Applied Soft Computing 61 (2017) 111–121

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A new approach for multiple attribute group decision making with interval-valued intuitionistic fuzzy information

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ARTICLE INFO

Article history:
Received 1 December 2016
Received in revised form 3 July 2017
Accepted 3 July 2017
Available online 12 July 2017

Keywords:
Multiple attribute group decision making
Interval-valued intuitionistic fuzzy information
Plant growth simulation algorithm (PGSA)
Optimal rally point
Aggregation

ABSTRACT

This paper proposes a novel method for multiple attribute group decision making (MAGDM) with interval-valued intuitionistic fuzzy information. The interval-valued intuitionistic fuzzy numbers of each expert preference matrix are first mapped into two dimensions. Thus, the values of each membership degree and non-membership degree are considered as points in the two-dimensional representation. Moreover, the distance between the points represents the variance among the different experts preferences. The preference points of the same character are considered as a point set. We employ the plant growth simulation algorithm (PGSA) to calculate the optimal rally points of every point set, the sum of whose Euclidean distances to other given points is minimal, and these optimal rally points reflect the preferences of the entire expert group. These points are used to establish an expert preference aggregation matrix. Suitable points from the matrix are chosen to constitute an ideal point matrix, a projection method is employed to calculate the sum of its Euclidean distance to the expert preference aggregation matrix, and the score of each alternative is evaluated. Finally, the overall ranking of alternatives is obtained. In addition, this study develops a process to evaluate the pros and cons of different aggregation methods. Two typical examples are presented to illustrate the feasibility and effectiveness of the proposed approach.

1. Introduction

Multi-attribute group decision making (MAGDM) is a classical decision-making construct applied in various areas such as emergency management, economics, society, etc. [1–5]. Herrera [6] classified the decision making process into three parts: translate experts preferences, aggregate experts opinions (i.e., establish the collective matrix) and select the best alternative (the aggregation of expert opinion is the core step among these three steps). The experts preference was described using distinct numbers in the early multi-criteria decision making problem. Yager [7] proposed the ordered weighted averaging operators (OWA) to integrate the experts preference, which provided a broader idea for future research on the multi-attribute decision making problem. However, with the increasing complexity of decision making, it is difficult to make the right decision by analyzing the opinion of a single expert. In addition, on account of their limited knowledge related to the problem research area and lack of adequate information as well as time, the experts cannot express their opinions with distinct numbers. Therefore, in recent years, many researchers have begun to develop various new operators to solve the MAGDM problem with attribute values that define interval numbers or linguistic variables. Yu [8] developed the uncertain linguistic power weighted geometric (ULPWG) operator and the uncertain linguistic power ordered weighted geometric (ULPOWG) operator. In his research, he also utilized the proposed operators to develop approaches to solve the uncertain linguistic MAGDM problems. Merigo [9] presented an overview of fuzzy research with bibliometric indicators, which provided a general overview identifying some of the most influential research in this area. However, the novel indicators have certain limitations due to the differences between researchers, the specificity of research area, and the difficulty faced in quantification of key information. Yu [10] developed an effective interval-valued multiplicative intuitionistic fuzzy preference relation, which analyzed the basic operations for interval-valued multiplicative intuitionistic preference information and its aggregation techniques. Further, the aggregation operators for interval-valued multiplicative intuitionistic fuzzy sets (IMIFs) were also developed in this study. Hashemi [11] developed an enhanced version of the ELECTRE method, called ELECTRE III, for multicriteria group decision making under the interval-valued

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http://dx.doi.org/10.1016/j.asoc.2017.07.008
1568-4946/© 2017 Published by Elsevier B.V.

intuitionistic fuzzy environment. And it had an advantage in handling a data set with a high degree of uncertainty. Sahin [12] proposed a novel function of interval-valued intuitionistic fuzzy numbers (IVIFNs) and established a new process by using the proposed function to rank the IVIFNs. However, the proposed method still used the aggregation operators developed by Xu in 2007 to establish the collective matrix, which may not deliver a precise outcome. In addition, certain straightforward methods were proposed to calculate experts weights. Yue [13–15] proposed several excellent methods for determining weights of experts in MAGDM based on an extended technique for order preference by similarity to an ideal solution (TOPSIS). Liu and LI [16] developed an approach to determine the integrated weights of experts based on interval-valued preference matrices. They used the generalized Fermat point to determine the weights of experts, which delivered good simulation results.

The purpose of this study is to develop a novel method for the accurate aggregation of interval-valued intuitionistic fuzzy matrices (IVIFMs), so that the MAGDM problems with IVIFN can be solved effectively. An iterative algorithm, called plant growth simulation algorithm (PGSA), is used to replace the traditional operators for determining the aggregation of experts preferences. The two-dimensional model of experts preference is established, which places expert preference points into their respective points set. Using the PGSA to calculate the optimal rally points of every point set, the collective matrix is established with the optimal rally point set. Then, the projection method is used to calculate the sum of Euclidean distances between the expert preference aggregation matrix and ideal point matrix. Finally, the best alternative is chosen by obtaining the ranking of alternatives after evaluating the score of each alternative.

The remainder of this paper is organized as follows: Section 2 briefly describes the research problem statement and contributions to this paper. Section 3 introduces the preliminaries, including the interval-valued intuitionistic fuzzy set, the concept of optimal rally point and the principle of the PGSA. Section 4 details the decision making process of the proposed method and focuses on the core steps of the expert preference aggregation. In Section 5, two practical examples are presented to illustrate the efficiency of the proposed method. Section 6 compares the solution results yielded by other relevant methods with those of the proposed method. Section 7 presents the conclusions and prospects for further research.

2. Research problem statement and contributions

This research investigates the MAGDM problem based on interval-valued intuitionistic fuzzy sets (IVIFS). In the recent past, owing to the fact that the IVIFN can express expert preference more accurately than a crisp number, interval-valued number, and linguistic value, several researchers have attempted to develop various operators for aggregating IVIFS.

2.1. The related work

He [17] defined interval-valued hesitant fuzzy 2nd-order central polymerization degree function and interval-valued hesitant fuzzy 2nd-order dispersive central polymerization degree function to compare different interval-valued hesitant fuzzy sets further. Then, he developed the interval-valued hesitant fuzzy power Bonferroni mean (IVHFPBM) and the interval-valued hesitant fuzzy weighted power Bonferroni mean (IVHFWPBM), and proposed a new ranking method for interval-valued hesitant fuzzy information. Guo [18] proposed intervalvalued intuitionistic fuzzy weighted exponential aggregation (IIFWEA) operator and the dual interval-valued intuitionistic fuzzy weighted exponential aggregation (DIIFWEA) operator for aggregating IVIFNs. Xu, an eminent scholar in this field, has proposed several well-known operators for the MAGDM problem with IVIFS in the past ten years [19–23], such as interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator, interval-valued intuitionistic fuzzy geometric-ordered weighted averaging (IIFWG) operator, interval-valued intuitionistic fuzzy hybrid weighted averaging (IIFHA) operator, and interval-valued intuitionistic fuzzy hybrid geometric-ordered weighted averaging (IIFHG) operator. These operators have been improved by a number of researchers to solve various MAGDM problems and have delivered positive experimental results. However, since a large number of continued products are used in the formulas, these operators have the drawback that they establish an unreasonable collective matrix in cases when the values of some IVIFNs are equal to zero.

In order to resolve this problem, Chen [24] proposed a new MAGDM method based on the proposed IVIFWA, IVIFOWA and IVIFHWA operators of IVIFs, that can accurately deal with the situation in which the membership degrees or non-membership degrees of some evaluating IVIFVs are equal to [0, 0]. However, the calculation process is complex because the primitive expert preference matrices are pretreated multiple times. In addition, Chen's research pays little attention to the situation in which only the lower limit of IVIFN is equal to zero.

2.2. Contributions

A simple and efficient iterative algorithm is proposed to solve the MAGDM problem with IVIFS. The main advantage of our proposed method is that it can overcome the drawbacks of Xu's operators and Chen's method, and accurately deal with practical MAGDM problems. The traditional IVIFNs of expert preferences are projected as preference points onto the two-dimensional coordinates and the differences between the expert preferences are expressed as the distances between preference points in our study. Zero-valued elements in expert preferences do not affect the accuracy of the collective matrix because the optimal rally point is obtained by calculating the distances between expert preference points.

