Economic Modelling 43 (2014) 42–56

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

"Little Science" and "Big Science": The institution of "Open Science" as a cause of scientific and economic inequalities among countries

Maria Rosaria Carillo [a,*], Erasmo Papagni [b]

[a] University of Naples, "Parthenope", Department of Businesses and Economics, Via Generale Parisi, 81033, Naples, Italy
[b] Second University of Naples, Department of Economics, Corso Gran Priorato di Malta, 81043 Capua, Italy

ARTICLE INFO

Article history:
Accepted 10 June 2014
Available online 24 August 2014

Keywords:
Scientific and economic inequalities
Social rewards in science
Institution of Open Science
Multiple equilibria and economic growth
Economic development and science

ABSTRACT

In this paper we analyse how the science sector's incentive structure strongly contributes to the development of science and of the economy, even if, in the same time, it can cause large disparities in size and productivity of scientific sectors of different countries. In order to show that, we adopt a Schumpeterian growth model where the resources allocated to science are endogenously determined within the economy and science is organised according to the institution of "Open Science". This latter consists in a self-reinforcing code of conduct, which comprises an incentive scheme based on the priority rule, and on the presence of both real rewards and social rewards. Social rewards take two main forms according to the source concerned: one is the social reward deriving from major innovations; another consists in high reputation enjoyed by researchers who put a high level of effort into their job, and devote themselves to the advancement of science. This set of rules causes the emergence of two locally stable steady-states: a low equilibrium, where the economy is endowed with a small science sector; and a high equilibrium, where the economy has a large science sector with rapid knowledge advancement. The two equilibria can account for the huge differences currently existing between scientific sectors of more developed and less developed countries. Comparative static results further characterise the two equilibria, since monetary and social rewards have different effects according to the type of equilibrium that emerges.

1. Introduction

The production of scientific knowledge is widely recognised as one of the key factors of the economic growth which has occurred in western countries since the Industrial Revolution. Economic historians, such as Ben-David (1964), Rosenberg and Birdzell (1986, 1990), Bekar and Lipsey (2004) and Mokyr (2005), maintain that one of the most important features that distinguishes the most industrialized countries from the rest is their scientific infrastructure. The influence of scientific advances on technological innovation has also been the subject of applied literature for a number of years and several authors, such as Jaffe (1989), Adams (1990), Mansfield (1991, 1995) and Cerruzzi (2003), reach the conclusion that scientific production has stimulated firms' technological innovation in several sectors. However, despite such broad consensus on the importance of science, it has not yet been sufficiently analysed how resources are allocated to the science sector and how its organization may affect the development process of the economy. To give a first answer to these questions, in this paper we will analyse the relationship between science and economic development by adopting a Schumpeterian growth model where the resources allocated to science are endogenously determined within the economy, science is organised according to the institution of "Open Science" and contributes to economic growth by producing new ideas freely accessible to firms. We will show that the institution of Open Science, introducing strong motivations to do research, does contribute to the development of science, but at the same time it causes, under certain conditions, multiple equilibria, each characterised by very different configurations of the science sector. Our approach is new since it is one of the first attempts to endogenise and analyse science within an endogenous growth framework. Indeed, science is generally considered by economists as exogenous to the economy, but such an approach is not well grounded either from a theoretical point of view, because the allocation of human capital to science considers other alternatives, and from an empirical point of view, because there is a clear evidence of a positive correlation between the development of science and the economic growth.

Our aim is to fill this lacuna. We will analyse how the organization of science and its incentive structure affects economic growth and whether there are feedback effects between the two. In this way we can also analyse the development process of science and give a preliminary answer as to why there are large differences between less developed and more developed countries in scientific productivity. Indeed, a clear

* Corresponding author.
E-mail addresses: carillo@uniparthenope.it (M.R. Carillo), papagni@unina.it (E. Papagni).

http://dx.doi.org/10.1016/j.econmod.2014.06.021
0264-9993/

empirical fact that emerges from international data on size and productivity of science (Cole and Phelan, 1999; Schofer, 2004) is the existence of large inequalities across science sectors of different countries. In the table below, we present a synopsis of international science to highlight how great such inequalities are.

Table 1 shows some descriptive statistics on two indicators of scientific output: the number of articles per 1000 population and the number of citations to articles per 1000 population during the years 1996–2007. Data are from the Scopus database[1] and refer to 204 countries. The last column in Table 1 presents statistics on real per capita GDP – the average in the same period – for 180 countries.

A first glance at Table 1 highlights striking inequalities in science across countries, which are even greater than income inequalities: in relative terms, the distance between maximum and minimum values of the number of articles and their citations is much greater than the same distance calculated for per capita GDP. This snapshot is also confirmed by the values of quartiles that synthesize the cross-country distributions of the indicators. Indeed, quartiles show that the right tail of the distribution of science indicators is longer than the left and the mass of the distribution is concentrated on the left. This phenomenon is also confirmed by the coefficient of variation which is greater for science indicators than for GDP. Moreover, recent trends in less developed economies are characterised by the migration of high-skill researchers towards those countries where basic research has a long standing tradition and an important infrastructure such as the USA, Japan and Europe (Weinberg, 2011). This phenomenon tends to reinforce the distance between more and less developed countries with respect to the production of scientific knowledge. Given that basic research is at the core of the education and innovation system, it is important to investigate whether the large differences in scientific productivity among less developed and more developed countries can explain, at least in part, the great divergence among countries in total factor productivity.

To analyse the science sector and its effects on economic growth, we take as a starting point the way in which science is organised. Science, in fact, follows a set of rules and norms of conducts which were first established in the eighteenth century in England (Ben-David, 1964; Mokyr, 2008). In that period, science was first recognised as an activity with an important social function, valued for its own sake, and norms were set to regulate conduct in the science sector, in a manner consistent with the realization of its aims, by giving to it autonomy from other activities.[2] This complex set of norms and rules, called "Open Science" (Dasgupta and David, 1994; David, 1998a), was the key to the development of science and to the rapid accumulation of scientific knowledge in Europe after the scientific revolution, and still nowadays regulates the modern organization of science all over the world.

The institution of "Open Science" consists in a sophisticated self-reinforcing code of conduct, whose aim is to produce and disseminate new ideas, and in a system of rewards whose main characteristics are the *originality rule*, which makes scientific production a "winner takes all" contest, since only the first discoverer of new knowledge obtains a reward (Merton, 1957),[3] and *multidimensionality*, since it comprises both real rewards, such as wages and monetary prizes, and social rewards. The latter take two main forms according to the source from which they derive: one is recognition in terms of *high social prestige* in the scientific community, which derives from major publications and innovations, another is the higher social esteem derived from *dedication to science*, i.e. the higher reputation recognised by peers to scientists who put a high level of effort into science and devote themselves to the advancement of the scientific discipline (Fox, 1983; Stephan and Levin, 1992). This set of rules causes a non-linear relation between the benefits of being a scientist and the size of the science sector. From such a non-linear dynamic a rich variety of equilibrium outcomes and multiple equilibria can be derived. Multiplicity of equilibria is able to explain the huge differences currently existing between the science sector in advanced countries and that in less developed countries shown by international data on scientific productivity. In our model we obtain two locally stable steady-states: a low equilibrium where the economy is endowed with a small science sector, whose productivity is low and competition for discoveries is weak; a high equilibrium, where the economy has a large science sector with rapid knowledge advancement and fierce competition among researchers.

Comparative static results allow us to further characterise the two equilibria, since monetary and social rewards have different effects according to the type of equilibrium that emerges. Indeed, in the high equilibrium, in order to increase scientific productivity, more effective policies are those raising the monetary reward linked to innovation, while, in the low equilibrium, the same policies are those raising peer pressure on researchers, for example, by introducing yardstick competition and/or by increasing accountability of researchers' activity. Monetary rewards can be the route to escape from low equilibrium, only if they are large enough. Indeed, marginal increases have a perverse effect on science productivity, since scientists may reduce the effort employed in the research activity. We also find empirical evidence for the existence of a different responsiveness to changes in the incentive structure according to the size of the science sector and hence the type of equilibria.

Multiplicity of equilibria may also explain the historical evolution of science. Historians of science generally agree that science was organised in two different ways in the last century, termed "Little Science" and "Big Science" (Price de Solla, 1963; Weinberg, 1967). Little Science was prevalent in the more developed countries before the Second World War and now it seems to characterise the less developed countries, while Big Science became established in the most developed countries after the Second World War, when a huge amount of resources was invested in basic research primarily for military purposes (Capshew and Rader, 1992). In particular the term Little Science indicates the case in which scant resources are invested in basic research and this latter is characterised by infrequent discoveries obtained mainly thanks to the effort of a few highly talented people who dedicate all their lives to the advancement of science. By contrast, the term Big Science defines modern large-scale science where a large amount of resources are employed in science. The state is deeply involved in the organization of science with the creation and development of institutions specialised in basic research and high-level (doctorate) education. Big Science is also characterised by a high arrival rate of new ideas and fierce competition among researchers. The

Table 1
Descriptive statistics on scientific publications and GDP per thousand population. Years 1996–2007.
Source http://www.scimagojr.com.

Statistics	Publications	Citations	GDP
Maximum	42.2	819.1	70,103.5
Minimum	0.002	0.006	409.4
Mean	4.4	56.7	9717
Standard deviation	8.0	127.6	11,552
Coefficient of variation	1.8	2.2	1.2
First quartile	0.2	1.2	1876.7
Second quartile	0.9	6.1	4873.6
Third quartile	3.3	30.4	13,110.7
Countries	204	204	180

Note: GDP, PPP constant 2005 international \$. Published documents and citations to documents published during 1996–2007.

[1] Available at the website: http://www.scimagojr.com/. Population is from the World Bank, World Development Indicators 2011.

[2] The independence of science from other fields of inquiry (such as teleology) and the recognition of the norms of science as independent of other norms, were part of the official programme of the Royal Society (Ben-David, 1964).

[3] This method of assignation of the reward gives scientists a powerful incentive to innovate because rewards invariably accrue only to those who discover things first (Dasgupta and David, 1994; Merton, 1957).

linkages between science and society are strong. In Big Science researchers are motivated and remunerated mainly for the attainment of an innovation, while in Little Science the reputation of scientists also stems from their devotion to science, considered a sort of religion on which to dedicate their own lives, reputation given by their peers through continuous personal contacts and strong forms of social interactions (Cotgrove, 1970).

The two equilibria that we find, with different effectiveness of rewards and different endogenous resources invested in science, may represent the two different configurations assumed by science during its evolution in more developed countries. Interestingly, in our model the two equilibrium configurations stem from a unique set of norms and rules on which the institution of Open Science is based. This set of rules does not change from one configuration to another. Nevertheless, it is able to generate both of them as a case of multiple equilibria. Hence, the evolution of science may not be due to a radical change in its organization or in the set of rules governing it, but to a transition from one equilibrium to another made possible by a large amount of investment in science.

1.1. Related literature

In the growth literature there is little attention paid to science. The focus is generally on the technological innovation of firms. An exception is Karl Shell (1967), who proposes a theory of exogenous economic growth in which basic research is endogenous. In his model, the state collects resources from the activities of private agents in order to finance basic research, which produces knowledge, which is a public input for the private sector.

A strand of the literature that has amply analysed science from an economic point of view is what has been termed the 'economics of science' (Dasgupta and David, 1987, 1994; Nelson, 1959; Stephan, 1996; Stephan and Levin, 1992). To this line of research belong Dasgupta and David (1987, 1994), who assert that the fundamental difference between science and technology concerns the dissemination of results: secret in technological research; immediate and complete in scientific research. In this latter case the paternity of a discovery is assured through the *priority rule*, according to which only those who first obtain an innovation have the paternity of it. From this the innovator derives recognition in monetary terms (career advancement, awards, etc.), and in terms of reputation and prestige in the scientific community. Other papers (Dasgupta, 1989; Lazear, 1997) stress that this incentive system means that individual scientists take part in contests belonging to the category of tournaments in which the *winner takes all*. Comparison with reality shows that this system efficiently provides incentives to academic researchers, in that they are generally highly motivated and committed to their research. In our model we build the microeconomics of science along such main lines, but we place it in a general equilibrium model of endogenous economic growth in order to analyse the effects on the development of science and of the economy.

