process. The firm has identified four potential suppliers for the same based on pre-screening process. Now, the objective is to select one better supplier out of the four selected suppliers. For this, we adopt the idea of MCDM. The implementation procedure is given below:

Step 1: Consider 3 DMs (D_1, D_2, D_3) evaluating 4 Suppliers (S_1, S_2, S_3, S_4) based on 5 criteria (C_1, C_2, C_3, C_4, C_5). The 5 criteria taken for evaluation are cost, quality, service performance, risk and profile of supplier respectively. Among these 5 criteria, C_1 and C_4 are cost factors and the remaining are benefit factors. A detailed description on these criteria and its corresponding sub-criteria can be found in [46]. The decision matrix is formed with linguistic preferences. The DMs also give their personal relative importance value for each of the alternatives linguistically. Table 2–5 presents this clearly.

Table 5
Aggregated decision matrix using LBA operator.

Aggregated Matrix	Weights	C_1	C_2	C_3	C_4	C_5
S_1	L_{-1}	L_1	L_0	L_1	L_1	L_{-1}
S_2	L_0	L_2	L_0	L_1	L_1	L_1
S_3	L_1	L_2	L_2	L_0	L_0	L_1
S_4	L_0	L_1	L_1	L_2	L_0	L_0

Table 6
Results obtained by applying preference function.

Criteria	Alternatives	Terms							
		μ				υ			
		S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4
C_1	S_1	0	0	0	0	0	0.17	0.17	0
	S_2	0.7	0	0	0.7	0	0	0	0
	S_3	0.7	0	0	0.7	0	0	0	0
	S_4	0	0	0	0	0	0.17	0.17	0
C_2	S_1	0	0	0	0	0	0	0.38	0.23
	S_2	0	0	0	0	0	0	0.38	0.23
	S_3	1	1	0	0.84	0	0	0	0
	S_4	1	1	0	0	0	0	0.15	0
C_3	S_1	0	0	1	0	0	0	0	0.14
	S_2	0	0	1	0	0	0	0	0.14
	S_3	0	0	0	0	0.21	0.21	0	0.35
	S_4	1	1	1	0	0	0	0	0
C_4	S_1	0	0	1	1	0	0	0	0
	S_2	0	0	1	1	0	0	0	0
	S_3	0	0	0	0	0.20	0.20	0	0
	S_4	0	0	0	0	0.20	0.20	0	0
C_5	S_1	0	0	0	0	0	0.42	0.42	0.24
	S_2	1	0	0	1	0	0	0	0
	S_3	1	0	0	1	0	0	0	0
	S_4	1	0	0	0	0	0.18	0.18	0

Step 2: Aggregate alternatives' weights and decision matrices of three DMs into the respective single matrix using LBA operator given in Definition 7. The aggregated value is shown in Table 5.

Step 3: Apply the proposed IFSP algorithm from Section 3.3 on the aggregated matrix to obtain a better supplier for the organization.

The Table 6 shown above is obtained by applying ((16)–(19)). The sample working is shown in Appendix A for a clear understanding of the decision making process. Now, we apply Step 4 of IFSP to obtain the IFPR from Table 6 based on the IFPR property of $\mu_{hk} = \nu_{kh}$. For brevity, we show the aggregated IFPR directly. The aggregation is done using IFHA operator given in Definition 4.

Table 7 shows the IFPRs R_1 to R_5 which are formed by using weight value as 0.5 and the resultant R matrix is formed using weight value as 0.2. As mentioned in Remark 1, if we deviate from the constraint of $\sum$ weight $= 1$, then the IFS property might fail to hold. In order to balance this issue, we introduce a factor n which divides the result from IFHA operator. For brevity, in this paper we however maintain (not deviate from) the constraint but follow the procedure as in. Now, we apply ((20)–(23)) to obtain the partial

Table 7
Estimation of IFPRs using IFHA Operator.