3. Preliminaries

3.1. Interval-valued intuitionistic fuzzy set (IVIFS)

IVIFS$\tilde{A}$, a concept introduced by Atanassov and Gargov [25], can be described as follows:

Let a set X be fixed:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\} \quad (1)$$

where

$$\mu_{\tilde{A}} = [\mu_{\tilde{A}}^L(x), \mu_{\tilde{A}}^U(x)] \subset [0, 1] \quad (2)$$

$$\nu_{\tilde{A}} = [\nu_{\tilde{A}}^L(x), \nu_{\tilde{A}}^U(x)] \subset [0, 1] \quad (3)$$

$\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are intervals, $\mu_{\tilde{A}}^L(x) = \inf \mu_{\tilde{A}}(x)$, $\mu_{\tilde{A}}^U(x) = \sup \mu_{\tilde{A}}(x)$, $\nu_{\tilde{A}}^L(x) = \inf \nu_{\tilde{A}}(x)$, $\nu_{\tilde{A}}^U(x) = \sup \nu_{\tilde{A}}(x)$

and

$$\mu_{\tilde{A}}^U(x) + \nu_{\tilde{A}}^U(x) \le 1, \ x \in X \quad (4)$$

Additionally,

$$\pi_{\tilde{A}}(x) := [\pi_{\tilde{A}}^L(x), \pi_{\tilde{A}}^U(x)], \ x \in X \quad (5)$$

where

$$\pi_{\tilde{A}}^L(x) = 1 - \mu_{\tilde{A}}^U(x) - \nu_{\tilde{A}}^U(x) \quad (6)$$

$$\pi_{\tilde{A}}^U(x) = 1 - \mu_{\tilde{A}}^L(x) - \nu_{\tilde{A}}^L(x) \quad (7)$$

$\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ represent the membership degree and the non-membership degree, respectively, of the element of x to $\tilde{A}$, $\pi_{\tilde{A}}(x)$ is the degree of indeterminacy of x to $\tilde{A}$. IVIFS can be defined as follows:

Definition 1. Let X be a fixed set, $\tilde{B} = \langle x, \mu_{\tilde{B}}(x), \nu_{\tilde{B}}(x)\rangle$, $\tilde{B_1} = \langle x, \mu_{\tilde{B_1}}(x), \nu_{\tilde{B_1}}(x)\rangle$, and $\tilde{B_2} = \langle x, \mu_{\tilde{B_2}}(x), \nu_{\tilde{B_2}}(x)\rangle$ be three IVIFSs, and $x \in X$, Then

$$\tilde{B}^c = \{\langle x, \nu_{\tilde{B}}(x), \mu_{\tilde{B}}(x)\rangle\} \tag{8}$$

$$\begin{aligned}\tilde{B_1} \subseteq \tilde{B_2} \ \Leftrightarrow\ & \mu^L_{\tilde{B_1}}(x) \le \mu^L_{\tilde{B_1}}(x), \mu^U_{\tilde{B_1}}(x) \le \mu^U_{\tilde{B_1}}(x), \\ & \text{and } \nu^L_{\tilde{B_1}}(x) \ge \nu^L_{\tilde{B_1}}(x), \nu^U_{\tilde{B_1}}(x) \ge \nu^U_{\tilde{B_1}}(x)\end{aligned} \tag{9}$$

$$\tilde{B_1} = \tilde{B_2} \Leftrightarrow \tilde{B_1} \subseteq \tilde{B_2} \text{ and } \tilde{B_1} \supseteq \tilde{B_2} \tag{10}$$

$$\begin{aligned}\tilde{B_1} \bigcap \tilde{B_2} = \ & \{\langle x, [\min\{\mu^L_{\tilde{B_1}}(x), \mu^L_{\tilde{B_2}}(x)\}], [\min\{\mu^U_{\tilde{B_1}}(x), \mu^U_{\tilde{B_2}}(x)\}], \\ & [\max\{\mu^L_{\tilde{B_1}}(x), \mu^L_{\tilde{B_2}}(x)\}], [\max\{\mu^U_{\tilde{B_1}}(x), \mu^U_{\tilde{B_2}}(x)\}]\rangle\}\end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{B_1} \bigcup \tilde{B_2} = \ & \{\langle x, [\max\{\mu^L_{\tilde{B_1}}(x), \mu^L_{\tilde{B_2}}(x)\}], [\max\{\mu^U_{\tilde{B_1}}(x), \mu^U_{\tilde{B_2}}(x)\}], \\ & [\min\{\mu^L_{\tilde{B_1}}(x), \mu^L_{\tilde{B_2}}(x)\}], [\min\{\mu^U_{\tilde{B_1}}(x), \mu^U_{\tilde{B_2}}(x)\}]\rangle\}\end{aligned} \tag{12}$$

Definition 2. Let X be a fixed set, $\tilde{B_1} = \langle x, \mu_{\tilde{B_1}}(x), \nu_{\tilde{B_1}}(x)\rangle$, $\tilde{B_2} = \langle x, \mu_{\tilde{B_2}}(x), \nu_{\tilde{B_2}}(x)\rangle$, and $\tilde{B_3} = \langle x, \mu_{\tilde{B_3}}(x), \nu_{\tilde{B_3}}(x)\rangle$ be three IVIFSs, where $x \in X$, and $\lambda > 0, m > 0$, then

$$\begin{aligned}\lambda\tilde{B_1} = \ & \{\langle x, [1-(1-\mu^L_{\tilde{B_1}}(x))^{\lambda}, 1-(1-\mu^U_{\tilde{B_1}}(x))^{\lambda}], \\ & [(\nu^L_{\tilde{B_1}}(x))^{\lambda}, (\nu^U_{\tilde{B_1}}(x))^{\lambda}]\rangle\}\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{B_1^{\lambda}} = \ & \{\langle x, [(\mu^L_{\tilde{B_1}}(x))^{\lambda}, (\mu^U_{\tilde{B_1}}(x))^{\lambda}], \\ & [1-(1-\nu^L_{\tilde{B_1}}(x))^{\lambda}, 1-(1-\nu^U_{\tilde{B_1}}(x))^{\lambda}]\rangle\}\end{aligned} \tag{14}$$

$$\begin{aligned}\tilde{B_2} + \tilde{B_3} = \ & \{\langle x, [\mu^L_{\tilde{B_2}}(x) + \mu^L_{\tilde{B_3}}(x) - \mu^L_{\tilde{B_2}}(x)\cdot\mu^L_{\tilde{B_3}}(x), \\ & \mu^U_{\tilde{B_2}}(x) + \mu^U_{\tilde{B_3}}(x) - \mu^U_{\tilde{B_2}}(x)\cdot\mu^U_{\tilde{B_3}}(x)], \\ & [\nu^L_{\tilde{B_2}}(x)\cdot\nu^L_{\tilde{B_3}}(x), \nu^U_{\tilde{B_2}}(x)\cdot\nu^U_{\tilde{B_3}}(x)]\rangle\}\end{aligned} \tag{15}$$

$$\begin{aligned}\tilde{B_2} \cdot \tilde{B_3} = \ & \{\langle x, [\mu^L_{\tilde{B_2}}(x)\cdot\mu^L_{\tilde{B_3}}(x), \mu^U_{\tilde{B_2}}(x)\cdot\mu^U_{\tilde{B_3}}(x)], \\ & [\nu^L_{\tilde{B_2}}(x) + \nu^L_{\tilde{B_3}}(x) - \nu^L_{\tilde{B_2}}(x)\cdot\nu^L_{\tilde{B_3}}(x), \\ & \nu^U_{\tilde{B_2}}(x) + \nu^U_{\tilde{B_3}}(x) - \nu^U_{\tilde{B_2}}(x)\cdot\nu^U_{\tilde{B_3}}(x)]\rangle\}\end{aligned} \tag{16}$$

In the MAGDM problem, assuming that there are experts applying IVIFNs $\tilde{d}^t_{ij}(1 \le t \le k)$ to evaluate ith alternative with jth characteristic, the expert preference matrices $[\tilde{D}_t]$ are constructed as follows:

$$[\tilde{D}_t] = [\tilde{d}^t_{ij}]_{i\times j} = \begin{bmatrix} ([\mu^L_{11}, \mu^U_{11}], [\nu^L_{11}, \nu^U_{11}]) & \cdots & ([\mu^L_{1j}, \mu^U_{1j}], [\nu^L_{1j}, \nu^U_{1j}]) \\ \vdots & \ddots & \vdots \\ ([\mu^L_{i1}, \mu^U_{i1}], [\nu^L_{i1}, \nu^U_{i1}]) & \cdots & ([\mu^L_{ij}, \mu^U_{ij}], [\nu^L_{ij}, \nu^U_{ij}]) \end{bmatrix} \tag{17}$$

Aggregating all the expert preference matrices into a single response matrix is the key step of decision-making. Most of the traditional aggregation operators use continued products during the calculation process. When the preference value of an expert is zero, the aggregation result will be set to zero, which leads to the neglect of the other experts preferences.

To solve this problem, this paper projects the expert preference information onto the two-dimensional coordinates, and constructs the expert preference aggregation matrix by calculating the optimal rally point.

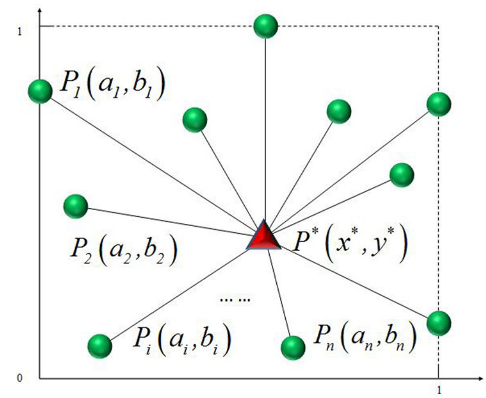


Fig. 1. The optimal rally point.