More recently, other theoretical papers that show the substantial differences between the activities of basic research and those of technological innovation have appeared. Among these, Carraro and Siniscalco (2003) analyse the race between public and private research units for a discovery with potential economic application under the hypothesis of knowledge externalities. Aghion et al. (2008) introduce creative freedom, a form of non-real incentive, within a model of scientific research. Comparing academic and private firm organization – where research has an economic focus – they show that relying on academic organizations in the early stages and on private firms in later stages of research is socially optimal. Bramoullé and Saint-Paul (2010) put forward a model of fundamental research where researchers can improve existing fields or invent new fields. They are motivated by the scientific reputation they obtain from the intrinsic value of a paper and from academic citations made by subsequent papers written in a given field. This model produces research cycles which are characterised by the creation of new scientific fields. None of these papers, however, analyse the interaction between the development of science and the economic growth, as in our paper.

Another important strand of the literature to which our paper is related analyses the role of institutions in economic development. Acemoglu et al. (2001, 2002) and Acemoglu et al. (2014) provide extensive evidence that differences in institutions are one of the fundamental causes of economic growth. The literature on this topic defines institutions in a rather broad sense as rules, regulations and laws that affect economic incentives to invest in technology, physical and human capital (Acemoglu, 2009). The institution of Open Science falls right into that definition: it is an institution which regulates the production of new scientific ideas and shapes the incentive scheme faced by scientists when they decide whether or not to enter the scientific sector and how much to invest in research. Acemoglu et al. (2005) distinguish "contracting institutions", which support private contracts, from "property rights institutions", which protect agents against the power of elites and privileged groups. Open Science can be considered a "property rights" institution since it arose to establish the autonomy of scientists with respect to their employers and to protect them from the power of the aristocracy and clergy (Ben-David, 1964; Zilsel, 1942).

Finally, our paper is also related to the literature dealing with the effects on economic growth of social reward, such as social status and social esteem. Most of these studies (Cole et al., 1992; Cooper et al., 2001; Corneo and Jeanne, 2001; Hopkins and Kornienko, 2006) investigate social status in terms of agent's concern for relative ranking in wealth and consumption and show that incorporation of concern for social status deeply alters the results of traditional growth models. Cole et al. (1992), for example, show that such an extension of growth theory is able to produce a poverty trap. Fershtman et al. (1996) apply this framework to occupational status and show that social concern may cause a mismatch of talent that reduces the growth rate. On the other hand, Corneo and Jeanne (2001) show that, when the initial wealth distribution is more equal, the status seeking motive may increase the steady-state growth rate. Recently, Moav and Neeman (2012) found that status concern, by increasing conspicuous consumption, may be a cause of poverty trap. The present paper is complementary to this strand, since it recognizes the paramount role of social reward and status concern in scientific production (Howitt, 2000) and investigates its implications for economic growth.

The paper is organised as follows. In the second section we present the model. In the third section we analyse the model's equilibrium solution and the implied economic dynamics. Comparative-statics results and empirical evidence for them are presented in Section 4. The conclusions follow in Section 5.

2. The model

2.1. Basic assumptions

In this paper we adopt a Schumpeterian growth model[4], where the production side of the economy is made up by two sectors: consumption goods and the science sector. There is no technology to convert resources at one date into resources at another date. Hence there is no capital and the only inputs to production are knowledge and workers, time is continuous and we distinguish calendar time, v, and the state of knowledge that is indexed by t.

The economy is populated by a continuum of individuals, of measure 1, who can find employment in one of the two sectors: l_{vt} work in a competitive sector that produces a consumption good c_{vt}; while n_{vt} are employed in a basic research sector which produces knowledge R_{vt} used in the production of the final good. Manufacturing firms are

[4] Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990. A recent survey of the literature can be found in Aghion et al. (2013).

owned by all agents in the economy, and labour and credit markets are perfectly competitive. The state owns and organises the science sector.

Each individual has an infinite life-span and we assume that every agent derives utility from consumption, from social prestige and suffers a loss of utility from the effort applied to his/her job. Formally the instantaneous utility function is given by:

$$u_{i,\nu,t} = c_{i,\nu,t} + P_{i,\nu,t} - D_{i,\nu,t}, \tag{1}$$

where index i indicates where the agent works: $i = S$ in the case of research and $i = y$ for good production; $c_{i,\nu,t}$ stands for consumption, $P_{i,\nu,t}$ denotes social prestige and $D_{i,\nu,t}$ is the disutility of effort. Consumption is the unique form of expenditure of income $I_{i,\nu,t}$.[5] The consumption good, which is the numeraire, is produced using the following technology:

$$Y_{\nu,t} = R_t l_{\nu,t}^{\alpha} Z^{1-\alpha} \tag{2}$$

with $0 < \alpha < 1$, where R_t is a technological parameter which measures the productivity of the basic knowledge freely available in the technological era t, and Z is an input available with fixed supply that in the following we normalize to 1.

In this economy, innovation is made of new knowledge, R_{t+1}, that is produced in the science sector and increases the productivity of final good workers by a constant parameter $\gamma > 1$. That is to say, we assume that:

$$R_t = R_0 \gamma^t. \tag{3}$$

Consequently t denotes both the state of basic knowledge and the technological era that comes to an end with a scientific discovery.[6]

2.2. The science sector

In this model the science sector is modelled according to the rules of the institution of Open Science characterised by: originality rule; full disclosure of findings and methods and multidimensionality nature of rewards. In order to capture the originality rule we model scientific research as a sequence of races: in each race the winner takes the whole prize and the previous innovator loses what he/she gained[7] To take account of the full disclosure of findings and methods, the new basic knowledge is assumed to be a public good freely available for the production of the final good. Moreover, we assume that the occurrence of new basic knowledge is uncertain and the probability of success for each agent follows a Poisson distribution whose parameter depends on the effort of the researcher:

$$\theta(x_t) = \theta x_t \tag{4}$$

where x_t is the effort and $\theta > 0$ is a productivity parameter. The Poisson distribution is often used to approximate the probability of rare events in a given time interval. In particular, this is the case of the literature on innovation and patent races because these events depend on important firms' investment in R&D and do not occur frequently (Reinganum, 1989). The same context characterises the world of science where significant advances of knowledge require long lasting efforts of researchers on ambitious projects.

Business research and science differ significantly because of the collective nature of scientific research. Indeed, scientific networks see diffused collaboration among researchers. They share and discuss ideas that in the community acquire validity as scientific propositions and are recombined to generate new ideas. The peculiar forms of information diffusion in basic research bring about important externalities. In a companion paper, Carillo and Papagni (2013), we show how social interactions in science can be the cause of multiple steady-state equilibria in a model of economic growth. Here, we pursue another line of research which investigates the effects of prestige and social reward, another source of externalities, on steady-state growth. Accordingly, we assume that each researcher works in isolation and discoveries are independent events across individuals. Hence, the aggregate arrival rate is given by:

$$\Theta(n_t x_t) = \theta n_t \bar{x}_t \tag{5}$$

where $\bar{x}_t$ is the average effort of the researchers' group. This assumption has the significant consequence that the probability of success of each researcher does not depend on the effort of other researchers. The number of researchers in the race is large and there is no strategic interaction[8] among the competitors.

Another important characteristic of the Open Science is the multidimensional nature of the reward system, since it consists of both real rewards and social rewards assigned by peers. In the following we explain in greater depth both types of rewards.

2.2.1. Social rewards

Many sociologists of science (Fox, 1983; Merton, 1973) have pointed out that social prestige is the most substantial part of scientists' total reward and is one of the main factors leading scientists to do research.[9] Social prestige derives, above all, from innovation. Indeed, the prime motive for which a scientist obtains recognition is for contributing to the advancement of science. However, as we have already stressed, scientific races, unlike many other races, do not award second and third prizes and assign recognition and fame only to the first to make a discovery (Gaston, 1978).[10] The recognition that scientists obtain, however, is not public acclaim, but rather recognition from their peers[11] which usually takes the forms of citation of their work, the respect of one's colleagues, honorific awards, titles etc. Coleman (1990) and Bramoullé and Saint-Paul (2010) noted that to establish one's social status the opinion of peers is far more valuable than those of other members of society, since it rests principally on a consensus within a group. This is even truer in the case of scientists, who consider the recognition of their peers highly valuable, while underestimating the opinion of other social groups.[12] In a recent study on the determinants of reputation in academe, Hamermesh and Pfann (2012) find that the major determinant of reputation is the interest that a scholar's work generates among his/her peers. According to the authors of this study "these results suggest that generating the respect that influences the direction of a field, and thus scientific progress, comes from creating works that are viewed as important."

[5] We have assumed a utility function linear in consumption for analytical simplicity, however this assumption is not relevant for the results we get, since we can obtain the same results also by assuming an instantaneous utility function non linear in consumption. For example, by assuming $u = \frac{(c)^{1-\eta}}{1-\eta} + \frac{(P)^{1-\eta}}{1-\eta} - \frac{D^{1-\eta}}{1-\eta}$, with $0 < \eta < 1$, we have results similar to those obtained with the linear utility function. Proof is available upon request to authors.

[6] Since any variable that defines the economy, and therefore the choices made by the agents, remains constant during each technological era, henceforth we simplify the notation by omitting the time index t when it is not indispensable.

[7] This is a simplifying assumption, since for our results it is sufficient to assume that the previous innovator bears only a reduction of his prize, after an innovation occurs.

[8] This assumption is often maintained in basic Schumpeterian growth models such as Aghion and Howitt, 1992.

[9] A reason that might explain the importance of such a system of scientists' reward may lie in the need to solve problems posed by externalities which arise in research activity (Weiss and Fershtman, 1998).

[10] As a result, scientists are obsessed with establishing who has "reached the pinnacle first", Merton (1957) amply showed how frequent and "hard" have been disputes over priority in the history of science.

[11] Charles Darwin once said, "My love of natural science…has been much aided by the ambition to be esteemed by my fellow naturalists", cit. in Merton (1957).

[12] David Raup (1986) coined the phrase "saganization" to describe the loss of professional reputation that scientists (such as Carl Sagan) suffer after receiving continued mass media attention.

In order to capture these features of scientists' social reward, we assume that the social prestige deriving from innovation is awarded only when a new find occurs. Formally:

$$P_{S,t+1} = P_0 R_{t+1} n_{t+1}^{\beta} \tag{6}$$

where: $0 < \beta < 1$, $P_0 > 0$. According to Eq. (6), the social prestige of a researcher increases with the prestige of science in the society, captured by parameter P_0, with the importance of the discovery, captured by the parameter R_{t+1}, and increases with the size of her/his scientific community, but at a decreasing rate.

Although the professional prestige deriving from being an innovator is the main type of social reward for a scientist, it is not the only one. Another is the reputation deriving from the so called "dedication to science", which finds its 'raison d'être' in the rules which govern the institution of Open Science, and includes the idea that science is a mission to which a scientist is dedicated.[13] There are some cultural and psychological characteristics that are valued for their own sake and are enough to keep many scientists working hard at their research. The degree of a scientist's dedication to science contributes to the formation of her reputation, especially in contexts where scientists are a group of highly motivated workers. Scientists who work in environments where there is a strong "science ethos" attach considerable social esteem to colleagues who put a high level of effort into their job and devote themselves wholeheartedly to the advancement of science.[14] In order to consider this dimension of the social reward, which links psychological attitudes and group norms, we hypothesize that reputation deriving from devotion to science interacts with the capacity to bear a high level of effort by reducing the disutility deriving from it,[15] but this reduction occurs only if a researcher employs a higher than average level of effort. To capture this aspect we assume that the cost of effort will be:

$$D(x_t, \bar{x}_t) = R_t d x_t^{1+\sigma} \left(\frac{x_t}{s\bar{x}_t}\right)^{-\epsilon}, \quad d>0, s>0, \sigma>0, \epsilon>0, \tag{7}$$

where $\bar{x}_t$ is the average effort of researchers, and $(\sigma - \epsilon) > 0$.[16] In Eq. (7) we distinguish two components of the cost of effort: the first, $dx_t^{1+\sigma}$, is the traditional concave relation with x, while the second, $\left(\frac{x_t}{s\bar{x}_t}\right)^{-\epsilon}$, represents the peer effect for which the pain of effort is reduced by improvements in the relative comparison with respect to the average of colleagues' effort. The last effect corresponds to recognition of the individual in the community because of dedication to science, captured also by parameter s that approximates the importance of the target represented by the average effort: greater s means keeping up with the scientific community is harder.[17] It can be thought, changes of this parameter could be caused by government policies aimed at increase yardstick competition and researchers' evaluation in science, for example by implementing forms of research evaluation that link evaluation to high reference standard or by enhancing the peers' pressure, or they can be the outcome of sociological modification of preferences for effort in research, that signify a more favourable disposition towards competition among scientists. Disutility increases with existing knowledge R_t because scientists need to learn the existing knowledge to do research, and this task becomes harder as R_t increases. The parameter d captures factors which make research more unpleasant as effort increases, finally parameter σ is the elasticity of marginal disutility of effort.