Matrices	Alternatives	S_1	S_2	S_3	S_4
R_1	S_1		(0.04, 0.22)	(0.04, 0.22)	(0, 0)
	S_2	(0.22, 0.04)		(0, 0)	(0.22, 0.04)
	S_3	(0.22, 0.04)	(0, 0)		(0.22, 0.04)
	S_4	(0, 0)	(0.04, 0.22)	(0.04, 0.22)	
R_2	S_1		(0, 0)	(0.11, 0.5)	(0.06, 0.5)
	S_2	(0, 0)		(0.11, 0.5)	(0.06, 0.5)
	S_3	(0.5, 0.11)	(0.5, 0.11)		(0.3, 0.04)
	S_4	(0.5, 0.06)	(0.5, 0.06)	(0.04, 0.3)	
R_3	S_1		(0, 0)	(0.5, 0.05)	(0.04, 0.5)
	S_2	(0, 0)		(0.5, 0.05)	(0.04, 0.5)
	S_3	(0.05, 0.5)	(0.5, 0.5)		(0.1, 0.5)
	S_4	(0.5, 0.04)	(0.5, 0.04)	(0.5, 0.1)	
R_4	S_1		(0, 0)	(0.5, 0.05)	(0.5, 0.05)
	S_2	(0, 0)		(0.5, 0.5)	(0.5, 0.05)
	S_3	(0.05, 0.5)	(0.5, 0.5)		(0, 0)
	S_4	(0.05, 0.5)	(0.05, 0.5)	(0, 0)	
R_5	S_1		(0.12, 0.5)	(0.12, 0.5)	(0.06, 0.5)
	S_2	(0.5, 0.12)		(0, 0)	(0.5, 0.05)
	S_3	(0.5, 0.12)	(0, 0)		(0.5, 0.05)
	S_4	(0.5, 0.06)	(0.05, 0.5)	(0.05, 0.5)	
Resultant R	S_1		(0.01, 0.03)	(0.06, 0.06)	(0.03, 0.07)
	S_2	(0.03, 0.01)		(0.05, 0.03)	(0.06, 0.05)
	S_3	(0.06, 0.06)	(0.03, 0.05)		(0.05, 0.03)
	S_4	(0.07, 0.03)	(0.05, 0.06)	(0.03, 0.05)	

Table 8
Partial and complete ranking estimates.

Alternatives	Outranking		
	ϕ^+	ϕ^-	ϕ^{net}
S_1	(0.03, 0.24)	(0.06, 0.23)	(0.09, 0.05)
S_2	(0.04, 0.07)	(0.04, 0.079)	(0.09, 0.006)
S_3	(0.07, 0.01)	(0.1, 0.01)	(0.16, 0)
S_4	(0.08, 0.08)	(0.07, 0.08)	(0.14, 0.006)

(positive, negative) and complete (net) ranking order for the alternatives. The Table 8 shows the values for positive, negative and net outranking.

Using the net outranking values from Table 8, we obtain the preference order by applying scheme c of Definition 5. The order follows: $L(A_1) = 0.5108$, $L(A_2) = 0.5221$, $L(A_3) = 0.5435$and $L(A_4) = 0.5361$ with $S_3 > S_4 > S_2 > S_1$. So, now the feasible supplier is S_3.

Step 4: Check the consistency of the proposed LBA-IFSP framework with other methods by using Spearman correlation [48].

In order to maintain homogeneity in the comparison process, we compare the proposed LBA-IFSP framework with other MCDM methods under IFS environment. The Table 9 shows the ranking of different suppliers using IF-ELECTRE, IF-VIKOR, IF-TOPSIS and IF-PROMETHEE

5. Results and discussion

5.1. Theoretical analysis of the proposed framework

From the illustrative example, we observe that IFSP clearly forms a unique preference order which is sensible and rationally coherent to the DMs' viewpoint. To the best of our knowledge, the proposed LBA-IFSP framework is the only framework that considers personal relative importance (personal opinion) for each of the alternative given by the DM. This is an essential factor to be considered during decision making process as it gives the selection panel a general view of what motivated a particular decision? On the other hand, the state of the art methods still keep this question unanswered. The proposed framework utilizes the personal opinion of each DM in its formulation and also coherently matches the expectation of the DM. From Fig. 2 we observe that, the IFSP method is moderately correlated with IF-VIKOR and IF-TOPSIS (80%) and weakly correlated with IF-ELECTRE method (40%). The reason for such variation is implicit from the nature of formulation used by each of these methods. To further demonstrate the strength of our proposed LBA-IFSP framework, we compare the method to its close

Table 9
Comparison of IFSP Vs other methods.