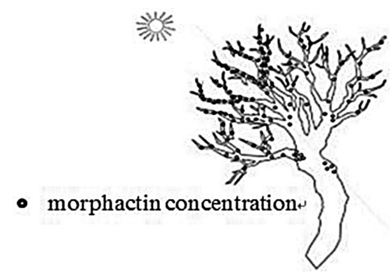


Fig. 2. Morphactin concentration distribution.

Fig. 3. Process of plant growth.

3.2. The optimal rally point in MAGDM

Definition 3. There are n ($n \ge 3$) weighted points in a bounded closed box in a two-dimensional plane, whose corresponding positive weights are $\xi_i \in [0, 1](1 \le i \le n)$, and $\sum_{i=1}^{n}\xi_i = 1$. If a point P^* exists, whose Euclidean distances to the other given points meet the following condition:

$$\begin{aligned} D &= \min \sum_{i=1}^{n} |P^* P_i| \\ &= \min\left(\sqrt{(x^* - a_1)^2 + (y^* - b_1)^2} + \cdots + \sqrt{(x^* - a_n)^2 + (y^* - b_n)^2}\right)\end{aligned} \tag{18}$$

then P^* can be defined as the optimal rally point (see Fig. 1).

For this research, we assume that each experts preference information is considered as a point in the two-dimensional coordinates and the distance between any two points indicates the similarity of the two experts preference. The closer the two points are, the more similar the two experts preference information is. The optimal rally point achieves Pareto optimality, which means that the optimal rally point can precisely represent a comprehensive opinion for all experts.

Finding the optimal rally point is an NP-hard problem. As the number of points on the plane increases, the difficulty faced in

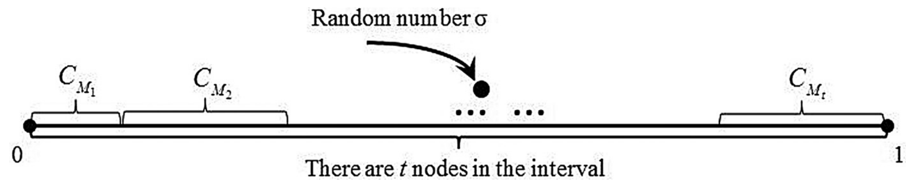


Fig. 4. The morphactin concentration state map.

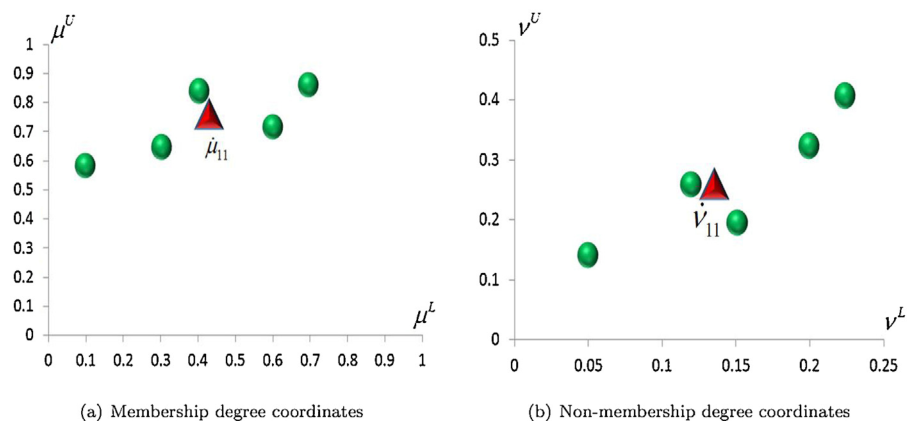


Fig. 5. Aggregation idea.

solving the problem will increase exponentially. This research employed the PGSA to solve this problem.

3.3. Plant growth simulation algorithm (PGSA)

The PGSA is a heuristic algorithm based on the plant growth mechanism first proposed by the Chinese scholar Li [26]. PGSA is less sensitive to parameter sets and its data does not need coding and decoding. Compared to classic intelligent algorithms, such as genetic algorithm (GA), artificial bee colony algorithm (ABC), artificial fish-swarm algorithm (AFS), ant colony algorithm (AC), and simulated annealing algorithm (SA), PGSA can find satisfactory solutions to certain problems [27]. Recently, it has been widely applied in numerous fields such as distribution system optimization planning, facility location, transmission network optimal planning, optimal capacitor placement, and Radio Frequency Identification (RFID) network planning [28–30]. However, in this study, we use PGSA to find the optimal rally point.

3.3.1. The principle of plant growth

A plant grows a trunk from its root, and most of the nodes on the trunk will grow new branches. The branches closer to the light source have higher morphactin concentration, and thus, they have more opportunities to grow new branches. Such growth mechanism will be repeated until a plant is formed. The morphactin concentration distribution and the process of plant growth are shown in Figs. 2 and 3, respectively.

Additionally, the morphactin concentration of each node and branch in the plant is updated as new stems grow. During the entire growth process, the morphactin concentration of each branch is determined by the environmental information of the node from which it grows, which depends on the branch's relative position on the plant and is not determined in advance.

3.3.2. The probability model of PGSA

The entire growing space of a plant is regarded as the feasible domain in the probability model of PGSA. First, one point in the feasible region is randomly selected as the root; a trunk M grows from x^0. Let us suppose there are t nodes $S_{M_1}, S_{M_2}, \ldots, S_{M_t}$ on the trunk. The growth hormone concentration of each node is $C_{M_1}, C_{M_2}, \ldots, C_{M_t}$. $C_{M_t}(1 \le i \le t)$ can be calculated by Eq. (19)

$$C_{M_i} = \frac{f(x_0) - f(S_{M_i})}{\Sigma_{i=1}^{t}(f(x_0) - f(S_{M_i})}, \quad (1 \le i \le t) \tag{19}$$

$f(X)$ is a backlight function for describing the environment of the node X on the plant. The closer the distance between X and the light source, the smaller is the value of $f(X)$ is. The function value decreases as the illumination of the growing point increases.

We can derive $\Sigma_{i=1}^{t} C_{Mi} = 1$ from Eq. (19), and then establish the state map of the morphactin concentration shown in Fig. 4, which means that there are t nodes in the interval [0,1] and the sum of their morphactin concentration is 1. A random number σ is selected in the interval [0,1], which is like a ball being thrown on the state map, and it will fall into any one of $C_{M_1}, C_{M_2}, \ldots, C_{M_t}$. The corresponding node that is called the preferential node will take the priority to grow a new branch in the next step.

It is assumed that a new branch m grows from $S_{M_k}(1 \le k \le t)$, which has r nodes, namely, $S_{m_1}, S_{m_2}, \ldots, S_{m_r}$ on it. The growth hormone concentrations of each node are $C_{m_1}, C_{m_2}, \ldots, C_{m_r}$. The morphactin concentration of each node in the plant will be updated after each new round of the branch growth. After growing the branch m, the nodes on trunk M (except S_{M_k}) and branch m need to be recalculated. Meanwhile, C_{M_i} and C_{m_j} can be calculated by Eq. (20).

$$\begin{cases} C_{M_i} = \dfrac{f(x_0) - f(S_{M_i})}{\sum_{i=1}^{t}(f(x_0) - f(S_{M_i})) + \sum_{j=1}^{r}(f(x_0) - f(S_{m_j}))}, & (1 \le i \le t, i \ne k) \\ C_{m_j} = \dfrac{f(x_0) - f(S_{m_j})}{\sum_{i=1}^{t}(f(x_0) - f(S_{M_i})) + \sum_{j=1}^{r}(f(x_0) - f(S_{m_j}))}, & (1 \le j \le r). \end{cases} \tag{20}$$

From Eq. (20), we can also derive $\sum_{i=1, i \ne k}^{t} C_{M_i} + \sum_{j=1}^{r} C_{m_j} = 1$. Then, a new preferential node will be selected in a similar way as S_{M_k} A new branch grows in the next step. The growth process is repeated until the new branch reaches the light source position, following which the plant stops growing.

The growth process is repeated until the new branch reaches the light source position, and then a plant stops growing.

4. Key methods

4.1. The process of aggregation of expert preferences based on PGSA

4.1.1. Aggregation idea

In MAGDM, the preference information of the tth $(1<t<k)$ expert can be expressed using the following matrix:

$$D^{(t)} = [d_{ij}^{(t)}]_{m\times n}, \; (1 \le i \le m, 1 \le j \le n, 1 \le t \le k). \tag{21}$$

In this study, expert preference is expressed by interval-valued fuzzy numbers. Eq. (21) can be expressed as follows:

$$D^{(t)} = [d_{ij}^{(t)}]_{m\times n} = [\mu_{ij}^{(t)}, \nu_{ij}^{(t)}] = [(\mu_{ij}^{(t)L}), (\mu_{ij}^{(t)U}), (\nu_{ij}^{(t)L}), (\nu_{ij}^{(t)U})], \; (1 \le i \le m, 1 \le j \le n, 1 \le t \le k). \tag{22}$$

We establish two coordinates to aggregate membership degree and non-membership degree information separately. As can be seen from Fig. 5(a), the membership degree interval-valued numbers of characteristics C_1 in alternative A_1, $\mu_{11}^{(k)}$ proposed by k experts are projected into a two-dimensional coordinate, where $\mu^L(x)$ is considered as abscissa axis and $\mu^U(x)$ is considered as vertical axis. Fig. 5(b) shows that non-membership degree interval-valued numbers $\nu_{11}^{(k)}$ can be projected as well.