2.2.2. Real rewards

In addition to social rewards, scientists find incentives from real rewards as well. These often consist of higher salary, monetary awards, royalties, consulting and speaking fees, which can be considered as prizes for new finds. There is substantial evidence that scientists' income is related to their productivity. Fulton and Trow (1974), for example, found that publications "sharply enhance scientists' chances of high salary and earnings outside the universities". Diamond (1986), in his study on mathematicians employed at Berkeley, saw that salary was positively related to productivity, while Tuckman (1976) found the same relation between publications and financial awards for academic engineering and physics.

In addition to this, the scientist's income also includes a component that is not strictly related to success in research.[18] In order to capture these important characteristics of scientists' incentive schemes, we assume that each researcher receives a real prize m_{t+1} if he/she produces new knowledge and a fixed salary F_t, not related to innovation and obtained just for entering the science sector.

Following the literature on Schumpeterian, we assume creative destruction in science: both prizes – real and social – last until a discovery produces new knowledge and a new technology, R_{t+1}. Creative destruction lies at the core of the Schumpeterian growth models. The innovative process is represented as a race between firms competing for a patent that ensures a monopoly over the profits from innovation. Drastic innovations bring about economic dynamics where today monopolist is replaced by the next innovative firm, while under incremental innovations both firms share the market. Hence, the return from an innovation does not last forever but is reduced to zero because of the success of other innovative firms. Several features of the world of academic research seem coherent with creative destruction. Indeed, one of the most important reward of a scientist is peer recognition. This can be reliably measured by the number of citation of a paper. In the bibliometric literature, the typical citation life-cycle pattern shows a growing trend followed, after a peak, by a steady fall. A recent paper by Larivière et al. (2008) analyses the obsolescence of scientific literature over more than 100 years: 1900–2004. According to this study, the median age of cited literature in the fields of natural sciences, engineering, and medicine is lower than 10 years. Interestingly, in the last fifty years the age of cited publications has risen. Evidence of a decline of citations of articles in the field of economics comes from McDowell (1982) and Aizenman and Kletzer (2011). Commenting the findings of their paper, Aizenman and Kletzer write "The gradual decline in citations with age fits the exhaustion of opportunities to use the paper's content in original ways or the role of creative destruction as the ideas or techniques in the paper are superseded".

2.2.3. Optimal effort in research

Researchers engaged in the race of period t – who did not win the previous race[19] – choose effort x_t to maximize the expected value of the total net reward in terms of utility. Indeed, the winner of the contest

[13] Some sociologists of science attribute this scientists' behaviour to the presence of an "inner compulsion" which exists even in the absence of external reward. Indeed, this approach has been called the "sacred spark" theory (Cole and Cole, 1973). In a similar vein, Mary Frank Fox (1983), a sociologist of science, notes that "productive scientists, and eminent scientists especially, are a strongly motivated group of researchers…and have the stamina or the capacity to work hard and persist in the pursuit of long-range goals" (1983, p. 287).

[14] Crane (1965) reports that the social environment (college, department, etc.) is crucial in determining norms, values, attitudes and style of work of scientists.

[15] This hypothesis is confirmed by empirical data on scientists (e.g. Cole and Cole, 1967), which suggests that in general they are highly absorbed, committed and strongly identified with their work and are able to work hard and to persist in a line of research even if the results are uncertain and long-term (see also Aghion, Dewatripoint and Stein, 2008).

[16] The last hypothesis ensures the utility function is concave with respect to x.

[17] This assumption is not necessary in order to obtain the main results of the paper. Also a more simpler and standard convex cost function gives the same qualitative results. However we have assumed this particular form since it better captures the influence of peer effects that could be relevant in determining the capacity to bear a high level of effort.

[18] Often this salary is connected with some other activity not directly linked to research (for example teaching).

[19] The winner of the $(k-1)_{th}$ race has a different incentive to participate in the k_{th} race since he/she would replace himself. Below, we will show that he/she prefers not to enter the present race.

is awarded both a real prize (m_{t+1}) and social reward (P_{t+1}). The winner scientist spends the prize m_{t+1} on consumption goods, while prestige directly increases his/her utility. Both forms of reward are gained after new knowledge is produced and therefore are enjoyed in the period $t+1$. Denote with V_{t+1} the value of the increase in satisfaction that accrues to the scientist who makes a discovery. Then:

$$V_{t+1} = \int_{\nu_0}^{\infty} e^{-\left[r+\theta n_{t+1}\overline{x}_{t+1}\right](\nu-\nu_0)}\left(m_{t+1}+P_{S,t+1}\right)d\nu, \tag{8}$$

where r is the intertemporal preference rate.

The agent who joins the science sector earns a salary F_t independent of the outcome of the race. Hence, the expected intertemporal flow of utility that derives from participating in the t_{vh} race in the research sector is given by:

$$\begin{aligned} U_{S,t} &= \int_{\nu_0}^{\infty} e^{-\left[r+\theta n_t\overline{x}_t\right](\nu-\nu_0)}\left[\theta x_t V_{t+1} + F_t - D(x_t)\right]d\nu \\ &= \frac{\theta x_t V_{t+1} + F_t - D(x_t)}{r + n_t\theta\overline{x}_t}. \end{aligned} \tag{9}$$

Given the above assumptions, a scientist does not adopt a strategic behaviour, meaning that he/she does not consider the effect of his/her effort on the arrival rate of discoveries in the economy $n_t\theta\overline{x}_t$. The value of effort that maximizes the total expected net benefits $U_{s,t}$ deriving from participation in a race satisfies the following first order equilibrium condition:

$$\theta V_{t+1} - (1+\sigma-\epsilon)D(x_t)^{-1} = 0. \tag{10}$$

According to Eq. (10), each researcher chooses the optimal value of effort by equating the expected discounted marginal utility of one more unit of effort to the marginal disutility that derives from effort.

2.3. The consumption good sector

For the sake of simplicity, we make two further assumptions: work in the goods sector does not produce any social prestige; the disutility of work is constant and we normalize it to zero. Hence, the utility function of this category of workers is: $u_{y,t} = c_{y,t}$.

Workers in the consumption sector receive technology from academic research at no cost, but they pay taxes on wages that the state uses to finance basic research. Considering the production function Eq. (2) and the hypothesis of perfect competition, profit maximization yields wages in the consumption good sector given by:

$$w_t = (1-\tau_t)\alpha R_t l_t^{\alpha-1} \tag{11}$$

where $\tau_t \in (0,1)$ is the tax rate.

2.4. The public sector

To finance production of knowledge by the research sector, the state levies taxes T_t on the consumption sector. Wages of workers in the consumption good sector are taxed according to a flat rate:

$$T_t = \tau_t \alpha Y_t, \tag{12}$$

with $\tau_t \in (0,1)$.

Our hypothesis on scientists' real reward implies that the public expenditure for the science sector is made of two components: the amount of real income awarded only to those who win a scientific discovery contest, and the fixed real income that does not depend on the outcome of scientific races.

Hence, the state's budget constraint is:

$$m_t + F_t n_t = \tau_t \alpha Y_t. \tag{13}$$

Given tax revenues, the state applies the following simple rule to assign these resources to the two forms of real reward of scientists:

$$m_t = \tau_1 \alpha Y_t, \tag{14}$$

$$F_t = \tau_2 \alpha Y_t, \tag{15}$$

with $\tau_1 \in (0,1)$ and $\tau_2 \in (0,1)$. Hence, $\tau_t = \tau_1 + n_t\tau_2$, where τ_1 and $n_t\tau_2$ represent the shares of private income that go to finance respectively the prize of scientific races and the fixed salary of researchers.

3. Equilibrium dynamics of the model economy

3.1. Equilibrium

The timing of the model is as follows. At the beginning of each technological age t the state sets up a contest in research and announces the prize m_{t+1} to be awarded to the winner. Individuals choose the sector of employment: research or good production. Scientists choose the amount of effort they will devote to research. Firms operate in competitive markets where they hire the non-researcher workers and produce and sell a consumption good.[20] The government collects taxes from workers' wages and pays researchers a fixed salary and the prize to the innovator scientist. Age t lasts until an innovation occurs. At age $t+1$ the economy is endowed with a better technology R_{t+1} and the agents revise all their decisions accordingly. Hence, the economy grows with a pace that depends on the uncertain time of innovation in science. Given individual choices, equilibrium in this model economy depends on the allocation of workers in the two sectors:

$$n_t + l_t = 1. \tag{16}$$

This decision involves any worker who evaluates the maximum expected utility he/she could attain working in research with that in good production and joins the sector with the greater incentive. As far as the choice of the innovator in the $(t-1)_{th}$ race is concerned, we can prove the following:

Lemma 1. *The winner of the previous $(t-1)_{th}$ race does not participate in the t_{th} race because he/she finds a greater incentive joining the other sector.*

Proof. See Appendix A.

For the rest of the agents in the economy, the incentive to work in good production during the technological age t is given by the expected intertemporal flow of utility from consumption, $U_{y,t}$, given by:

$$U_{y,t} = \int_{\nu_0}^{\infty} e^{-\left[r+\theta n_t\overline{x}_t\right](\nu-\nu_0)} w_t = \frac{w_t}{r + n_t\theta\overline{x}_t.} \tag{17}$$

Since individuals can choose to participate in the labour market either as workers in the consumption sector or as researchers in the science sector, in equilibrium the maximum expected utility yielded by the two types of activity should be the same. From Eqs. (9), (10) and (17) we have the following equilibrium condition for the labour market:

$$\theta x_t V_{t+1} + F_t - D(x_t) = w_t \tag{18}$$

[20] Equilibrium in the market for consumption goods is assured by the equality of demand and supply. Indeed, every agent spends his/her income on consumption. Income includes both revenues from work (researcher salary, F_k, wage, w_k, research prize, m_k) and rents from the fixed factor Z, which are a share $(1-\alpha)Y_k$ of total income.

which yields the equilibrium value of employment in research n_t. Since individuals are homogeneous, equilibrium will be symmetric, which implies $\bar{x}_t = x_t$.

3.2. Dynamics and equilibria

Eqs. (10), (16) and (18) characterise the equilibrium of this economy. From these equations, after some algebra, we obtain a monotone increasing function of employment in science with respect to effort given by:

$$n_t = 1 - x_t^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1-\alpha}} \qquad (19)$$

where $B = \frac{\alpha(1-\tau_1-\tau_2)}{s^{\epsilon} d(\sigma-\epsilon)}$.[21]

We have obtained a first interesting result: in equilibrium there is a positive relation between the two dimensions of the research sector, number of researchers and effort. The intuition of this result is quite simple, when there is a large number of researchers, in order to win a race a single scientist has to employ more effort, because the competition among scientists becomes fiercer.