MCDM methods	Suppliers				Preference Order	Feasible Choice
	S_1	S_2	S_3	S_4		
IFSP (proposed)	4	3	1	2	$S_3 > S_4 > S_2 > S_1$	S_3
IF-ELECTRE [49]	4	1	2	3	$S_2 > S_3 > S_4 > S_1$	S_2
IF-VIKOR [50]	4	3	2	1	$S_4 > S_3 > S_2 > S_1$	S_4
IF-TOPSIS [51]	4	3	2	1	$S_4 > S_3 > S_2 > S_1$	S_4
IF-PROMETHEE [1]	4	1	2	3	$S_2 > S_3 > S_4 > S_1$	S_2

Note: IFSP uses the IF-AHP [38] criteria weights as its preference value and its indifference value as equal to zero. IF-ELECTRE uses 0.333 for concordance and discordance weights (strong, medium and weak) with criteria weights taken from [38]. IF-VIKOR uses strategy weight as 0.5 and the weight of the criteria is also taken from [38], IF-PROMETHEE uses zero for indifference and preference values are taken from criteria values given in [38].

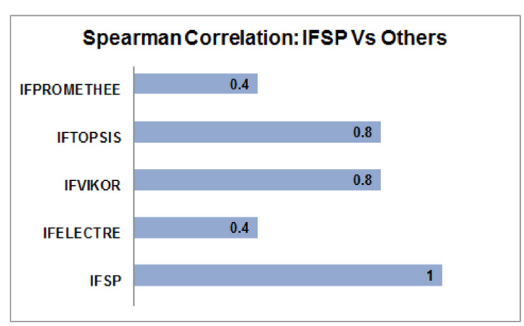

Fig. 2. Correlation inference.

Table 10
Comparison between IFSP and IF-PROMETHEE.

Context	Proposed IFSP	IF-PROMETHEE [1]
Data (input)	Linguistic term to IFVs	Crisp values
Aggregation	LBA and IFHA	IFWA
Preference Function	V-shaped	V-shaped
Preference and Indifference values	IFVs- preference values are estimated using IF-AHP. Indifference values are set to zero.	Single valued- preference values are given by DM. Indifference values are set to zero.
Ranks Calculated	Both partial and net ranking	Both partial and net ranking
Ranking Method	Scheme c of Definition 5	Scheme b of Definition 5
Net Outranking	Apply (4) to positive and negative outranking	Apply difference of positive and negative outranking
Weight/Relative Importance Estimation	Relative importance of the alternatives is given linguistically by DMs and aggregated using LBA operator.	Relative importance of the criteria is given by the DM and these are intuitionistic fuzzy values.
Modeling of fuzziness	Fuzziness is better modeled by allowing IFVs throughout the formulation process. Initially, linguistic terms are used to ease the process of rating and then, IFVs are used completely for better handling of vagueness. Unlike the counterpart method, this method preserves the IFPR property till the final stage	Fuzziness is only moderately modeled as the crisp values are used initially for the input and IFPR concept is only used, which is also relaxed at the final stage.
PROMETHEE Family Extended	PROMETHEE-II	PROMETHEE-II
Critics	The IFPR property is maintained throughout the decision making process	The IFPR property is liberalized during certain occasions in the decision making process

counterpart IF-PROMETHEE [1]. Table 10 shows the comparison of these two methods which clarifies the power of LBA-IFSP better.