The membership degree optimal rally point $\dot{\mu}_{11}$ and the non-membership degree optimal rally point $\dot{\nu}_{11}$ can be found by using PGSA (see Fig. 5(a), (b)). From the inverse mapping relationship, the coordinate value of the optimal rally point can be translated easily into the interval-valued number of aggregation preference information.

Using this method, we can find the optimal rally point of all expert preferences and generate the expert preference aggregation matrix $[\dot{D}]$:

$$[\dot{D}] = [\dot{d}_{ij}]_{i\times j} = \begin{bmatrix} (\dot{\mu}_{11}, \dot{\nu}_{11}) & \cdots & (\dot{\mu}_{1j}, \dot{\nu}_{1j}) \\ \vdots & \ddots & \vdots \\ (\dot{\mu}_{i1}, \dot{\nu}_{i1}) & \cdots & (\dot{\mu}_{ij}, \dot{\nu}_{ij}) \end{bmatrix} \tag{23}$$

4.1.2. Core steps for selecting optimal rally point based on PGSA

Suppose there are n known membership degree points $(\mu_1, \mu_2, \ldots, \mu_n) \in E$, where E is the bounded closed box in R^N and its length is l. The corresponding positive weights of these points are known as $\zeta_1, \zeta_2, \ldots, \zeta_n$. To identify the optimal rally point $\dot{\mu}$, the core steps of PGSA are as follows:

- Determine the initial growing point $x^0 \in x$ and the step length λ (it is set as $l/200$ in this study), where $x=(x_1, x_2, \ldots, x_m)$ is the vector group of box E. Set $X_{min}=x^0$, $F_{min}=f(x^0)$, where $f(x^0)$ is the backlight function of x^0.
- Set x^0 as a thought center, drawing a line segment parallel to the x-axis and the y-axis, then extending $a_1 \le x_1^0 \le b_1, a_2 \le x_2^0 \le b_2, a_m \le x_m^0 \le b_m$ as new branches. Find $S_{i_1j_1}^0 (1 \le i_1 \le m, 1 \le j_1 \le k_1)$ from branches in λ where $S_{i_1j_1}^0$ is the j_1th germination point in the i_1th branch.
- Compare $f(S_{i_1j_1}^0)$ with F_{min}. If $f(S_{i_1j_1}^0) < F_{\min}$, then $X_{\min} = S_{i_1j_1}^0$, $F_{\min} = f(S_{i_1j_1}^0)$. Otherwise, keep X_{min} and F_{min} unchanged.
- If $f(x^0) < f(S_{i_1j_1}^0)$, then its growth hormone concentration $C_{S_{i_1j_1}^0} = 0$. Otherwise, use Eq. (24) to calculate $C_{S_{i_1j_1}^0}$:

$$C_{S_{i_1j_1}^0} = \frac{f(x^0) - f(S_{i_1j_1}^0)}{\sum_{i_1=1}^{m}\sum_{j_1=1}^{k_1}[f(x^0) - f(S_{i_1j_1}^0)]} \tag{24}$$

- Use growth hormone concentrations of all germination points to establish a morphactin concentration state map between 0 and 1. A random number δ_0 is selected in this interval. If

$$\sum_{i_1=1}^{r_1}\sum_{j_1=1}^{t_1-1} C_{S_{i_1j_1}^0} < \delta_0 \le \sum_{i_1=1}^{r_1}\sum_{j_1=1}^{t_1} C_{S_{i_1j_1}^0} \tag{25}$$

then select $S_{r_1t_1}^0$ as the new growing point, set $x^1 = S_{r_1t_1}^0$, $X_{min}=x^1$ and $F_{min}=f(x^1)$.

- Set x^1 as a thought center, drawing a line segment parallel to the x-axis and the y-axis, then extending $a_1 \le x_1^1 \le b_1, a_2 \le x_2^1 \le b_2, \ldots, a_m \le x_m^1 \le b_m$, as new branches. Find $S_{i_1j_1}^1 (1 \le i_2 \le m, 1 \le j_2 \le k_1)$ from branches in λ.
- Compare $f(S_{i_1j_1}^1)$ with F_{min}. If $f(S_{i_1j_1}^1) < F_{\min}$, then $X_{\min} = S_{i_1j_1}^1$, $F_{\min} = f(S_{i_1j_1}^1)$. Otherwise, keep X_{min} and F_{min} unchanged.
- Calculate $C_{S_{i_1j_1}}^0$ and $C_{S_{i_2j_2}}^1$.

  If $f(x^0) \le f(S_{i_1j_1}^1)$, then $C_{S_{i_1j_1}}^0 = 0$. Otherwise, use Eq. (26) to calculate $C_{S_{i_1j_1}}^0$:

$$C_{S_{i_1j_1}}^0 = \frac{f(x^0) - f(S_{i_1j_1}^0)}{\sum_{i_1=1}^{m}\sum_{j_1=1}^{k_1}[f(x^0) - f(S_{i_1j_1}^0)] + \sum_{i_1=1}^{m}\sum_{j_1=1}^{k_2}[f(x^0) - f(S_{i_2j_2}^1)]} \tag{26}$$

  If $f(x^0) \le f(S_{i_2j_2}^1)$, then $C_{S_{i_2j_2}}^0 = 0$. Otherwise, use Eq. (27) to calculate $C_{S_{i_2j_2}}^1$:

$$C_{S_{i_2j_2}}^1 = \frac{f(x^0) - f(S_{i_2j_2}^1)}{\sum_{i_1=1}^{m}\sum_{j_1=1}^{k_1}[f(x^0) - f(S_{i_1j_1}^0)] + \sum_{i_1=1}^{m}\sum_{j_1=1}^{k_2}[f(x^0) - f(S_{i_2j_2}^1)]} \tag{27}$$

- Use growth hormone concentrations of all germination points to establish a morphactin concentration state map between 0 and 1. A random number δ_1 is selected in this interval, and if

$$\sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2-1} C_{S_{i_1j_1}^0} < \delta_1 \le \sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2} C_{S_{i_1j_1}^0} \tag{28}$$

then select $S_{r_1t_1}^0$ as the new growing point, set $x^2 = S_{r_1t_1}^0$, $X_{min}=x^2$ and $F_{min}=f(x^2)$. Otherwise, if

$$\sum_{i_1=1}^{m}\sum_{j_1=1}^{k_1} C_{S_{i_1j_1}^0} + \sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2-1} C_{S_{i_2j_2}^1} < \delta_1 \le \sum_{i_1=1}^{m}\sum_{j_1=1}^{k_1} C_{S_{i_1j_1}^0} + \sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2} C_{S_{i_2j_2}^1} \tag{29}$$

then select $S_{r_2t_2}^1$ as the new growing point, set $x^2 = S_{r_1t_1}^1$, $X_{min}=x^2$ and $F_{min}=f(x^2)$.

- Repeat Step 6 to Step 9, until F_{min} remains unchanged. Then $x^* = X_{min}$ is the globally optimal solution, set $\dot{\mu} = x^*$, and the calculation is stopped.

4.2. Scores of alternatives

4.2.1. Projection method for scoring programs with intuitionist fuzzy information

Xu proposed several definitions (Definitions 4–6) for uncertain multiple attribute decision making as follows:

Definition 4. Let $\alpha=(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta=(\beta_1, \beta_2, \ldots, \beta_n)$ be two vectors, then

Table 1
Inter-valued intuitionistic fuzzy decision matrix proposed by e_1.

	C_1	C_2	C_3	C_4	C_5
A_1	([0.3,0.4], [0.4,0.6])	([0.5,0.6], [0.1,0.2])	([0.6,0.7], [0.2,0.3])	([0.7,0.8], [0.0,0.1])	([0.6,0.7], [0.2,0.3])
A_2	([0.6,0.8], [0.1,0.2])	([0.6,0.7], [0.2,0.3])	([0.2,0.3], [0.4,0.6])	([0.5,0.6], [0.1,0.3])	([0.7,0.8], [0.0,0.2])
A_3	([0.5,0.8], [0.1,0.2])	([0.7,0.8], [0.0,0.1])	([0.5,0.5], [0.4,0.5])	([0.2,0.3], [0.2,0.4])	([0.4,0.6], [0.2,0.3])
A_4	([0.2,0.3], [0.4,0.5])	([0.5,0.7], [0.1,0.3])	([0.6,0.7], [0.1,0.2])	([0.4,0.5], [0.1,0.3])	([0.6,0.9], [0.0,0.1])
A_5	([0.6,0.8], [0.1,0.2])	([0.3,0.5], [0.4,0.5])	([0.4,0.6], [0.3,0.4])	([0.6,0.8], [0.1,0.2])	([0.5,0.6], [0.2,0.3])

Table 2
Inter-valued intuitionistic fuzzy decision matrix proposed by e_2.