A consequence of Eq. (19) is that equilibrium dynamics of the model can be represented by a difference equation in x_t since the dynamics in n_t follow from those in x_t. Indeed, the system given by Eqs. (16), (18) and (19) provides the following implicit difference equation that summarises the equilibrium dynamics of the economy:

$$\Psi^x(x_{t+1}) = \Omega^x(x_t), \qquad (20)$$

where

$$\Psi^x(x_{t+1}) \equiv \frac{\theta\gamma\left[\alpha\tau_1 B^{\frac{\alpha}{1-\alpha}} x_{t+1}^{\frac{\alpha(1+\sigma)}{\alpha-1}} + P_0\left(1 - B^{\frac{1}{1-\alpha}} x_{t+1}^{\frac{(1+\sigma)}{\alpha-1}}\right)^{\beta}\right]}{r + \theta x_{t+1}\left(1 - B^{\frac{1}{1-\alpha}} x_{t+1}^{\frac{(1+\sigma)}{\alpha-1}}\right)},$$
$$\Omega^x(x_t) \equiv s^{\epsilon} d(1+\sigma-\epsilon) x_t^{\sigma}.$$

The function $\Psi^x(x_{t+1})$ can be considered the marginal benefits derived from greater effort of a scientist. While $\Omega^x(x_t)$ stands for the marginal costs of the same choice. The shape of these functions depends on the structure of the incentive scheme that we introduced, and in order to study the dynamics of the model, we need a complete characterisation of both functions, that we summarise in the following lemma:

Lemma 2. *The function $\Psi^x(x_{t+1})$, defined for $x_{t+1} \in \left[B^{\frac{1}{1+\sigma}}, \infty\right)$, assumes non-negative values and is continuous. It is shaped like an inverted U with a first branch increasing and then decreasing. In the increasing branch, the second derivative is negative. Moreover, $\Psi^x(x^{t+1})$ takes the following two limit values:*

$$\lim_{x_{t+1} \to B^{\frac{1}{1+\sigma}}} \Psi^x(x_{t+1}) = \frac{\alpha\theta\gamma\tau_1}{r};$$
$$\lim_{x_{t+1} \to \infty} \Psi^x(x_{t+1}) = 0.$$

The function $\Omega^x(x_t)$, defined for $x_{t+1} \in \left[B^{\frac{1}{1+\sigma}}, \infty\right)$, assumes positive values and is continuous. It is monotone increasing and concave in x_t and takes limit values:

$$\lim_{x_{t+1} \to B^{\frac{1}{1+\sigma}}} \Omega^x(x_t) = d s^{\epsilon}(1+\sigma-\epsilon) B^{\frac{\sigma}{1+\sigma}};$$
$$\lim_{x_{t+1} \to \infty} \Omega^x(x_t) = +\infty.$$

[21] This equilibrium relation implies $n \geq 0$ if $x \geq B^{\frac{1}{1+\sigma}}$.

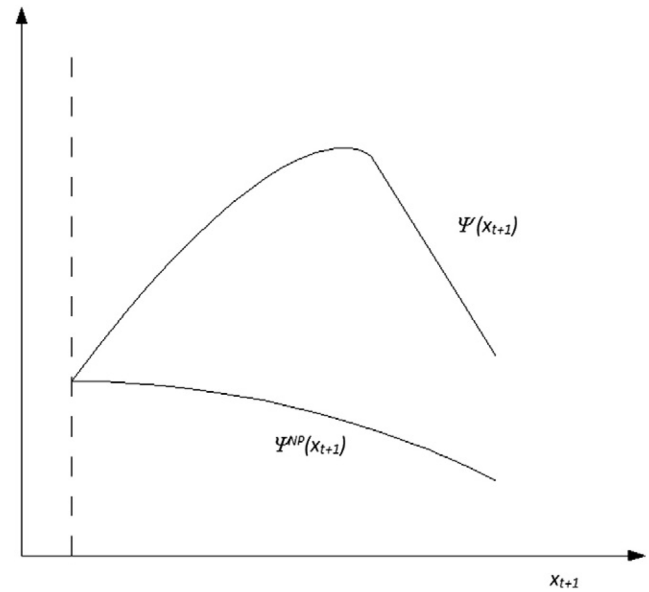

Fig. 1. The marginal benefit curve with and without social prestige.

Proof. See Appendix A.

As might be expected, the innovative structure of incentives in science that we introduce greatly affects the relation between benefits and size of effort in research. In Fig. 1 we represent marginal benefits curves in the presence of prestige deriving from an innovation – $\Psi^x(x_{t+1})$ – and in its absence – $\Psi^{NP}(x_{t+1})$. As shown by the lemma, the marginal benefits curve $\Psi^x(x_{t+1})$ has an inverse U shape. While, in the absence of social reward, the curve will always be decreasing and it is always beneath the marginal benefits curve in the presence of prestige. At the extreme values of x_{t+1} the two curves have the same value. The inverted *U* shape of the marginal benefits curve derives from the interplay of two effects with different sign. On the one hand, greater effort goes along with an enlarged science sector and this causes increasing prestige obtainable from a discovery. On the other hand, an increasing value of effort from future researchers reduces the real reward obtainable from a new find and the period during which both benefits deriving from discovery last. The latter effect implies a negative relation between current effort and future effort. The function of marginal cost of effort is always increasing because greater x_t implies greater n_t and a positive effect on the workers' wage in the consumption sector, which is the alternative sector.

From the proof of Lemma 2 it can be verified that: $\frac{\partial\Omega^x(x_t)}{\partial x_t} \neq 0$. Hence, we can apply the implicit function theorem to Eq. (20) and write the difference equation:

$$x_t = \Gamma^x(x_{t+1}), \quad B^{\frac{1}{1+\sigma}} \leq x \leq \infty; \qquad (21)$$

which, given the relation Eq. (19), summarises the dynamics of equilibrium of the economy under the assumption of perfect foresight.[22] Indeed, we define a dynamic equilibrium with perfect foresight as an infinite sequence of scientists' employment $\{n_0, n_1,. n_t \ldots\}$ and scientists' effort $\{x_0, x_1,.. x_t \ldots\}$ that satisfy Eqs. (19) and (22).

From lemma (2), it seems clear that function $x_t = \Gamma^x(x_{t+1})$ has an inverse U shape, as Fig. 2 shows. The difference equation can be characterised by one or two rest points (Fig. 2), and it can intersect the 45° line both when the curve is increasing and when it is decreasing. Of course, the two curves may not intersect at all, and in this case equilibrium implies nil research and economic growth. See Fig. 3.

[22] Since $\frac{\partial\theta^n(n_k)}{\partial n_k} \neq 0$ also holds, a similar difference equation in n_k can be defined: $n_k = \Gamma^n(n_{k+1})$.

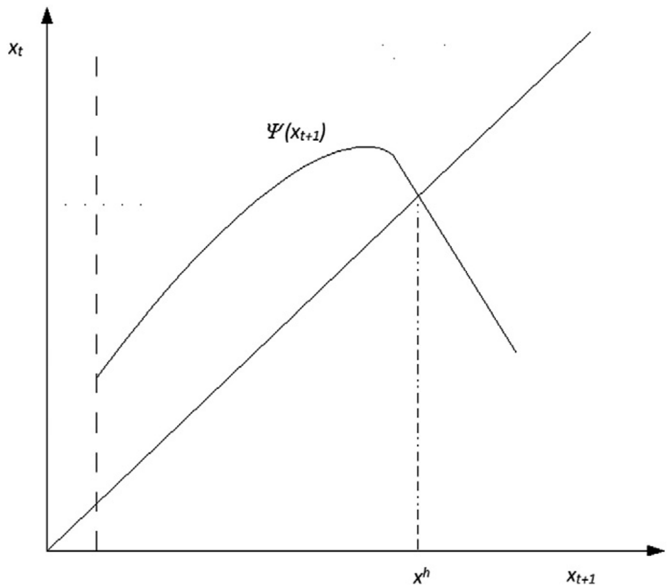

Fig. 2. The dynamics of equilibrium effort. The case of a unique equilibrium.

Also, apart from stationary points, the difference Eq. (21) should allow rich dynamics, e.g. cycles. However, we restrict our investigation to steady states because they represent equilibria that contain many of the interesting results of the model. A steady state is defined as the value of x such that $x = \Gamma^x(x)$. Stability properties of such stationary points will be analysed in terms of their local forward perfect foresight dynamics. We summarise the cases that we investigate in the following proposition:

Proposition 1. *By solving the difference equation $x_t = \Gamma^x(x_{t+1})$, we have three possible equilibrium configurations:*

1) *If $\frac{\alpha\theta\gamma\tau_1}{r} > ds^{\epsilon}(1+\sigma-\epsilon)B^{\frac{\sigma}{1+\sigma}}$, then the system may have one rest point x^*, n^*. This equilibrium point is locally stable in the forward dynamics if it occurs in the decreasing section of the $\Gamma^x(x)$ curve and the condition $\Psi_x^x(x^*) + \Omega_x^x(x^*) < 0$ holds. In this case x_t and n_t converge to x^* and n^* non-monotonically.*

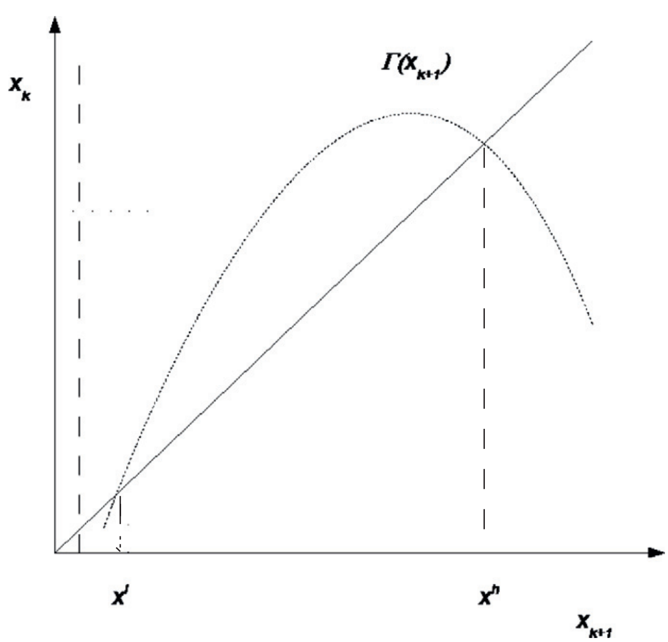

Fig. 3. The dynamics of equilibrium effort. The case of multiple equilibria.

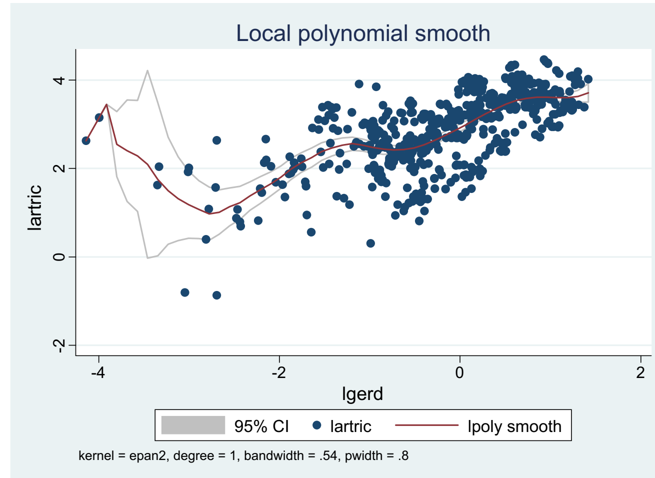

Fig. 4. Local polynomial regression of the number of articles per re-searcher on research expenditure on GDP.

2) *If $\frac{\alpha\theta\gamma\tau_1}{r} < ds^{\epsilon}(1+\sigma-\epsilon)B^{\frac{\sigma}{1+\sigma}}$, then the system has two stable equilibria: (x^l, n^l) and (x^h, n^h). (x^l, n^l), occurs in the increasing section of the $\Gamma^x(x)$ curve and is locally stable in the forward dynamics. (x^h, n^h) is locally stable in the forward dynamics if it occurs in the decreasing section of the $\Gamma^x(x)$ curve and the condition $\Psi_x^x(x^*) + \Omega_x^x(x^*) < 0$ holds. While convergence to (x^l, n^l) is monotone, that towards (x^h, n^h) is non-monotone.*

3) *The two rest points are characterised by the following relations: $x^h > x^l$ and $n^h > n^l$.*

Proof. In Appendix A. ■

In the first case of Proposition 2 there is one stable equilibrium which occurs when the growth potential of the science sector is completely exploited. In fact, in this context, the steady state will be locally stable only if it occurs when the relationship between $x_{t=1}$ and x_t becomes negative and, hence, it occurs when the prestige effect, although it has contributed to the development of the sector by raising the marginal benefits in the first stages, becomes so weak that it does not counterbalance the creative destruction effect.