From the theoretical analysis conducted above, following ***innovations/strengths*** of the proposed LBA-IFSP framework can be realized:

(1) The LBA operator is proposed with a view of smoothening the decision process, by offering the DMs freedom to rate alternatives in linguistic fashion. Further, the operator aggregates the preferences in a sensible and rational manner, without losing originality of the rating.
(2) The proposed IFSP method is an outranking method, which uses intuitionistic fuzzy values as input. Unlike IF-PROMETHEE [1], the proposed method sets a new formulation for evaluating deviation separately for both membership and non-membership terms.
(3) The proposed IFSP method also extends the formulation of preference function to both membership as well as non-membership terms. This enables the method to separately handle membership and non-membership terms effectively, without loss of information. The motivation for such innovations are derived from Geldermann et al. [36] who claim that the success of any decision framework is not realized only from the fuzzy input values but also from the use of fuzzy concepts at all possible phases of decision making.
(4) On the other hand, IF-PROMETHEE [1] method takes crisp or classical fuzzy values as input and evaluates deviation and preference function values as a single valued entity. This aggravates vagueness and imprecision in the decision making process. Further, the conversion of preference valued matrix (using preference function) into IFPR leads to unreasonable intuitionistic fuzzy values, which are again aggregated to form a matrix which is no longer enjoying the property of IFPR.
(5) To circumvent such issues of [1], in this paper, we make efforts to maintain intuitionistic fuzzy values throughout the ranking process and aggregation is done only for the upper triangular matrix, using which lower triangular matrix is constructed based on the IFPR property. As mentioned earlier, IFS is used for better handling imprecision and the use of linguistic terms for rating smoothens the decision process by giving a broad scope to DMs for evaluation. Thus, the conversion of linguistic terms to its corresponding intuitionistic fuzzy value can be justified.
(6) The proposed IFSP method also enjoys a unique idea of using the relative importance of the each alternative in its formulation, This gives a clear idea to the decision committee about the motivation behind any decision being made and this also gives the DMs an additional freedom for making initial judgments about each alternative. To the best of our knowledge, there is no ranking method which gives this freedom to the DMs and hence, the proposed IFSP method is more suitable for decision making process involving uncertainty and it also resembles closely to the real case decision making scenario.
(7) The final ranking values obtained using IF-PROMETHEE method [1] is often unrealistic and unreasonable. This prevents the DMs from forming a rank value set and assign suitable backup alternatives for the task. With the view of mitigating this issue, in this paper, efforts are made to evaluate final ranking values in a new fashion using (4) and scheme c of Definition 5. This setup ensures that the ranking values are sensible and hence, rank value set and suitable backup alternatives can be easily formed. Thus, the proposed LBA-IFSP framework is a powerful framework for scientific decision making involving uncertainty and vagueness.

5.2. Numerical analysis of the proposed framework

In the previous section, we admired the strength of the proposed LBA-IFSP framework in a theoretic manner. While in this section, we realize the strength of the proposed framework using numerical analysis. Such analysis is carried out with the view of understanding the numerical difference between proposed LBA-IFSP framework and framework of Liao and Xu [1]. For maintaining homogeneity in the analysis process, we confine our comparison between these two methods. In order to achieve this goal, we consider 6 parameters viz., adequacy to alternatives changes, adequacy to criteria changes, adequacy to preference values changes, agility of the consensus process, number of suppliers and criteria, construction of rank value set and backup alternatives. The motivation for conducting numerical analysis with these parameters is driven from [37]. The investigation on the numerical difference between proposed IFSP method and IF-PROMETHEE [1] method is presented below:

5.2.1. Adequacy to alternatives changes

This parameter is mainly used to validate the stability of the preference order. The rank reversal issue is also explored in this section from the alternatives' point of view. As per the example, we have 4 alternatives and each alternative's preference value is repeated to check rank reversal issue. Since there are 4 alternatives, 4 new test cases are constructed for testing. The inference from the study is made below:

- Initially, the test cases are given to IF-PROMETHEE [1] method. The normal ranking order is $S_2 > S_3 > S_4 > S_1$ and repetition of suppliers 2 and 3 creates rank reversal issue when IF-PROMETHEE [1] method is used. This breaks the stability of IF-PROMETHEE [1] method. When supplier 2 is repeated, ranking order changes to $S_3 > S_4 > S_2 > S_5 > S_1$ and when supplier 3 is repeated, ranking order changes to $S_2 > S_4 > S_3 > S_5 > S_1$.
- Similarly, when proposed IFSP method is used on these test cases, we observe that the normal ranking order $S_3 > S_4 > S_2 > S_1$ is changed for the test cases 2 and 4 which shows that, the proposed IFSP method is also affected by rank reversal issue.
- Finally, the crux of this analysis is that both IF-PROMETHEE method [1] and the proposed IFSP method are affected by rank reversal issue when adequate changes are made to the alternatives. This may be considered as a weakness of the proposed method and could be addressed in the future.