	C_1	C_2	C_3	C_4	C_5
A_1	([0.4,0.5], [0.3,0.4])	([0.5,0.6], [0.1,0.2])	([0.6,0.7], [0.2,0.3])	([0.7,0.8], [0.1,0.2])	([0.7,0.8], [0.0,0.2])
A_2	([0.6,0.8], [0.1,0.2])	([0.5,0.6], [0.3,0.4])	([0.4,0.5], [0.3,0.4])	([0.4,0.6], [0.3,0.4])	([0.4,0.7], [0.1,0.3])
A_3	([0.5,0.6], [0.3,0.4])	([0.5,0.7], [0.1,0.2])	([0.5,0.6], [0.3,0.4])	([0.3,0.4], [0.2,0.5])	([0.6,0.7], [0.2,0.3])
A_4	([0.5,0.6], [0.3,0.4])	([0.7,0.8], [0.0,0.1])	([0.4,0.5], [0.2,0.4])	([0.5,0.7], [0.1,0.2])	([0.5,0.7], [0.2,0.3])
A_5	([0.4,0.7], [0.2,0.3])	([0.5,0.6], [0.2,0.4])	([0.3,0.6], [0.3,0.4])	([0.6,0.8], [0.1,0.2])	([0.4,0.5], [0.2,0.3])

Table 3
Inter-valued intuitionistic fuzzy decision matrix proposed by e_3.

	C_1	C_2	C_3	C_4	C_5
A_1	([0.4,0.6], [0.3,0.4])	([0.5,0.7], [0.0,0.2])	([0.5,0.6], [0.2,0.4])	([0.6,0.8], [0.1,0.2])	([0.4,0.7], [0.2,0.3])
A_2	([0.5,0.8], [0.1,0.2])	([0.3,0.5], [0.2,0.3])	([0.3,0.6], [0.2,0.4])	([0.4,0.5], [0.2,0.4])	([0.3,0.6], [0.2,0.3])
A_3	([0.5,0.6], [0.0,0.1])	([0.5,0.8], [0.1,0.2])	([0.4,0.7], [0.2,0.3])	([0.2,0.4], [0.2,0.3])	([0.5,0.8], [0.0,0.2])
A_4	([0.5,0.7], [0.1,0.3])	([0.4,0.6], [0.0,0.1])	([0.3,0.5], [0.2,0.4])	([0.7,0.9], [0.0,0.1])	([0.3,0.5], [0.2,0.2])
A_5	([0.7,0.8], [0.0,0.1])	([0.4,0.6], [0.0,0.2])	([0.4,0.7], [0.2,0.3])	([0.3,0.5], [0.1,0.3])	([0.6,0.7], [0.1,0.2])

Table 4
Inter-valued intuitionistic fuzzy decision matrix proposed by e_4.

	C_1	C_2	C_3	C_4	C_5
A_1	([0.3,0.4], [0.4,0.5])	([0.8,0.9], [0.1,0.1])	([0.7,0.8], [0.1,0.2])	([0.4,0.5], [0.3,0.5])	([0.2,0.4], [0.3,0.6])
A_2	([0.5,0.7], [0.1,0.3])	([0.4,0.7], [0.2,0.3])	([0.4,0.5], [0.2,0.2])	([0.6,0.8], [0.1,0.2])	([0.2,0.3], [0.0,0.1])
A_3	([0.2,0.4], [0.1,0.2])	([0.4,0.5], [0.2,0.4])	([0.5,0.8], [0.0,0.1])	([0.4,0.6], [0.2,0.3])	([0.5,0.6], [0.2,0.3])
A_4	([0.7,0.8], [0.0,0.2])	([0.5,0.7], [0.1,0.2])	([0.6,0.7], [0.1,0.3])	([0.4,0.5], [0.1,0.2])	([0.7,0.8], [0.1,0.2])
A_5	([0.5,0.6], [0.2,0.4])	([0.5,0.8], [0.0,0.2])	([0.4,0.7], [0.2,0.3])	([0.3,0.6], [0.2,0.3])	([0.7,0.8], [0.0,0.1])

$$\mathrm{Prj}_{\beta}(\alpha) = |\alpha|\cos(\alpha,\beta) = |\alpha|\frac{\alpha\beta}{|\alpha||\beta|} = \frac{\alpha\beta}{|\beta|} \tag{30}$$

is called the projection of the vector α on the vector β.

In general, $\mathrm{Prj}_{\beta}(\alpha)$ shows the approaching degree of the vector α to the vector β, its value rises with an increase in the two vectors approaching degree.

Definition 5. Let $\ddot{\mu}_j = (\dot{\mu}^L_{j\min}, \dot{\mu}^U_{j\max})$, then $\ddot{\mu} = (\ddot{\mu}_1, \ddot{\mu}_2, \ldots, \ddot{\mu}_j)$ is called the positive ideal vector of the membership degree values of all alternatives.

Definition 6. Let $\ddot{\nu}_j = (\dot{\nu}^L_{j\max}, \dot{\nu}^U_{j\min})$, then $\ddot{\nu} = (\ddot{\nu}_1, \ddot{\nu}_2, \ldots, \ddot{\nu}_j)$ is called the positive idea vector of the non-membership degree values of all alternatives.

With reference to the above definitions, we propose the projection method for scoring programs with intuitionistic fuzzy information.

Definition 7. Let $[\dot{D}] = [\dot{d}_{ij}]_{i\times j}$ be the expert preference aggregation matrix, then the projection of membership degree values of each alternative vector $\mathrm{Prj}_{\ddot{\mu}}(d_i)$ can be calculated as follows:

$$\mathrm{Prj}_{\ddot{\mu}}(d_i) = \frac{\mu_i\ddot{\mu}}{|\ddot{\mu}|} \tag{31}$$

Similarly, the projection of non-membership degree values of each alternative vector $\mathrm{Prj}_{\ddot{\nu}}(d_i)$ can be calculated as follows:

$$\mathrm{Prj}_{\ddot{\nu}}(d_i) = \frac{\nu_i\ddot{\nu}}{|\ddot{\nu}|} \tag{32}$$

The larger the value of $\mathrm{Prj}_{\ddot{\mu}}(d_i)$ is, the higher is the degree of the ith alternative approaching the positive ideal vector $\ddot{\mu}$ is $\mathrm{Prj}_{\ddot{\nu}}(d_i)$ can be described similarly.

4.2.2. Scoring process

A process of selecting the membership degree positive ideal point $\ddot{\mu}_j$ and non-membership degree negative ideal point $\ddot{\nu}_j$ of each character from $[\dot{D}_k]$, $\ddot{\mu}_j$ and $\ddot{\nu}_j$ was employed to establish the ideal point matrix $[\ddot{D}]$:

$$[\ddot{D}] = ((\ddot{\mu}_1, \ddot{\nu}_1), (\ddot{\mu}_2, \ddot{\nu}_2), \ldots, (\ddot{\mu}_j, \ddot{\nu}_j)) \tag{33}$$

$\mathrm{Prj}_{\ddot{\mu}}(d_i)$ and $\mathrm{Prj}_{\ddot{\nu}}(d_i)$ can be obtained by using a projection method as shown in the following formulas:

$$\mathrm{Prj}_{\ddot{\mu}}(d_p) = \frac{\sum_{q=1}^{j}(\dot{\mu}^L_{pq}\cdot\dot{\mu}^L_{p\min}\cdot\omega_q + \dot{\mu}^U_{pq}\cdot\dot{\mu}^U_{p\max}\cdot\omega_q)}{\sqrt{\sum_{q=1}^{j}[(\dot{\mu}^L_{q\min})^2\cdot\omega_q + (\dot{\mu}^U_{q\max})^2\cdot\omega_q]}} \tag{34}$$

$$\mathrm{Prj}_{\ddot{\nu}}(d_p) = \frac{\sum_{q=1}^{j}(\dot{\nu}^L_{pq}\cdot\dot{\nu}^L_{p\max}\cdot\omega_q + \dot{\nu}^U_{pq}\cdot\dot{\nu}^U_{p\min}\cdot\omega_q)}{\sqrt{\sum_{q=1}^{j}[(\dot{\nu}^L_{q\max})^2\cdot\omega_q + (\dot{\nu}^U_{q\min})^2\cdot\omega_q]}} \tag{35}$$

where ω_q is the positive weight of character q.