The second case is characterised by the existence of multiple equilibria. This result is due to the non-monotonic shape of the marginal benefits curve which derives from social prestige awarded by the scientific community to the winner of a race.[23] Hence, even if the effect of prestige introduced in agents' preferences contributes strongly to the development of science, it causes the emergence of multiple equilibria: one equilibrium (n^l, x^l), characterised by low values of n and x, which occurs when the science sector has still unexploited growth potential; another equilibrium (n^h, x^h), with high values of both n and x, occurs in the decreasing section of the marginal benefits curve. This latter rest point has the same qualitative characteristics as the rest point that emerges in the case of the unique equilibrium.[24] In the low equilibrium (n^l, x^l), the research sector is small, scientists have low productivity and there is scant investment in knowledge advances, hence it seems to aptly describe the science sector in the less developed countries. The opposite picture derives from the model at the high equilibrium (n^h, x^h). This describes an economy with a large academic community, whose incentive structure shows high social prestige awarded to scientists for their discoveries. The science sector is very productive and can transfer new knowledge rapidly to the sector of goods production which becomes very efficient. In this state of equilibrium, creative destruction is strong.

[23] This type of social prestige is captured by parameters P_0 and β.
[24] Hence we will refer below to both n^h, x^h and n^*, x^* with the notation n^h, x^h.

In order to complete the characterization of the model at steady states we have to focus on the growth rate of aggregate output. Growth proceeds over time according to a stochastic process with leaps in scientific and technological knowledge. At steady states the expected growth rate of goods production may be derived as follows:

$$g \equiv E(lnY_{\nu} - lnY_{\nu-1}) = ln(\gamma)\theta xn. \tag{22}$$

According to Eq. (22), the growth rate depends positively on the number of researchers and on their level of effort. Hence the two types of equilibria can also be distinguished by the growth rate: low at low equilibrium and high at high equilibrium. In this respect the low equilibrium may be interpreted as a low development trap.

4. Comparative statics

As we have seen in the previous section, given the shape of the marginal benefit curve, a sufficient condition for the emergence of multiple equilibria is $\frac{\alpha\theta\gamma\tau_1}{r} - ds^{\epsilon}(1+\sigma-\epsilon)B^{\frac{\sigma}{1+\sigma}} < 0$. Hence, changes in parameters that reverse the sign of the above inequality, may cause a shift from an equilibrium configuration with multiple equilibria, to a configuration where the only possible stable equilibrium is the high one. At this regard, we can show that:

Proposition 2. *An increase in real reward parameters τ_1 and τ_2, sufficiently large to modify the sign of the following condition: $\frac{\alpha\theta\gamma\tau_1}{r} - ds^{\epsilon}(1+\sigma-\epsilon)B^{\frac{\sigma}{1+\sigma}} < 0$, causes a shift from an equilibrium configuration with multiple equilibria, to an equilibrium configuration where the only stable equilibrium is the high one. Among these parameters, the monetary prize which remunerates the attainment of an innovation (τ_1) is the most effective one.*

Proof. See Appendix A. ■

A policy indication that follows from the above proposition is that if the economy is trapped in the low equilibrium, a strong increase of scientists' real rewards is very effective in order to escape from low science equilibrium. However, a rise in the innovation reward parameter (τ_1) can be particularly effective.

Nevertheless, the effectiveness of principal parameters of the model economy must be evaluated by considering also their effects when changes are only marginal and leave the economy in a neighbourhood of the same type of steady state. At this regard, we analyse first the case of low equilibrium. The results are summarised in the following:

Proposition 3. *Let us consider the case of low equilibrium (x^l, n^l);*

1) *Positive changes in social reward parameters P_0, and β have negative effects on the level of effort, on the number of researchers, and on the average growth rate of real output;*
2) *Positive changes in real reward parameters τ_1, and τ_2 have negative effects on the level of effort, on the number of researchers and on the average growth rate of real output;*
3) *Positive changes in parameter s have positive effects on the number of researchers, while the effects on the level of effort will be positive if the increase in the number of researchers, following a rise in s, is large enough (i.e. $\frac{\partial n}{\partial s} > \frac{\epsilon}{s(1-\alpha)}$). In this case the effect on the average growth rate is positive.*

Proof. In Appendix A. ■

Proposition 3 provides a support for the view of low equilibrium as a low development trap, since it shows that in this case changes which are only marginal in some policy instruments might fail (Azariadis and Stachurski, 2005). Indeed, marginal increases both in social prestige awarded to innovative researchers (P_0 and β) and in their real rewards (τ_1 and τ_2) have negative effects on both dimensions of the research sector: the number of researchers and the level of effort.

These effects can be explained by considering that an increase in the reward for an innovation can also be used by scientists to increase the utility by reducing the level of effort employed in the research activity. This reduction can be explained by a sort of income effect, for which a researcher prefers to reduce the effort supply when the revenue of research increases. Given the positive strategic complementarity between the number of researchers and the level of effort, this reduction may cause a reduction also in the number of researchers.

To better understand this aspect, let us refer to Eq. (19), the derivative of the number of researchers with respect to the principal parameters is:

$$\frac{\partial n_t}{\partial z} = \frac{\partial B}{\partial z}x_x + \frac{1-\alpha}{1+\sigma}B^{\frac{1}{1+\sigma}}\frac{\partial x_t}{\partial z} \gtrless 0,$$

with $z = P_0, \tau_1, \tau_2, \beta$. The first term, which captures the direct effects of parameters on the number of researchers, is always positive and is higher when the level of effort is higher, while the sign of the second term depends on the effect of changes in the level of effort (indirect effect), and is negative. The latter overcomes the first term in the low equilibrium. Hence the derivative with respect to $P_0, \tau_1, \tau_2, \beta$ parameters is negative.

In the same way, it can be justified that in this equilibrium only a strengthening of the reputation deriving from a *dedication to science* motive has a positive effect on the level of effort and on the number of researchers. This can be obtained by increasing interactions among researchers, in order to raise the peer effect, and/or by strengthening researchers' accountability (in our model both policies are captured by an increase in the *s* parameter). Greater interactions may be obtained for example by promoting researchers' mobility, while greater accountability can be obtained by introducing yardstick competition among researchers, as what occurs through assessment exercises. However, such policies are difficult to implement, since they involve changes in the organization of the research activity. It is interesting to note that Aghion et al. (2010) reach similar results by analysing empirically the differences among scientific productivity of European countries and the United States.

A different picture of comparative statics effects of parameter changes derives from the model at high equilibrium. The results are summarised in the following:

Proposition 4. *Let us consider comparative statics at high steady state (x^h, n^h). Then:*

1) *Positive changes in parameters P_o and β have positive effects on the level of effort, on the number of researchers and on the average growth rate of output;*
2) *Positive changes in parameters τ_1 and τ_2 have positive effects on the number of researchers, while these have ambiguous effects on the level of effort. In these cases, the consequences on the average growth rate are indeterminate;*
3) *Positive changes in parameter s decrease the number of researchers, the level of effort and the average growth rate of output.*

Proof. In Appendix A. ■

Proposition 4 highlights the fact that, in high equilibrium, changes in parameters P_0 and β, specific to the social reward given to the innovator, have a positive impact on both dimensions of the science sector: size and effort, while strengthening dedication to science reduces the number of people that may join this sector and their level of effort. At this regard, it should be considered that in the high equilibrium effort is already high. Since the marginal effect of *s* on disutility is an increasing function of *x*:

$$\frac{\partial D}{\partial s} = dx_t^{(1+\sigma-\epsilon)}\overline{x}_t^{\epsilon}\epsilon s^{\epsilon-1},$$

Table 2
Simulated effects of parameters τ_1, τ_2, on endogenous variables at high steady state.

	n^h	x^h	g^h
$\tau_1 = 0.2$	0.396	3.884	9.354
$\tau_1 = 0.25$	0.508	3.689	11.399
$\tau_1 = 0.3$	0.625	3.502	13.314
$\tau_2 = 0.4$	0.396	3.884	9.354
$\tau_2 = 0.45$	0.495	3.668	11.045
$\tau_2 = 0.5$	0.608	3.469	12.830

the effect of greater competition in the scientific community should be strong and could bring about a reduction of effort and employment in research.

Different outcomes derive from variations in real rewards parameters, since their increase will raise the number of scientists, even if the effects on the level of effort are indeterminate. Summing up, in the case of high equilibrium, social reward deriving from innovation strongly contributes to the development of the science sector. Also real incentives increase the number of scientists, but have ambiguous effects on the level of effort.

An answer to the question of indeterminate sign of some comparative statics effects on g^h in the high equilibrium can be obtained by a simulation of the model, whose results are presented in Table 2.[25]

The above simulations show that at the high equilibrium the two tax rates may have positive effects on growth, even if they have negative effects on the level of effort. Accordingly, at high growth steady state, policies aimed at increasing real benefits deriving from research can contribute positively to economic growth.

4.1. Comparison of theoretical results with empirical evidence

The main result of our analysis is that the institution of Open Science, by bearing on a social reward positively linked to the size of science, introduces a sort of "market size" effect in the scientific reward. This allows a rapid growth of science and a rapid accumulation of new scientific knowledge, after science has reached a critical size, but at the same time it can cause large inequalities among different scientific sectors. The evidence on the international distribution of science showing persistent and marked differences across countries is in accordance with our theoretical results. Our case of low equilibrium may represent countries with some degree of development, but a long way from the technological frontier. Data reveal that in such economies, a minority of the population works in academia, and its marginal position in the international scientific community brings about low prestige deriving from an innovation (Drori, 1993). By contrast, our high equilibrium may represent economies which are close to the technological frontier and have a well-developed science sector.

Another result of our model is that real incentives could work well in the high equilibrium, but may have perverse effects in the low equilibrium. In order to find empirical confirmation for this result, we estimate the relation between science productivity and research expenditures, by using data on research expenditure on GDP (GERD) and the number of articles and of researchers in academic and government institutions for 76 countries in the period 1995–2009 drawn from UNESCO. From our theoretical results we would expect a non-linear relation between the two variables, showing persistent low productivity in countries which invest less real resources in science and high productivity in those countries most involved in research. Indeed, in Fig. 4 we present the results of the non-parametric regression of the log of the number of articles per researcher on the log of GERD. The local linear estimate in Fig. 4 shows a U-shaped relation between the two variables. This result highlights significant difficulties suffered by countries with low involvement in basic research in improving their productivity: when GERD is lower than a threshold, greater investment has negative effects on productivity. On the other hand, the same relation works well in countries where GERD has reached a value sufficient to trigger a virtuous process in basic research.