5.2.2. Adequacy to criteria changes

The parameter is used to test the stability of the ranking methods. Here, the rank reversal issue is addressed from the viewpoint of criteria. As per the example, there are 5 criteria taken for evaluation. So, we form 5 new test cases with each criterion repeated once in the study. The inference from the study is made below:

- Next, the proposed IFSP method is tested with all these 5 test cases. Unlike IF-PROMETHEE [1], the proposed IFSP method maintains the ranking order and hence, proves to be highly stable and robust against rank reversal issue.
- The crux of the study is that, unlike IF-PROMETHEE method [1], the IFSP method retains the ranking order as $S_3 > S_4 > S_2 > S_1$, and stays stable for all 5 test cases and thus, remains robust against rank reversal issue even when adequate changes are made to the criteria.

5.2.3. Adequacy to preference values changes

This parameter is used to test the stability of the ranking methods from the preference values perspective. In this study, the rank reversal issue is addressed from the exclusion category rather than the inclusion, which means that, the preference values are iteratively removed and the respective criteria are dropped from the analysis. The new preference values for the existing criteria are computed using the procedure given in [38]. The preference value with the least rank is excluded from the study in an iterative manner. Let us now investigate the process using 3 test cases.

- The example taken for the study has 5 criteria with each criterion having a preference value. Now, these values are ranked using scheme c of Definition 5 and the order is given by $C_1 > C_2 > C_3 > C_4 > C_5$. This clearly shows that, C_5 must be removed from the study and using the procedure given in [38], new preference values are calculated for the remaining criteria. In the next iteration, C_4 is removed and preference values for C_1, C_2andC_3 are calculated. Similar procedure is followed until we are left with top two criteria. Both IFSP method and IF-PROMETHEE method [1] are applied and results are inferred.
- When IF-PROMETHEE [1] is applied, the ranking order changes to $S_4 > S_3 > S_2 > S_1$ when C_4 is removed. Similarly, when criterion C_3 is removed, the order changes to $S_3 > S_2 > S_4 > S_1$. Finally, the removal of criterion C_5, has no effect on the ranking order and it retains the normal ranking order.
- When proposed IFSP method is applied, the ranking order remains unchanged for all 3 test case studies and the normal ranking order $S_3 > S_4 > S_2 > S_1$ is retained throughout the study confirming that, the proposed IFSP method is highly stable and robust against rank reversal issue even after adequate changes to preference values.

5.2.4. Number of suppliers and criteria

This parameter tests the scalability of the ranking methods. Since both the methods adopt IFPR concept, which is a typical pairwise comparison, the order of the matrix increases with the increase of suppliers. The number of such IFPR matrices increase with respect to increase in the number of criteria. Thus, with a view of maintaining a balance in decision making process, we follow the concept discussed in [52], which states that the maximum number of items that can be handled by humans is only 9 and hence, we confine our decision matrix to this order for both the ranking methods.

5.2.5. Construction of rank value set and backup alternatives

This parameter is used to test the sensibility and rationality of the preference order obtained from both the ranking methods. The following inferences can be made from the study:

- When IF-PROMETHEE [1] method is applied to the example taken for study (Section 6), the final net outranking value is given by, $\rho(A_1) = -0.5112$, $\rho(A_2) = 0.4317$, $\rho(A_3) = 0.0833$ and $\rho(A_4) = 0.0319$, with ranking order $S_2 > S_3 > S_4 > S_1$. When we multiply these values by a factor 100, we clearly observe that, the IF-PROMETHEE [1] method produces a rank value set which is unrealistic and unreasonable. Thus, the DMs cannot make any sensible decisions based on such rank value set and no backup alternatives for the task can be considered. This can be realized as a critical weakness of IF-PROMETHEE [1] method which arises due to the loss of information and weak estimation of net outranking values.
- On the other hand, when proposed IFSP method is applied for the example given in Section 6, we observe that the preference order is highly sensible and rational (see step 4 of Section 6). When these values are multiplied by a factor 100, a reasonable and rational rank value set is constructed. This clearly helps the DMs in making a sensible decision and also provides backup alternatives for effective management of the task under consideration. The proposed IFSP method eradicates negative preference values from the rank value set and also adds more sense to the decision making process by preventing potential loss of information. The proposed IFSP method also yields a decision which is highly coherent with the DMs' viewpoint and hence, clear motivation for the decision can be easily driven.
- Finally, the crux of the study is that, unlike IF-PROMETHEE [1] method, the proposed IFSP method produces more sensible and rational rank value set along with clear justification for backup alternatives, which helps DMs to make apt decisions at critical times.