Finally, the score of alternative P can be evaluated as follows:

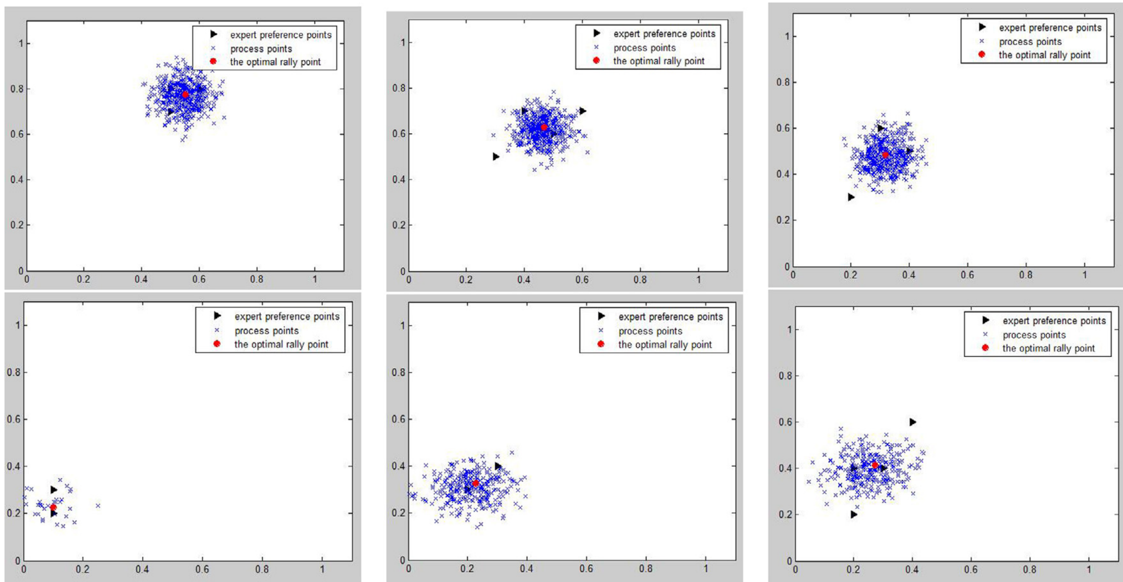
$$S(A_p) = \mathrm{Prj}_{\ddot{\mu}}(d_p) + \mathrm{Prj}_{\ddot{\nu}}(d_p) \tag{36}$$

4.2.3. Evaluation principle

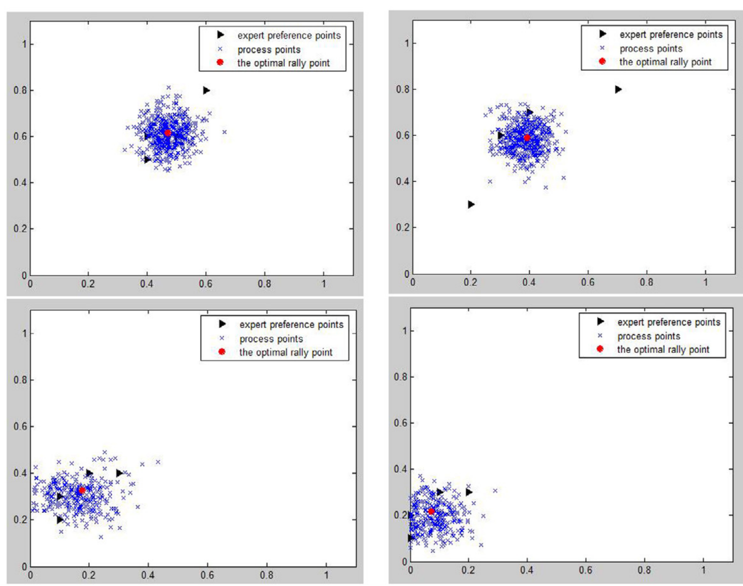
The process of group decision making can be explained in three stages: translating the experts preferences, aggregating experts opinions and selecting the best alternative. The preference aggregation is the critical stage, which greatly influences the decision-making. This paper proposes a method to evaluate the quality of aggregation matrices derived using different aggregation methods.

Firstly, the IVIFNs in the preference matrices are considered as the corresponding points on the two-dimensional coordinates. The preference points of the same character are considered as a point set. Therefore, the distance between two points can be regarded as a degree of difference between the preferences of two experts. The farther the distance is, the larger the degree of difference is, and vice versa. Then, the sum of the Euclidean distance from the aggregation point to the other points in its point set is evaluated as follows:

$$S_{\mu} = \sum_{t=1}^{k}|\dot{\mu}_{ij}\mu^{(t)}_{ij}| = \sqrt{(\dot{\mu}^L_{ij} - \mu^{(1)L}_{ij})^2 + (\dot{\mu}^U_{ij} - \mu^{(1)U}_{ij})^2} + \cdots + \sqrt{(\dot{\mu}^L_{ij} - \mu^{(k)L}_{ij})^2 + (\dot{\mu}^U_{ij} - \mu^{(k)U}_{ij})^2} \tag{37}$$



(a) The aggregation process of $\tilde{d}_{21}^{(k)}$ (b) The aggregation process of $\tilde{d}_{22}^{(k)}$ (c) The aggregation process of $\tilde{d}_{23}^{(k)}$



(d) The aggregation process of $\tilde{d}_{24}^{(k)}$ (e) The aggregation process of $\tilde{d}_{25}^{(k)}$

Fig. 6. The aggregation process of $\tilde{d}_{2}^{(k)}$.

Table 5
The collective matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	([0.350,0.481], [0.349,0.473])	([0.558,0.692], [0.067,0.180])	([0.598,0.698], [0.181,0.302])	([0.605,0.740], [0.105,0.228])	([0.463,0.655], [0.185,0.345])
A_2	([0.551,0.774], [0.100,0.226])	([0.467,0.630], [0.227,0.327])	([0.318,0.484], [0.272,0.413])	([0.471,0.617], [0.175,0.328])	([0.392,0.589], [0.071,0.218])
A_3	([0.439,0.626], [0.132,0.233])	([0.526,0.719], [0.095,0.213])	([0.465,0.648], [0.234,0.335])	([0.259,0.412], [0.200,0.378])	([0.497,0.685], [0.134,0.266])
A_4	([0.469,0.604], [0.196,0.348])	([0.503,0.686], [0.048,0.171])	([0.478,0.603], [0.147,0.316])	([0.510,0.657], [0.073,0.191])	([0.527,0.743], [0.113,0.193])
A_5	([0.548,0.737], [0.122,0.241])	([0.423,0.617], [0.142,0.320])	([0.373,0.652], [0.248,0.348])	([0.449,0.681], [0.126,0.247])	([0.545,0.646], [0.127,0.227])

Table 6
Inter-valued intuitionistic fuzzy decision matrix proposed by e_1.

	C_1	C_2	C_3	C_4
A_1	([0.5,0.5],[0.5,0.5])	([0.6,0.7],[0.1,0.2])	([0.5,0.6],[0.2,0.3])	([0.3,0.5],[0.2,0.4])
A_2	([0.1,0.2],[0.6,0.7])	([0.5,0.5],[0.5,0.5])	([0.4,0.6],[0.1,0.2])	([0.6,0.7],[0.1,0.3])
A_3	([0.2,0.3],[0.5,0.6])	([0.1,0.2],[0.4,0.6])	([0.5,0.5],[0.5,0.5])	([0.3,0.4],[0.5,0.6])
A_4	([0.2,0.4],[0.3,0.5])	([0.1,0.3],[0.6,0.7])	([0.5,0.6],[0.3,0.4])	([0.5,0.5],[0.5,0.5])

Table 7
Inter-valued intuitionistic fuzzy decision matrix proposed by e_2.

	C_1	C_2	C_3	C_4
A_1	([0.5,0.5],[0.5,0.5])	([0.2,0.3],[0.5,0.6])	([0.5,0.7],[0.1,0.2])	([0.2,0.4],[0.1,0.3])
A_2	([0.5,0.6],[0.2,0.3])	([0.5,0.5],[0.5,0.5])	([0.5,0.7],[0.1,0.2])	([0.3,0.6],[0.2,0.3])
A_3	([0.1,0.2],[0.5,0.7])	([0.1,0.2],[0.5,0.8])	([0.5,0.5],[0.5,0.5])	([0.4,0.6],[0.1,0.4])
A_4	([0.1,0.3],[0.2,0.4])	([0.2,0.3],[0.3,0.6])	([0.1,0.4],[0.4,0.6])	([0.5,0.5],[0.5,0.5])

Table 8
Inter-valued intuitionistic fuzzy decision matrix proposed by e_3.

	C_1	C_2	C_3	C_4
A_1	([0.5,0.5],[0.5,0.5])	([0.4,0.5],[0.2,0.3])	([0.6,0.7],[0.1,0.2])	([0.5,0.7],[0.2,0.3])
A_2	([0.2,0.3],[0.4,0.5])	([0.5,0.5],[0.5,0.5])	([0.5,0.6],[0.2,0.4])	([0.7,0.8],[0.1,0.2])
A_3	([0.1,0.2],[0.6,0.7])	([0.2,0.4],[0.5,0.6])	([0.5,0.5],[0.5,0.5])	([0.6,0.7],[0.1,0.3])
A_4	([0.2,0.3],[0.5,0.7])	([0.1,0.2],[0.7,0.8])	([0.1,0.3],[0.6,0.7])	([0.5,0.5],[0.5,0.5])

Table 9
The collective matrix established by the PGSA.