Our results find confirmation not only from the cross-countries empirical evidence, but also from the evolution of science in the last century. The two equilibria we have found seem to aptly describe the two types of science organizations which alternated in the last century in more industrialized countries, named Little Science and Big Science. The former characterised the first stages of science development during the 18th and 19th centuries, while the latter characterised the science sector, after the Second World War, particularly in the USA (Price de Solla, 1963; Weinberg, 1967).[26] According to sociologists and historians of science, Little Science and Big Science differ not only in the size and in productivity of the science sector, low in the first and high in the second, but also in motivations and in the behaviour of scientists. According to Cole and Cole (1967, 1973), in Little Science, researchers usually take part in the so called "Invisible Colleges" (Crane, 1972; David, 1998b) characterised by strong interactions among researchers and strong peer pressure, and have as a major motivation for doing research, the so called "sacred spark", a sort of inner compulsion that ensures total dedication of researchers to the advancement of science (Fox, 1983). By contrast, in Big Science people are attracted mainly by high prestige and pay derived from being an innovator, but they give less importance to their peers' reputation stemming from dedication to science[27] Also Box and Cotgrove (1966), and Cotgrove (1970) find empirical confirmation of this change in the reactiveness to different types of reward. According to Cotgrove (1970) scientists can be classified in three groups: a first group of scientists for whom recognition from innovation is of major importance; a second group of scientists who uses their skills instrumentally as a means of achieving non-scientific rewards, and a third group which "…differs from the first group of scientists only in that they do not attach importance to publication but gained their satisfactions from practising science and from recognition of colleagues" (Cotgrove, 1970, p. 4). Box and Cotgrove (1966) find empirical evidence that the third group of scientists is more present when the science sector is in the initial stage of its development (Little Science), while in Big Science the first group is more present. Further indirect empirical evidence of the greater importance of prestige reward given for innovation in Big Science stems from the fact that in recent decades in the most developed countries there has been a large increase in the number of scientific awards. Zuckerman (1992) observes that this proliferation especially concerns scientific disciplines that were previously excluded from the established prestigious scientific awards (such as the Nobel Prizes) and begins when they become more important.[28]

This picture is quite consistent with our comparative static results, according to which in the high equilibrium rewards that award innovations are highly effective for enlarging academia and economic growth, while in the low equilibrium, the same aim can be reached through a

[25] We performed model simulations by assuming the following values of parameters: $\theta = 0.15$; $\gamma = 1.5$; $\alpha = 0.7$; $\beta = 0.5$; $r = 0.1$; $s = 0.5$; $\tau_1 = 0.2$; $\tau_2 = 0.4$; $\sigma = 0.4$; $d = 0.6$; $P = 2.4$; $\epsilon = 0.2$. In this case, the model produces two positive rest points that satisfy conditions for local stability.

[26] The growth of Big Science started in the 1930 in the USA to cope with problems of providing hydroelectric power, but it was after World War II that it grew rapidly and spread to all western countries, even if the United States was the power-house behind this process. Its main features are large scale, which is large not only with respect to science, but also with respect to the economy, and the major involvement of public finance (Galison and Hevly, 1992). The most famous examples of Big Science research projects are the Manhattan Project, Cape Canaveral Rocketry, the European Centre for Nuclear Research (CERN), and, more recently, the Human Genome Project.

[27] Cotgrove (1970) noted that "Big science offers increasing incentives in the shape of pay, status and power. But it may no longer attract mainly those who previously embraced the role of science as a source of personal emotional gratification. Moreover, the pleasure to be bought with higher incomes may seduce the scientists from the monastic devotion to the pursuit of knowledge" (p. 9, 1970).

[28] According to Zuckerman (1992), in North America in 1990, there were 3000 scientific awards, five times more than thirty years before.

strengthening of the dedication to science motive, by increasing interactions among researchers, and/or the researchers accountability.

5. Conclusions

In this paper we put forward a model of basic research and long-run economic growth in which the system of incentives to scientific work heavily relies on social rewards and may produce positive feedbacks and traits of increasing returns in scientific production. In the model presented here the state finances production of new knowledge – a public good that improves firms' technology – with resources taken from the private sector. Scientists compete with one another to attain priority over a discovery and be awarded both a real prize and prestige in the scientific community. Also, scientists derive utility from doing research. The dynamic of the model economy shows that two locally stable stationary equilibria can be obtained. These equilibria may explain the huge differences existing between scientific sectors of less developed and more developed countries, and the evolution of modern science in the last century. From this model we can also gain some useful insights on which policies are more effective at increasing the efficiency of the science sectors both in more and less developed countries and at reducing the gap between the two.

Appendix A

A.1. Proof of Lemma 1

We prove this proposition by showing how the previous innovator has a greater incentive to join the good production sector rather than research. Let us distinguish between the scientist who made a discovery in age $t-1$ and the rest of the participants in the t_{th} scientific contest. The incentive of the first category of researchers to enter the t_{th} race is given by:

$$U^v_{S,t} = \frac{\theta x^v_t\left(V_{t+1} - V_t\right) + F_t - D\left(x^v_t\right)}{r + n_t\theta \overline{x}_t}$$

where the superscript v refers to the winner of the previous race. This researcher would get from winning the present race a prize V_{t+1} but he/she would lose V_t. The choice of the sector depends on the maximum expected utility attainable in each alternative. His/her optimal value of effort is:

$$x^v_t = \left[\frac{\theta\left(V_{t+1} - V_t\right)}{d(1+\sigma-\epsilon)R_t s^\epsilon \overline{x}^\epsilon_t}\right]^{\frac{1}{\sigma-\epsilon}}.$$

By substituting the optimal effort function in $U^v_{S,t}$, we get the maximum expected utility that the previous discoverer might obtain on winning the present race:

$$\widetilde{U}^v_{S,t} = \frac{\left[\theta\left(V_{t+1} - V_t\right)\right]^{\frac{1+\sigma-\epsilon}{\sigma-\epsilon}}\left[\frac{1}{d(1+\sigma-\epsilon)R_t s^\epsilon \overline{x}^\epsilon_t}\right]^{\frac{1}{\sigma-\epsilon}}\frac{\sigma-\epsilon}{1+\sigma-\epsilon} + F_t}{\left(r + n_t\theta\overline{x}_t\right).}$$

On the other hand, the rest of the participants in the scientific contest expect from winning the race an increase in utility equal to:

$$U^e_{S,t} = \frac{\theta x^e_t V_{t+1} + F_t - D\left(x^e_t\right)}{r + n_t\theta\overline{x}_t},$$

where the superscript e distinguishes effort specific to this kind of scientist. Winning the race, they lose nothing, hence have a greater incentive to participate. When the optimal effort is chosen, the maximum incentive is given by:

$$\widetilde{U}^e_{S,t} = \frac{\left(\theta V_{t+1}\right)^{\frac{1+\sigma-\epsilon}{\sigma-\epsilon}}\left[\frac{1}{d(1+\sigma-\epsilon)R_t s^\epsilon \overline{x}^\epsilon_t}\right]^{\frac{1}{\sigma-\epsilon}}\frac{\sigma-\epsilon}{1+\sigma-\epsilon} + F_t}{\left(r + n_t\theta\overline{x}_t\right).}$$

The occupational choice of agents at the beginning of the t_{th} age satisfies the equilibrium condition:

$$\widetilde{U}^e_{S,t} = U_{y,t} = \frac{w_t}{\left(r + n_t\theta\overline{x}_t\right),}$$

from which the inequality:

$$\widetilde{U}^v_{S,t} < U_{y,t}$$

follows. Here, the proof is complete: at the beginning of the t_{th} age, the innovator prefers working in the good production sector.

A.2. Proof of Lemma 2

In this proof we take advantage of the possibility of representing the dynamics of the economy in a difference equation of n_t. We prove this lemma first by characterising the shape of the function $\Psi^n(n_{t+1})$ and then we obtain the derivatives of $\Psi^x(x_{t+1})$ using the relation $n_t = 1 - x_t^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1-\alpha}}$. In particular, $\Psi^n(n_{t+1})$ is the function:

$$\Psi^n\left(n_{t+1}\right) \equiv \frac{\theta\gamma\left[\alpha\tau_1\left(1-n_{t+1}\right)^\alpha + P_0 n^\beta_{t+1}\right]}{r + \theta n_{t+1}\left(1-n_{t+1}\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}.}$$

By considering the first derivative of the marginal benefits function with respect to n_{t+1}, we have:

$$\begin{aligned}
&\frac{\partial\Psi^n\left(n_{t+1}\right)}{\partial n_{t+1}} \gtreqless 0 \Longleftrightarrow \beta P_0 n^{\beta-1}_{t+1} - \tau_1\alpha^2\left(1-n_{t+1}\right)^{\alpha-1} r + \\
&-\theta P_0 n^\beta_{t+1}\left(1-n_{t+1}\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\left\{1-\beta + \frac{1-\alpha}{(1+\sigma)}\left(1-n_{t+1}\right)^{\frac{(\alpha-1)}{1+\sigma}-1} B^{\frac{1}{1+\sigma}}\right\} + \\
&-\frac{\tau_1\alpha\left(1-n_{t+1}\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\theta\left(1-n_{t+1}\right)^{\alpha-1}}{1+\sigma}\left[1+\sigma-n_{t+1}\sigma(1-\alpha)\right] \gtreqless 0.
\end{aligned}$$

This expression is composed of a positive term – the first, which is decreasing in n_{t+1} and for $n_{t+1} \epsilon (0, 1)$ assumes values from $+\infty$ to $\beta P_o r$ – and of three other terms, all negative, whose absolute values are increasing in n_{t+1} and tend to $-\infty$ for $n_{t+1} = 1$. This means that for $n_{t+1} \epsilon (0, 1)$ the above expression is first increasing and then decreasing with one stationary point.

To check the concavity of the increasing section of the marginal benefits function, we consider the sign of its second derivative with respect to n_{t+1}. In this respect we have:

$$\begin{aligned}
&\frac{\partial^2\Psi^n\left(n_{t+1}\right)}{\partial n_{t+1}\partial n_{t+1}} \gtreqless 0 \Longleftrightarrow \\
&-\left[(1-\alpha)\alpha^2\tau_1\left(1-n_{t+1}\right)^{\alpha-2} + (1-\beta)\beta P_0 n^{\beta-2}_{t+1}\right]\left(r + \theta n_{t+1}\left(1-n_{t+1}\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\right) + \\
&-\left[\alpha\tau_1\left(1-n_{t+1}\right)^\alpha + P_0 n^\beta_{t+1}\right] - 2\theta\frac{\left(1-n_{t+1}\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}} + n_{t+1}\frac{\partial x_{t+1}}{\partial n_{t+1}}}{\left(r + \theta n_{t+1}\left(1-n_{t+1}\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\right)}\frac{\partial\Psi^n\left(n_{t+1}\right)}{\partial n_{t+1}} \gtreqless 0.
\end{aligned}$$

This expression is composed by three terms which are all negative. Also the last term is negative in the increasing section of the marginal benefits curve. Hence, we are sure that in this section the second derivative assumes negative values and the curve is concave.

The two limit values of the marginal benefits function can be trivially derived from an inspection of the marginal benefits function.

We can use these results to prove the lemma in the case of the function $\Psi^x(x_{t+1})$. Indeed, let us define the function

$$n_t = 1 - x_t^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1-\alpha}} = f(x_t).$$

Then, applying the chain rule of differentiation, we have:

$$\frac{\partial \Psi^x(x_{t+1})}{\partial x_{t+1}} = \frac{\partial \Psi^n(n_{t+1})}{\partial n_{t+1}} \frac{\partial f(x_t)}{\partial x_t},$$

and

$$\frac{\partial^2 \Psi^x(x_{t+1})}{\partial x_{t+1} \partial x_{t+1}} = \frac{\partial^2 \Psi^n(n_{t+1})}{\partial n_{t+1} \partial n_{t+1}} \left(\frac{\partial f(x_t)}{\partial x_t}\right)^2 + \frac{\partial \Psi^n(n_{t+1})}{\partial n_{t+1}} \frac{\partial^2 f(x_t)}{\partial x_t \partial x_t}.$$

Since $\frac{\partial f(x_t)}{\partial x_t} > 0$, and $\frac{\partial^2 f(x_t)}{\partial x_t \partial x_t} < 0$, the sign of the first derivative of $\Psi^x(x_{t+1})$ is the same as that of $\Psi^n(n_{t+1})$, while in the increasing section of $\Psi^x(x_{t+1})$ the second derivative is negative like that of $\Psi^n(n_{t+1})$. The function $\Omega^x(x_t)$ is a simple concave function.

The two limit values of the marginal benefits function can be trivially derived from an inspection of the function.