5.2.6. Agility to consensus process

This parameter is used to determine the number of judgments required by an expert to arrive at a suitable consensus. We now make efforts to estimate the agility of both the methods. Since proposed IFSP method and IF-PROMETHEE method [1] follow preference relations, they resemble closely to the AHP, and hence, the judgments are given by $\left(i\left(\frac{i-1}{2}\right)\right)$, where i refers to the order of preference relation.

- The agility of IF-PROMETHEE [1] method is estimated for m criteria and n alternatives with preference relations of order $(n \times n)$.

This method forms m matrices of order $(n \times n)$. Thus, the agility of judgment is given by $\left(m\left(n\left(\frac{n-1}{2}\right)\right)\right)$.

- On the other hand, the agility of judgment for proposed IFSP method is estimated for same m criteria and n alternatives. But, here the information loss is avoided by retaining the membership and non-membership values throughout the evaluation process. Thus, the agility of judgment is given by $\left(2m\left(n\left(\frac{n-1}{2}\right)\right)\right)$.
- Clearly, for the investigation made above, we infer that, proposed IFSP method takes more time to converge to a suitable consensus than the IF-PROMETHEE [1] method. Though the loss of information is prevented in IFSP, the agility to consensus is slower than IF-PROMETHEE [1].
- Considering the example discussed above, there are 5 criteria and 4 alternatives and hence, for IF-PROMETHEE [1] method, there are 5 matrices of order (4×4). So, the agility to judgment is given by $\left(5\left(4\left(\frac{4-1}{2}\right)\right)\right) = 30$. Similarly, when proposed IFSP method is applied, the agility to judgment becomes $\left((2 \times 5)\left(4\left(\frac{4-1}{2}\right)\right)\right) = 60$.
- The final crux of the analysis is that, proposed IFSP method is slower in arriving at consensus than IF-PROMETHEE [1] method. This could be considered as a weakness of proposed IFSP method, which needs to be addressed in the future.

6. Conclusion

In this paper, we have proposed a new framework for decision making which consist of two tiers. In the first tier, we proposed a new aggregation operator (LBA operator) which aggregates DMs' linguistic preferences directly without adopting any conversion mechanism. Further, in the next tier, we rank the alternatives using IFSP method, which is a new extension to PROMETHEE under IFS environment. The applicability of the proposed framework is tested using SS problem and consistency is verified by comparison with other methods (follow Spearman correlation). Further, the strength and weakness of the proposal are realized from both theoretic as well as numeric perspective. Finally, as a part of future work, we address the weakness of LBA-IFSP framework that is pointed out in Section 5.2 (see parameter 6) and have planned to design new frameworks under hesitant fuzzy environment. Also, new aggregation operators for effectively handling linguistic terms and new outranking schemes for better judgments are planned for future development in the field of MCDM.

Acknowledgements

We, the authors would like to thank the funding agencies, University Grants Commission for their financial support for the research from Rajiv Gandhi National Fellowship of Grant no: F./2015-17/RGNF-2015-17-TAM-83 and Department of Science and Technology, India for their financial support for providing cloud environment through FIST programme (Grant no: SR/FST/ETI-349/2013). We also thank the SASTRA University, for providing an excellent infrastructure to carry out our research work. We also thank the editor and the anonymous reviewers for their constructive effort in making the research a successful work.

Appendix A.