	C_1	C_2	C_3	C_4
A_1	([0.500,0.500], [0.500,0.500])	([0.399,0.499], [0.265,0.365])	([0.517,0.680], [0.143,0.243])	([0.299,0.499], [0.174,0.326])
A_2	([0.200,0.290], [0.371,0.470])	([0.500,0.500], [0.500,0.500])	([0.462,0.631], [0.142,0.260])	([0.599,0.700], [0.120,0.283])
A_3	([0.144,0.244], [0.517,0.680])	([0.140,0.255], [0.462,0.631])	([0.500,0.500], [0.500,0.500])	([0.401,0.599], [0.193,0.433])
A_4	([0.174,0.326], [0.299,0.499])	([0.120,0.283], [0.537,0.673])	([0.163,0.352], [0.401,0.599])	([0.500,0.500], [0.500,0.500])

$$S_\nu = \sum_{t=1}^{k} |\dot{\nu}_{ij}\nu_{ij}^{(t)}| = \sqrt{\left(\dot{\nu}_{ij}^{L} - \nu_{ij}^{(1)L}\right)^2 + \left(\dot{\nu}_{ij}^{U} - \nu_{ij}^{(1)U}\right)^2} + \cdots + \sqrt{\left(\dot{\nu}_{ij}^{L} - \nu_{ij}^{(k)L}\right)^2 + \left(\dot{\nu}_{ij}^{U} - \nu_{ij}^{(k)U}\right)^2} \quad (38)$$

$$S = S_\mu + S_\nu \quad (39)$$

The closer the sum is, the nearer the aggregation point is to the Pareto optimality, and vice versa. Accordingly, by comparing the sum of the Euclidean distances of the aggregation matrices to the expert preference matrices, the pros and cons of aggregation methods can be established.

5. Practical examples

5.1. Example 1

In this section, the performance of the proposed method is evaluated through a case study, which was first presented by Herrera and subsequently modified by Xu [31].

- Let us assume that someone intends to buy a car and consults a set of experts. An expert $e_k(k=1, 2, 3, 4)$ uses an IVIFN $\tilde{d}_{ij}^{k}(i, j = 1, 2, 3, 4, 5)$ to describe the characteristics $C_j(j=1, 2, 3, 4, 5)$ of each supplier $A_i(i=1, 2, 3, 4, 5)$, The weighted vector of the four experts is $\zeta=(0.3, 0.2, 0.3, 0.2)^{\mathrm{T}}$, the weighted vector of the five characteristics is $\omega=(0.2, 0.15, 0.2, 0.3, 0.15)^{\mathrm{T}}$. The interval-valued intuitionistic fuzzy decision matrices proposed by the four experts $\tilde{D}_k = [\tilde{d}_{ij}^{(k)}]_{5\times 5}(k = 1, 2, 3, 4)$ are established in Tables 1–4.
- Map the interval-valued membership degree numbers and non-membership degree numbers into two-dimensional coordinates and employ the PGSA to aggregate the expert preference decision matrices into a single response matrix, as shown in Table 5. To demonstrate the search process of the optimal rally point, the MATLAB simulations of the aggregation process of $\tilde{d}_2^{(k)}$ are shown in Fig. 6.
- Choose ideal points of each attribute from the expert preference aggregation matrix to establish the optimal point matrix $[\dot{\tilde{D}}]$:

$$[\dot{\tilde{D}}] = \begin{bmatrix} ([0.350, 0.774], [0.226, 0.349]), \\ ([0.423, 0.692], [0.171, 0.227]), \\ ([0.318, 0.698], [0.272, 0.302]), \\ ([0.259, 0.740], [0.191, 0.200]), \\ ([0.392, 0.646], [0.185, 0.193]) \end{bmatrix}^{\mathrm{T}} \quad (40)$$

- Eqs. (34)–(36) are employed to obtain the alternative scores $S(\tilde{A}_i)(i = 1, 2, 3, 4, 5)$:

$$S(\tilde{A_1}) = 0.202771,\ S(\tilde{A_2}) = 0.181188,\ S(\tilde{A_3}) = 0.169267,\ S(\tilde{A_4}) = 0.234513,\ S(\tilde{A_5}) = 0.221834$$

Thus,

$$A_4 \succ A_5 \succ A_1 \succ A_2 \succ A_3$$

Therefore, the best alternative is A_4. The collective matrix established by the proposed method is accurate because zero-valued elements in expert preferences do not affect the calculation process of the optimal rally point (see Fig. 6(e)). The high precision collective matrix ensure the reasonable of the alternative ranking.

Table 10
The collective matrix for example 1 aggregated by the IIFHA operator.

	C_1	C_2	C_3	C_4	C_5
A_1	([0.348,0.471], [0.338,0.473])	([0.577,0.741], [**0.000**,0.162])	([0.595,0.698], [0.165,0.296])	([0.564,0.735], [**0.000**,0.254])	([0.536,0.716], [**0.000**,0.284])
A_2	([0.549,0.790], [0.100,0.210])	([0.420,0.616], [0.210,0.311])	([0.322,0.462], [0.288,0.382])	([0.447,0.670], [0.153,0.278])	([0.417,0.652], [**0.000**,0.243])
A_3	([0.470,0.630], [**0.000**,0.120])	([0.535,0.749], [**0.000**,0.192])	([0.489,0.667], [**0.000**,0.306])	([0.273,0.428], [0.200,0.360])	([0.493,0.675], [**0.000**,0.278])
A_4	([0.488,0.638], [**0.000**,0.340])	([0.520,0.707], [**0.000**,0.144])	([0.470,0.605], [0.145,0.325])	([0.498,0.678], [**0.000**,0.208])	([0.521,0.730], [**0.000**,0.185])
A_5	([0.564,0.756], [**0.000**,0.214])	([0.420,0.631], [**0.000**,0.324])	([0.388,0.650], [0.249,0.350])	([0.460,0.682], [0.109,0.249])	([0.574,0.678], [**0.000**,0.205])

Table 11
Euclidean distances between aggregation preferences and expert preferences.

	C_1		C_2		C_3		C_4		C_5	
	Xu	**PGSA**	Xu	**PGSA**	Xu	**PGSA**	Xu	**PGSA**	Xu	**PGSA**
A_1	0.744	0.537	1.052	0.653	0.590	0.405	1.420	0.842	1.769	1.068
A_1	0.376	0.302	0.704	0.587	1.075	0.782	1.022	0.674	1.421	1.073
A_1	1.298	0.967	1.159	0.695	1.629	0.852	1.046	0.604	1.112	0.578
A_1	1.798	1.137	0.794	0.603	1.018	0.755	1.174	0.798	1.352	0.900
A_1	1.248	0.753	1.386	0.944	0.538	0.385	1.044	0.833	1.214	0.721

Table 12
The collective matrix of example 2 established by the IIFHA operator.

	C_1	C_2	C_3	C_4
A_1	([0.512,0.512], [0.479,0.479])	([0.376,0.489], [0.272,0.392])	([0.532,0.691], [0.126,0.224])	([0.302,0.506], [0.145,0.328])
A_2	([0.372,0.475], [0.313,0.426])	([0.512,0.512], [0.479,0.479])	([0.486,0.746], [0.124,0.239])	([0.494,0.683], [0.150,0.278])
A_3	([0.133,0.231], [0.532,0.684])	([0.129,0.249], [0.481,0.704])	([0.512,0.512], [0.479,0.479])	([0.430,0.587], [0.157,0.421])
A_4	([0.150,0.326], [0.276,0.484])	([0.153,0.277], [0.605,0.668])	([0.220,0.436], [0.412,0.565])	([0.512,0.512], [0.479,0.479])

5.2. Example 2

In this example proposed by Xu [1], some interval-valued intuitionistic fuzzy decision matrices without preference value of zero are proposed to illustrate the advantage of aggregation accuracy of the PGSA.

Three experts evaluate four alternatives and their preference matrices are shown below:

We employ our method to establish the collective matrix, as shown in Table 9.

Then, we obtain the alternatives scores $S(\tilde{A}_i)(i = 1, 2, 3, 4)$:

$$S(\tilde{A_1}) = 0.08865,\ S(\tilde{A_2}) = 0.08673,\ S(\tilde{A_3}) = -0.10191,\ S(\tilde{A_4}) = -0.12958.$$

Thus,

$$A_1 \succ A_2 \succ A_3 \succ A_4.$$

The result is in full accord with the alternative ranking in Xu's monograph [31]. Obviously, the proposed method has the same accuracy as classical operators when dealing with the example without preference value of zero.

6. Discussion

The above two examples are solved by the IIFHA operator. This section will discuss the different results obtained by the proposed method and the IIFHA operator and illustrate the advantages of the proposed method.