A.3. Proof of Proposition 1

The proof of Proposition 1 follows from a characterisation of the dynamics of n_t which is embedded in the difference equation $n_t = \Gamma^n(n_{t+1})$ that can be derived by applying the implicit function theorem to equilibrium equation $\Psi^n(n_{t+1}) = \Omega^n(n_t)$ where

$$\Omega(n_t) \equiv ds^\epsilon (1 + \sigma - \epsilon)(1 - n_t)^{\frac{(\alpha-1)\sigma}{1+\sigma}} B^{\frac{\sigma}{1+\sigma}}.$$

From the implicit function theorem we know that: $\frac{d\Gamma^n(n_{t+1})}{dn_{t+1}} = \frac{\Psi^n_n(n_{t+1})}{\Omega^n_n(n_t)}$, but $\Omega^n_n(n_t) > 0$, hence: $\frac{d\Gamma^n(n_{t+1})}{dn_{t+1}}$ 0 if $\Psi^n_n(n_{t+1})$ 0. This means that as n_{t+1} increases starting from zero, $\Gamma^n(n_{t+1})$ is increasing and concave till it reaches a maximum, then it is decreasing until it intersects the horizontal axis. From the above lemma we know that the difference equation $n_t = \Gamma^n(n_{t+1})$ has a graph that intersects the vertical axis if $\Psi^n(0) > \Omega^n(0)$, i.e. if

$$\frac{\alpha\theta\gamma\tau_1}{r} > ds^\epsilon (1 + \sigma - \epsilon) B^{\frac{\sigma}{1+\sigma}}.$$

In this case, if the map intersects the 45° line at the decreasing branch and condition $\Psi^n_n(n^*) + \Omega^n_n(n^*) < 0$ holds, then we have $\left|\frac{d\Gamma^{-1}(n)}{dn}\right| < 1$, which is a sufficient condition for the local stability in the forward dynamics of the rest point n^*. The map $\Gamma^n(n_{t+1})$ intersects the horizontal axis twice if $\Psi^n(0) < \Omega^n(0)$, i.e. if

$$\frac{\alpha\theta\gamma\tau_1}{r} < ds^\epsilon (1 + \sigma - \epsilon) B^{\frac{\sigma}{1+\sigma}}.$$

Given the shapes of marginal benefits and cost functions, in this case there can be either no intersection at all or two intersections, which identify the two rest points, n^l and n^h with $n^l < n^h$. At the first stationary point n^l both $\Psi^n(n_{t+1})$ and $\Omega^n(n_t)$ are increasing and $\Psi^n_n(n^l) > \Omega^n_n(n^l)$ holds, hence $\frac{d\Gamma^{-1}(n)}{dn} < 1$, and n^l is locally stable in the forward dynamics. The higher steady state n^h has the same properties as the single equilibrium n^*, and the same arguments apply.

The properties of $x_t = \Gamma^x(x_{t+1})$ can be derived from the relation $n_t = f(x_t)$. Indeed, this means the rest points of $x_t = \Gamma^x(x_{t+1})$ satisfy $x = f^{-1}(x)$. Hence, under the conditions of Proposition 1 we have 1, 2 or 0 steady states. Stability analysis of steady states derives from that of $n_t = \Gamma^n(n_{t+1})$. We can express the equilibrium condition in n_t as

$$\Psi^n[f(x_{t+1})] = \Omega^n[f(x_t)],$$

and obtain the derivative

$$\frac{\partial x_t}{\partial x_{t+1}} = \frac{\frac{\partial \Psi^n(n_{t+1})}{\partial n_{t+1}}}{\frac{\partial \Omega^n(n_t)}{\partial n_t}} \left(\frac{x_{t+1}}{x_t}\right)^{\frac{1+\sigma}{\alpha-1}-1}.$$

At steady states $x_t = x_{t+1}$, and this means that the stability conditions of x are the same as that of n.

The last statement of the proposition derives from the monotone increasing relation between x and n.

A.4. Proof of Proposition 2

Let us consider the inequality: $\frac{\alpha\theta\gamma\tau_1}{r} < ds^\epsilon (1 + \sigma - \epsilon) B^{\frac{\sigma}{1+\sigma}}$.

It can be easily checked that the derivative of the first term with respect to τ_1 is positive, while the derivative of the second term is negative. Hence there exists a sufficiently large increase in τ_1 that reverses the sign of the above inequality.

The same argument applies to the case of τ_2. The difference in this case is that the derivative of the first term with respect to τ_2 is zero, while the derivative of the second term is negative.

A.5. Proof of Proposition 3

Our strategy in this proof is to obtain the effects of parameters on equilibrium n, and then derive the same effects on x by making use of the relation between the two variables. Hence, let us define the implicit function which identifies the two steady state equilibria: $F(n) \equiv \Psi^n(n) - \Omega^n(n) = 0$. This equation can be rewritten as:

$$F(n) \equiv \frac{\theta\gamma\left[\alpha\tau_1\left(1-n^l\right)^\alpha + P_0\left(n^l\right)^\beta\right]}{r + \theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}} - (1 + \sigma - \epsilon) ds^\epsilon \left(1-n^l\right)^{\frac{(\alpha-1)\sigma}{1+\sigma}} B^{\frac{\sigma}{1+\sigma}} = 0.$$

For the implicit function theorem we have

$$\frac{\partial n^l}{\partial z} = -\frac{\frac{\partial \Psi^n(n^l,z)}{\partial z} - \frac{\partial \Omega^n(n^l,z)}{\partial z}}{\frac{\partial \Psi^n(n^l,z)}{\partial n^l} - \frac{\partial \Omega^n(n^l,z)}{\partial n^l}}$$

where $z = \beta, P_0, \tau_1, \tau_2, s$. In the neighbourhood of n^l the denominator is positive because of the stability condition, hence $\frac{\partial n^l}{\partial z} \gtrless 0 \Leftrightarrow \frac{\partial \Psi^n(n^l,z)}{\partial z} - \frac{\partial \Omega^n(n^l,z)}{\partial z} \lessgtr 0$.

For $z = \beta, P_0$, we obtain: $\frac{\partial \Psi^n(n^l,z)}{\partial z} - \frac{\partial \Omega^n(n^l,z)}{\partial z} = \frac{\partial \Psi^n(n^l,z)}{\partial z}$; this expression, as can be easily checked, is always positive. Then, in an interval around the low equilibrium, an increase in these parameters always has negative effects on the number of researchers.

For $z = \tau_1$, we have:

$$\frac{\partial \Psi^n\left(n^l, \tau_1\right)}{\partial \tau_1} = \frac{\theta\gamma\left[r + \theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\right]\alpha\left(1-n^l\right)^\alpha}{\left[r + \theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\right]^2} +$$

$$+\frac{\theta\gamma\left[\alpha\tau_1\left(1-n^l\right)^\alpha + P_0\left(n^l\right)^\beta\right]\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}-1} \frac{1 \qquad \alpha}{1 + \sigma ds^\epsilon(\sigma-\epsilon)}}{\left[r + \theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}} B^{\frac{1}{1+\sigma}}\right]^2} > 0;$$

and

$$\frac{\partial \Omega^n\left(n^l,\tau_1\right)}{\partial \tau_1}=-\frac{\alpha}{(\sigma-\epsilon)}\frac{\sigma(1+\sigma-\epsilon)}{(1+\sigma)}\left(1-n^l\right)^{\frac{(\alpha-1)\sigma}{1+\sigma}}B^{\frac{\sigma}{1+\sigma}-1}<0.$$

Hence, $\frac{\partial \Psi^n(n^l,\tau_1)}{\partial \tau_1}-\frac{\partial \Omega^n(n^l,\tau_1)}{\partial \tau_1}>0$ and $\frac{\partial n^l}{\partial \tau_1}<0$.

For $z=\tau_2$, then:

$$\frac{\partial \Psi^n\left(n^l,\tau_2\right)}{\partial \tau_2}=\frac{\theta\gamma\left[\alpha\tau_1\left(1-n^l\right)^{\alpha}+P_0\left(n^l\right)^{\beta}\right]\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}-1}\frac{1}{1+\sigma ds^{\epsilon}}\frac{\alpha}{(\sigma-\epsilon)}}{\left[r+\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}\right]^2}>0;$$

and

$$\frac{\partial \Omega^n\left(n^l,\tau_2\right)}{\partial \tau_2}=-\frac{\alpha}{(\sigma-\epsilon)}\frac{\sigma(1+\sigma-\epsilon)}{(1+\sigma)}\left(1-n^l\right)^{\frac{(\alpha-1)\sigma}{1+\sigma}}B^{\frac{\sigma}{1+\sigma}-1}<0.$$

Hence, $\frac{\partial \Psi^n(n^l,\tau_2)}{\partial \tau_2}-\frac{\partial \Omega^n(n^l,\tau_2)}{\partial \tau_2}>0$ and $\frac{\partial n^l}{\partial \tau_2}<0$.

For $z=s$, then:

$$\frac{\partial \Psi^n\left(n^l,s\right)}{\partial s}=-\Psi^n\left(n^l,s\right)\frac{\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}\frac{1}{1+\sigma s}\epsilon}{r+\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}}<0;$$

and

$$\frac{\partial \Omega^n\left(n^l,s\right)}{\partial s}=\Omega^n\left(n^l,s\right)\frac{\epsilon}{s(1+\sigma)}>0.$$

Hence,

$$\frac{\partial \Psi^n\left(n^l,s\right)}{\partial s}-\frac{\partial \Omega^n\left(n^l,s\right)}{\partial s}=$$
$$-\Psi^n\left(n^l,s\right)\frac{\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}\frac{1}{1+\sigma s}\epsilon}{r+\theta n^l\left(1-n^l\right)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}}-\Omega^n\left(n^l,s\right)\frac{\epsilon}{s(1+\sigma)}=$$
$$-\Psi^n\left(n^l,s\right)\frac{r\epsilon}{s(1+\sigma)}<0.$$

This expression is always negative, hence $\frac{\partial n^l}{\partial s}>0$.

As regards the effects on the level of effort, we rewrite the equilibrium condition Eq. (19) as follows:

$$x_t=(1-n_t)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}$$

and take the derivatives of x_t with respect to $z=\{P_0,\beta,\tau_1,\tau_2,s\}$.

For $z=P_0\beta$, then:

$$\frac{\partial x_t^l}{\partial z}=\frac{1-\alpha}{1+\sigma}\left(1-n_t^l\right)^{\frac{\alpha-1}{1+\sigma}-1}B^{\frac{1}{1+\sigma}}\frac{\partial n_t^l}{\partial z}.$$

The sign of this expression depends on the sign of $\frac{\partial n_t^l}{\partial z}$, which, for the parameters at hand, is negative.

For $z=\tau_1,\tau_2$,

$$\frac{\partial x^l}{\partial z}\gtreqless 0\Leftrightarrow\frac{1-\alpha}{1-n^l}\frac{\partial n^l}{\partial z}-\frac{1}{1-\tau_1-\tau_2}\gtreqless 0.$$

For the parameters at hand $\frac{\partial n^l}{\partial z}<0$, hence this derivative is always negative.

For $z=s$,

$$\frac{\partial x^l}{\partial s}\gtreqless 0\Leftrightarrow\frac{\partial n^l}{\partial z}-\left(1-n^l\right)\frac{\epsilon}{s(1-\alpha)}\gtreqless 0.$$

Since $\frac{\partial n^l}{\partial s}$ is always positive, $\frac{\partial x^l}{\partial s}>0$ if $\frac{\partial n^l}{\partial z}\geq\frac{\epsilon}{s(1-\alpha)}$.

A.6. Proof of Proposition 4

This proposition concerns the comparative statics at the high equilibrium and it also holds for the case of unique equilibrium since the same arguments can be applied. Application of the implicit function theorem to the equation $\Psi^n(n^h,z)-\Omega^n(n^h,z)=0$ gives:

$$\frac{\partial n^h}{\partial z}=-\frac{\frac{\partial \Psi^n(n^h,z)}{\partial z}-\frac{\partial \Omega^n(n^h,z)}{\partial z}}{\frac{\partial \Psi^n(n^h,z)}{\partial n^h}-\frac{\partial \Omega^n(n^h,z)}{\partial n^h}}$$

where $z=\beta,P_0,\tau_1,\tau_2,s$.