Consider the instance $(S_3 \times S_4)$ for criteria C_2 from Table 6. It yields a value of 0.84 in μ and 0 in υ. Let us consider the working for this as an example. The p value is (0.1785, 0.6585) taken from criteria weights given in [38] and q value is (0, 0). Now, we look at membership part (0.84). From ((16) and (18)) it is clear that the deviation is 0.15 which is greater than q value and so $\frac{d-q}{p-q}$ form is adopted. This gives $\frac{0.15-0}{0.1785-0} = 0.84$. Similar approach is followed for the construction of Table 6.

Table A1
List of abbreviation and its expansion.

Abbreviation(s)	Expansion
AIFS	Atanassov's Intuitionistic Fuzzy Set
GDSS	Group Decision Support System
GAIA	Graphical Analysis for Interactive Aid
IFPR	Intuitionistic Fuzzy Preference Relation
IOWA	Intuitionistic Ordered Weighted Averaging
LBA	Linguistic Based Aggregation
IFHA	Intuitionistic Fuzzy Hybrid Aggregation
IFWA	Intuitionistic Fuzzy Weighted Averaging
PROMETHEE	Preference ranking organization method for enrichment evaluation
IFSP	IFS based PROMETHEE
AHP	Analytical Hierarchy Process
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
ELECTRE	ELimination Et ChoixTraduisant la REalité
VIKOR	VIseKriterijumskaOptimizacijaKompromisnoResenje
IFV	Intuitionistic Fuzzy Value

Now Table 7 is constructed in the following manner. Let us consider for example, $(A_3 \times A_4)$ of R_2. It yields a value of (0.4, 0.03) . We consider the working for 0.4 here. The IFPRs $(R_2,\ R_7)$ need to be paired to form aggregated R_2 using IFHA operator. In our example this is (0.84, 0) and (0, 0.15). Now aggregate the instance using IFHA (with weight as 0.5), we get (0.6004, 0.0780). We divide by the factor n = 2 (number of matrices being fused). We get, (0.3002, 0.0390) ≈ (0.3, 0.04). Similar approach is followed for the construction of Table 7.

Partial and net ranking is shown in Table 8. For an instance, the working is demonstrated with S_1. We consider the positive outranking of S_1 (membership part). $(1-((0.35 \times 0.01)\ (0.35 \times 0.06)\ (0.35 \times 0.03))) = 0.0347$. Similarly, we can construct the non-membership part. Also, negative outranking can be determined in the same fashion.

Net outranking is then estimated. For instance consider S_1 again. We again consider the membership part of S_1. The positive outrank instance is 0.03 and negative outrank instance is 0.06. Now, net outrank is given by $(0.03 + 0.06 - (0.03 \times 0.06)) = 0.0882$. Similar approach is adopted to form rest of the instances of Table 8. Please refer to Table A1 for abbreviations.

Appendix B.

Another illustrative example

In this section, we demonstrate another decision making example to better realize the power of the proposed framework. Consider an example of personnel selection from [53], where 4 DMs are considered with 4 alternatives and 8 criteria. For brevity, we confine our discussion on these 8 criteria and encourage readers to refer [53] for its details.

We get 4 decision matrices of order (4×8) from [53]. These matrices are aggregated into a single decision matrix of order (4×8) by using proposed LBA operator. Since the information in the matrices are linguistic in nature, LBA proves to be an effective tool for aggregation. For brevity, we directly display the aggregated matrix in Table B1 which is obtained by fusing the 4 decision matrices using proposed LBA operator.

The weight value of each alternative is also estimated using the LBA operator, by fusing different linguistic opinions of DMs. These opinions are shown in Table B2 and by applying LBA operator we obtain the weight vector as $(L_2, L_1.L_0.L_1)$.

The matrix from Table B1 and the weight vector are given as input to the proposed IFSP method along with p and q values.

Table B1
Matrices of [53] being fused using LBA operator.

D_{123}	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	L_1	L_2	L_2	L_2	L_2	L_1	L_1	L_2
A_2	L_0	L_1	L_2	L_3	L_3	L_1	L_3	L_2
A_3	L_1	L_2	L_2	L_2	L_1	L_2	L_0	L_3
A_4	L_3	L_0	L_2	L_2	L_1	L_{-2}	L_{-2}	L_0

Table B2
Criteria weight evaluation matrix.