Definition 8 *([1])*. Let $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i])(i = 1, 2, \ldots, n)$ be IVIFNs, $\dot{\tilde{\alpha}}_{\sigma(j)}$ be the jth largest element of weighted IVIFNs $\dot{\tilde{\alpha}}_i(i = 1, 2, \ldots, n)$, where $\dot{\tilde{\alpha}}_i = n\omega_i\tilde{\alpha}_i(i = 1, 2, \ldots, n)$ and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weighting vector of $\dot{\tilde{\alpha}}_i(i = 1, 2, \ldots, n)$. Let w_j be the weight of $\tilde{\alpha}_j$, $w_j \in [0, 1]$, $\sum_{j=1}^{n} = 1$, and n be the balance factor. Let $\dot{\tilde{\alpha}}_i = ([\dot{a}_i, \dot{b}_i], [\dot{c}_i, \dot{d}_i])$, $\dot{\tilde{\alpha}}_{\sigma(j)} = ([\dot{a}_{\sigma(j)}, \dot{b}_{\sigma(j)}], [\dot{c}_{\sigma(j)}, \dot{d}_{\sigma(j)}])$. The IIFHA operator of the IVIFVs is defined as follows:

Table 13
Modified inter-valued intuitionistic fuzzy decision matrix proposed by e_2.

	C_1	C_2	C_3	C_4
A_1	([0.5,0.5],[0.5,0.5])	([0.2,0.3],[0.5,0.6])	([0.5,0.7],[0.1,0.2])	([0.2,0.4],[0.1,0.3])
A_2	([0.5,0.6],[0.2,0.3])	([0.5,0.5],[0.5,0.5])	([0.5,0.7],[0.0~~(0.1)~~,0.2])	([0.3,0.6],[0.2,0.3])
A_3	([0.1,0.2],[0.5,0.7])	([0.1,0.2],[0.5,0.8])	([0.5,0.5],[0.5,0.5])	([0.4,0.6],[0.1,0.4])
A_4	([0.1,0.3],[0.2,0.4])	([0.2,0.3],[0.3,0.6])	([0.1,0.4],[0.4,0.6])	([0.5,0.5],[0.5,0.5])

$$\begin{aligned}\text{IIFHA}_{\omega,w}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_n) &= \sum_{j=1}^{n}\omega_j\dot{\tilde{\alpha}}_{\sigma(j)} \\ &= \left(\left[1-\prod_{j=1}^{n}(1-\dot{a}_{\sigma(j)})^{w_j},1-\prod_{j=1}^{n}(1-\dot{b}_{\sigma(j)})^{w_j}\right],\left[\prod_{j=1}^{n}\dot{c}_{\sigma(j)}^{w_j},\prod_{j=1}^{n}\dot{d}_{\sigma(j)}^{w_j}\right]\right)\end{aligned} \tag{41}$$

From Eq. (41), we can see that the continuous product is used in the entire process. When the IVIFN of any expert preference equals zero, other experts preferences will be rejected due to one vote. The following two examples illustrate this drawback of the IIFHA operator.

6.1. Example 1

Xu employed the IIFHA operator to process this case, and obtained the collective matrix shown in Table 10:

Xu obtained alternative scores as follows:

$$S(\tilde{A_1})=0.4678,\ S(\tilde{A_2})=0.4084,\ S(\tilde{A_3}) =0.3805,\ S(\tilde{A_4})=0.4672,\ S(\tilde{A_5})=0.4531.$$

The result obtained is $A_1 \succ A_4 \succ A_5 \succ A_2 \succ A_3$ establishing A_1 as the best alternative. However, there are many unreasonable preference values in the collective matrix. The IIFHA operator contains a large number of connected products. In case the IVIFN of any expert preference matrix contains a value zero, the IVIFN in the collective matrix will be directly set to zero (as shown by the bold font in Table 6). For example, the non-membership degrees given by all experts to the C_1 of the A_4 are $\tilde{d}_{41}^{(1)}=[0.4, 0.5]$, $\tilde{d}_{41}^{(2)}=[0.3, 0.4]$, $\tilde{d}_{41}^{(3)}=[0.1, 0.3]$, $\tilde{d}_{41}^{(3)}=[0.0, 0.2]$, and the non-membership degree of the fourth character of A_1 in the collective matrix has been directly set to zero only because the non-membership degree given by expert 4 for this character contains zero. It is clear that the opinions of other experts were rejected due to one vote (Tables 7–9).

Comparing Tables 5 and 10, we can see that the collective matrix established by the PGSA is more reasonable than the IIFHA operator. Expert preference values do not affect aggregate outcomes because the relationships between expert preferences are expressed as distances between preference points.

We also calculate the Euclidean distances between the collective matrices established by the two methods and the expert preference matrices respectively. Eqs. (37)–(39) are employed to calculate the Euclidean distances between the IVIFNs in the collective matrix and their corresponding IVIFNs in the expert preference matrices (see Table 11). Evidently, the collective matrix established by the proposed method is closer to the expert preference matrices, and the difference in distance is more obvious in the case of the expert preference matrix containing zero (see colored part in Table 11). Thus, according to the practical example, the result obtained by the proposed method is more credible.

6.2. Example 2

In this example, all IVIFNs of expert preference are not equal to zero. The collective matrix established by the IIFHA operator is shown in Table 12.

Finally, Xu obtained alternatives scores as follows:

$$S(\tilde{A_1})=0.2316,\ S(\tilde{A_2})=0.2274,\ S(\tilde{A_3})=-0.0925,\ S(\tilde{A_4})=-0.1274.$$

The result obtained is $A_1 \succ A_2 \succ A_3 \succ A_4$ establishing A_1 as the best alternative, which is consistent with the result obtained by the proposed method.

Now, we change one data in this example, adding a zero value to the second expert preference matrix (see Table 13).

The IIFHA operator and the proposed method are employed to aggregate new expert preference matrices.

The alternative scores obtained by the IIFHA operator are shown as follows:

$$S(\tilde{A_1})=0.2316,\ S(\tilde{A_2})=0.3410,\ S(\tilde{A_3})=-0.0925,\ S(\tilde{A_4})=-0.1274.$$

Thus, $A_2 \succ A_1 \succ A_3 \succ A_4$. The best alternative becomes A_2 due to the appearance of a 0 value.

Hence, we see that a small change in the preference matrix results in a large shift in alternative ranking when we employ the IIFHA operator. Many aggregation operators with continued products like the IIFWA, IIFWG and IIFHG operators are also beset with such drawbacks. However, The alternative scores obtained by the proposed method are shown as follows:

$$S(\tilde{A_1})=0.24971,\ S(\tilde{A_2})=0.21820,\ S(\tilde{A_3}) =-0.18566,\ S(\tilde{A_4})=-0.21372.$$

Thus, $A_1 \succ A_2 \succ A_3 \succ A_4$. Obviously, the alternative ranking obtained by the proposed method is not affected by a zero in the expert preference.

7. Conclusions and suggestions for future research

In this paper, a novel method based on the PGSA is proposed for MAGDM problems with IVIFS. The distinctive feature of the proposed method is its ability to handle IVIFSs with a value of zero. Unlike traditional operators, our method translates the IVIFNs of expert preference matrices as preference points instead of handling them directly. The differences between the experts preferences are expressed as distances between expert preference points, thereby ensuring that the preference aggregation result is not affected by any particular value of the expert preference. The proposed method can overcome the drawbacks of Xu's method [1,21,22] and Chen's method [24] considering the situations in which the IVIFNs of expert preference contain a 0 value. In addition, the proposed method provides a new method to improve the quality of the collective matrix. The collective matrix is composed of the optimal rally points obtained by the iterative algorithm instead of being calculated by the aggregation operator. The PGSA can obtain the global optimal result because each iteration will scan the nodes in the entire growth space. Thus, the collective matrices established by the proposed method are more reasonable.

The proposed method can effectively solve the aggregation problem of multiple points in two-dimensional coordinates. Thus, it can deal with a number of practical problems, such as facility location problem, compositive optimization, AHP-group decision making, RFID network planning and multiple-attribute group decision making. Because the proposed algorithm does not involve complex calculating process, the optimal rally point can be founded efficiently.

However, the proposed method also has certain limitations. The PGSA cannot effectively carry out distance computation when the number of preference points is too large (more than fifty points). Thus, the proposed method cannot directly handle the multiple-attributed large group decision-making (MALGDM) problem. The proposed method is suitable for dealing with the MAGDM problems with IVFN or IVIFN. When dealing with the MAGDM with linguistic values, the accuracy of the result obtained by the proposed method is affected by the quality of the language value conversion to IVFN or IVIFN.

In the future, we will focus on two issues: firstly, we will propose a novel method based on the PGSA for the MALGDM problem with IVIFN [32,33]. We will attempt to use a clustering algorithm to cluster the expert preferences so as to reduce the number of expert preference points. Secondly, we will investigate the MAGDM with incomplete weight information or preference information [34]. The Bias algorithm will be employed to complement continuous or discrete missing information.

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