Criteria weights	A_1	A_2	A_3	A_4
D_1	L_2	L_1	L_0	L_2
D_2	L_1	L_1	L_0	L_1
D_3	L_2	L_0	L_1	L_1
D_4	L_0	L_2	L_1	L_1

Table B3
Outranking values of IFSP method.

Outranking	Positive outrank		Negative outrank		Net outrank	
	μ	υ	μ	υ	μ	υ
A_1	0.1122	0.0038	0.1594	0.0027	0.2537	0
A_2	0.0673	0.0118	0.1215	0.0146	0.1806	0.0002
A_3	0.0588	0.746	0.0676	0.0816	0.1224	0.0061
A_4	0.1198	0.0139	0.1137	0.0156	0.2198	0.0002

Table B4
Outranking values of Liao and Xu [1] method.

Outranking	Positive outrank		Negative outrank		Net outrank
	μ	υ	μ	υ	$\rho(A_i)$
A_1	0.3166	0	0.4537	0	0.1588
A_2	0.1540	0	0.2628	0	0.1405
A_3	0.2957	0	0.3345	0	0.0460
A_4	0.3466	0	0	0	−0.4599

The p values are taken as (0.125, 0) for each criterion and the q value is taken as zero for all criteria. Since, we are concerned about the power of the proposed framework we skip the intermediate steps and depict only the resultant IFPR matrix, final outranking values and ranking order of the alternatives. The outranking value is depicted in Table B3.

$$R = \begin{pmatrix} - & (0.04, 0.06) & (0.03, 0.06) & (0.06, 0.06) \\ (0.06, 0.04) & - & (0.05, 0.06) & (0.06, 0.06) \\ (0.06, 0.03) & (0.06, 0.05) & - & (0.04, 0.06) \\ (0.06, 0.06) & (0.06, 0.06) & (0.06, 0.04) & - \end{pmatrix}$$

The similarity function $L(A_i)$ is estimated for different net outranking values and the results are obtained as (0.5726, 0.5496, 0.5311, 0.5617) with the ranking order of $A_1 > A_4 > A_2 > A_3$. Now, we compare the proposed framework with Liao and Xu [1] method for clearly understanding the power of proposed LBA-IFSP framework.

From Table B4, we infer that, the ranking order is $A_1 > A_2 > A_3 > A_4$ and clearly, we see that the ranking order changes and the reason is the implicit nature of these two methods. As the method proposed in [1] does not make full use of the IFS concept, it lacks in some aspect of handling uncertainty better and thus, intuitively, we prefer proposed LBA-IFSP framework for ranking alternatives. Since, the theoretic strength remains same as of the discussion made above in Section 5.1, we pay attention to numeric strength and it is depicted in Table B5.

Table B5
Numeric analysis of IFSP Vs IF-PROMETHEE [1] method.

Numeric analysis	LBA-IFSP (proposed)	Liao and Xu [1]
(1)	When alternative 2 is repeated, the ranking order changes and this indicates rank reversal issue.	When alternatives 2 and 4 are repeated the ranking order changes and this shows that the method is affected by rank reversal issue.
(2)	Even though adequate changes are made to the criteria the ranking order remains unchanged.	The method is affected by rank reversal issue when adequate changes are made to the criteria. Repetition of criterion 5–8 shows change in ranking order.
(3)	We take up 5 test cases by repeating 5 criteria. The ranking order remains unchanged.	When the same test cases are implemented using [1], the ranking order changes for criterion 5.
(4)	As our motivation is driven from [52], we confine our study within the bounds of 9 items.	The method [1] also obeys the rule specified in [52].
(5)	The proposed method is effective in providing a much sensible ranking set and backup for future task.	Since the final rank values yield negative result, the method becomes weak in producing proper rank value set and backup alternatives.
(6)	By following the idea described in section 5.2 (see point #6), the method reaches consensus at 96 judgments.	The method reaches consensus at 48 judgments. As the method uses single value (crisp) for the estimation, the agility of the method is better compared to IFSP method.

Note: Refer Section 5.2 for headings referring (1) to (6).

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