In an interval around n^h the denominator is negative because of the stability condition. Hence:

$$\frac{\partial n^h}{\partial z}\gtreqless 0\Leftrightarrow\frac{\partial \Psi^n\left(n^h,z\right)}{\partial z}-\frac{\partial \Omega^n\left(n^h,z\right)}{\partial z}\gtreqless 0.$$

This implies that in the n^h equilibrium, changes in the different parameters have the opposite sign with respect to those in the low equilibrium. Hence $\frac{\partial n^h}{\partial z}>0$ for $z=\{P_0\beta,\tau_1,\tau_2\}$ and $\frac{\partial n^h}{\partial s}<0$.

To find the effects of changes in the relevant parameters on the level of effort, we rewrite the equilibrium condition as a function of effort. In this case:

$$\Psi^n\left(x^h\right)-\Omega^n\left(x^h\right)=\frac{\theta\gamma\left[\alpha\tau_1\left(x^h\right)^{-\frac{\alpha(1+\sigma)}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}+P_0\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)^{\beta}\right]}{r+\theta x^h\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)}+$$
$$-\left(x^h\right)^{\sigma}(1+\sigma-\epsilon)ds^{\epsilon}$$

and it can be easy to derive:

$$\frac{\partial x^h}{\partial z}=-\frac{\frac{\partial \Psi^n(n^h,z)}{\partial z}-\frac{\partial \Omega^n(x^h,z)}{\partial z}}{\frac{\partial \Psi^n(x^h,z)}{\partial x^h}-\frac{\partial \Omega^n(x^h,z)}{\partial x^h}},$$

where $z=\beta,P_0,\tau_1,\tau_2,s$.

Close to x^h the denominator of the last equation is negative because of the stability condition. Hence:

$$\frac{\partial x^h}{\partial z}\gtreqless 0\Leftrightarrow\frac{\partial \Psi^n\left(x^h,z\right)}{\partial z}-\frac{\partial \Omega^n\left(x^h,z\right)}{\partial z}\gtreqless 0.$$

For $z=P_0$,

$$\frac{\partial \Psi^n\left(x^h,P_0\right)}{\partial P_0}-\frac{\partial \Omega^n\left(x^h,P_0\right)}{\partial P_0}=\frac{\theta\gamma\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)^{\beta}}{\left[r+\theta x\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)\right]^2},$$

which is always positive.

For $z=\beta$,

$$\frac{\partial \Psi^n\left(x^h,\beta\right)}{\partial \beta}-\frac{\partial \Omega^n\left(x^h,\beta\right)}{\partial \beta}=\frac{\theta\gamma P_0\beta ln\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)}{\left[r+\theta x\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)\right]^2}$$

which is always positive.

For $z=\tau_2$, we have $\frac{\partial \Psi^n(x^h,\tau_2)}{\partial \tau_2} - \frac{\partial \Omega^n(x^h,\tau_2)}{\partial \tau_2} \gtreqless 0 \Leftrightarrow$

$$+\gamma\beta P_0\left(1-\left(x^h\right)^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)^{\beta-1} - \gamma\alpha\tau_1\left(x^h\right)^{1+\sigma}B^{-1} - \Psi^n\left(x^h,\tau_2\right)x^h \gtreqless 0.$$

The first term is positive the other two are negative, hence the sign of the derivative is indeterminate.

For $z=\tau_1$,

$$\frac{\partial \Psi^n\left(x^h,\tau_1\right)}{\partial \tau_1} - \frac{\partial \Omega^n\left(x^h,\tau_1\right)}{\partial \tau_1} \gtreqless 0 \Leftrightarrow \gamma\alpha\left(x^h\right)^{1+\sigma}B^{-1}[(1-\alpha)(1-\tau_1-\tau_2)-\alpha\tau_1] + \\ + \gamma\beta P_0\left(1-x^{\frac{-(1+\sigma)}{1-\alpha}}B^{\frac{1}{1-\alpha}}\right)^{\beta-1} - \Psi^n\left(x^h,\tau_1\right)x^h \gtreqless 0.$$

The first two terms are positive while the last one is negative. Also in this case the sign is indeterminate.

As regards the effect of s on the level of effort, we take the derivative of the equilibrium condition Eq. (19):

$$x_t = (1-n_t)^{\frac{\alpha-1}{1+\sigma}}B^{\frac{1}{1+\sigma}}$$

with respect to s and, using the previous results, obtain:

$$\frac{\partial x_t^h}{\partial s} = \frac{1-\alpha}{1+\sigma}\left(1-n_t^l\right)^{\frac{\alpha-1}{1+\sigma}-1}B^{\frac{1}{1+\sigma}}\frac{\partial n_t^h}{\partial s} + \left(1-n_t^l\right)^{\frac{\alpha-1}{1+\sigma}}\frac{1}{1+\sigma}B^{\frac{1}{1+\sigma}-1}\frac{\partial B}{\partial s} < 0.$$

References

Acemoglu, D., 2009. Introduction to Modern Economic Growth. Princeton University Press, Princeton.
Acemoglu, D., Johnson, S., Robinson, J.A., 2001. The colonial origins of comparative development: an empirical investigation. Am. Econ. Rev. 91, 1369–1401.
Acemoglu, D., Johnson, S., Robinson, J.A., 2002. Reversal of fortune: geography and institutions in the making of the modern world income distribution. Q. J. Econ. 117, 1231–1294.
Acemoglu, D., Johnson, S., Robinson, J.A., 2005. Institutions as a fundamental cause of long run growth. In: Aghion, P., Durlauf, S.N. (Eds.), Handbook of Economic Growth. Elsevier, Amsterdam, North-Holland.
Acemoglu, D., Naidu, S., Restrepo, P., Robinson, J.A., 2014. Democracy does cause growth. NBER WP. No 20004.
Adams, J.D., 1990. Fundamental stocks of knowledge and productivity growth. J. Polit. Econ. 98, 673–702.
Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.
Aghion, P., Dewatripont, M., Stein, J.C., 2008. Academic freedom, private-sector focus, and the process of innovation. RAND J. Econ. 39, 617–635.
Aghion, P., Dewatripont, M., Hoxby, C., Mas-Colell, Sapir, A., 2010. The governance and performance of universities: evidence from Europe and the US. Econ. Policy 25 (61), 7–59.
Aghion, P., Akcigit, U., Howitt, P., 2013. What Do We Learn From Schumpeterian Growth Theory? National Bureau of Economic Research (wp n°. w18824).
Aizenman, J., Kletzer, K., 2011. The life cycle of scholars and papers in economics—the 'citation death tax'. Appl. Econ. 43, 4135–4148.
Azariadis, C., Stachurski, J., 2005. Poverty traps. In: Aghion, P., Durlauf, S.N. (Eds.), Handbook of Economic Growth. Elsevier, Amsterdam, North-Holland.
Bekar, C., Lipsey, R.G., 2004. Science institutions and the Industrial Revolution. J. Eur. Econ. Hist. 33, 709–753.
Ben-David, J., 1964. Scientific growth: a sociological view. Minerva 2 (4), 455–476.
Box, S., Cotgrove, S., 1966. Scientific identity, occupational selection, and role strain. Br. J. Sociol. 17.
Bramoullé, Y., Saint-Paul, G., 2010. Research cycles. J. Econ. Theory 145, 1890–1920.
Capshew, J., Rader, K., 1992. Big science: price to present. Osiris 7, 2–25.
Carillo, M., Papagni, E., 2013. Is the "globalization" of science always good for scientific productivity and economic growth? Metroeconomica 64, 607–644.
Carraro, C., Siniscalco, D., 2003. Science versus profit in research. J. Eur. Econ. Assoc. 1, 576–590.
Cerruzzi, P.E., 2003. A History of Modern Computing, Cambridge, Mass. MIT Press.
Cole, S., Cole, J., 1967. Scientific output and recognition: a study in the operation of the reward system in science. Am. Sociol. Rev. 32, 377–390.
Cole, S., Cole, J., 1973. Social Stratification in Science. University of Chicago Press, Chicago.
Cole, S., Phelan, T.J., 1999. The scientific productivity of nations. Minerva 37, 1–23.
Cole, H., Mailath, G.J., Postlewaite, A., 1992. Social norms, savings behavior, and growth. J. Polit. Econ. 100, 1092–1125.
Coleman, J., 1990. Foundations of Social Theory. Harvard University Press, Cambridge, MA.
Cooper, B., Garcia-Pignalosa, C., Funk, P., 2001. Status effects and negative utility growth. Econ. J. 111, 642–665.
Corneo, G., Jeanne, O., 2001. Status, the distribution of wealth, and growth. Scand. J. Econ. 103, 283–293.
Cotgrove, S., 1970. The sociology of science and technology. Br. J. Sociol. 21, 1–15.
Crane, D., 1965. Scientists at major and minor universities: a study of productivity and recognition. Am. Sociol. Rev. 30, 699–715.
Crane, D., 1972. Invisible Colleges: Diffusion of Knowledge in Scientific Communities. University of Chicago Press, Chicago.
Dasgupta, P., 1989. Patents, priority and imitation: the economics of races and waiting games. Econ. J. 98.
Dasgupta, P., David, P.A., 1987. Information disclosure and the economics of science and technology. In: Feiwell, G. (Ed.), Arrow and the Ascent of Modern Economic Theory. Macmillan Press, London, pp. 519–542.
Dasgupta, P., David, P.A., 1994. Toward a new economics of science. Res. Policy 23, 487–521.
David, P.A., 1998a. Clio and the economic organization of science, common agency contracting and the emergence of "open science" institutions. Am. Econ. Rev. Pap. Proc. 88 (2), 15–21.
David, P.A., 1998b. Communication norms and the collective cognitive performance of "invisible colleges". In: Barba Navaretti, G., Dasgupta, P., Maler, K.-G., Siniscalco, D. (Eds.), Creation and Transfer of Knowledge. Springer-Verlag, Berlin.
Diamond, A., 1986. What is a citation worth? J. Hum. Resour. 21, 200–215.
Drori, G., 1993. The relationship between science, technology, and the economy of lesser developed countries. Soc. Stud. Sci. 23, 201–215.
Fershtman, C., Murphy, K.M., Weiss, Y., 1996. Social status, education, and growth. J. Polit. Econ. 104, 108–132.
Fox, M.F., 1983. Publication productivity among scientists: a critical review. Soc. Stud. Sci. 13, 285–305.
Fulton, O., Trow, M., 1974. Research activity in American higher education. Sociol. Educ. 47, 29–73.
Galison, P., Hevly, B., 1992. Big Science: The Growth of Large-Scale Research. Standford University Press, Standford, USA.
Gaston, J., 1978. The Reward System in British and American Science. John Wiley, New York.
Grossman, G.M., Helpman, E., 1991. Innovation and Growth in the Global Economy. MIT Press, Cambridge.
Hamermesh, D.S., Pfann, G.A., 2012. Reputation and earnings: the roles of quality and quantity in academe. Econ. Inq. 50, 1–16.
Hopkins, E., Kornienko, T., 2006. Inequality and growth in the presence of competition for status. Econ. Lett. 93 (2), 291–296.
Howitt, P., 2000. The economics of science and the future of universities. 16th Timlin LectureUniversity of Saskatchewan, Saskatoon.
Jaffe, A.B., 1989. Real effects of academic research. Am. Econ. Rev. 79, 957–970.
Larivière, V., Archambault, É., Gingras, Y., 2008. Long-term variations in the aging of scientific literature: from exponential growth to steady-state science, 1900–2004. J. Am. Soc. Inf. Sci. Technol. 59, 288–296.
Lazear, E.P., 1997. Incentives in basic research. J. Labor Econ. 15, S167–S197.
Mansfield, E., 1991. Academic research and industrial innovation. Res. Policy 20, 1–12.
Mansfield, E., 1995. Academic research underlying industrial innovations: sources, characteristics and financing. Rev. Econ. Stat. LXXVII, 55–65.
McDowell, J.M., 1982. Obsolescence of knowledge and career publication profiles: some evidence of differences among fields in costs of interrupted careers. Am. Econ. Rev. 72, 752–768.
Merton, R.K., 1957. Priorities in scientific discovery: a chapter in the sociology of science. Am. Sociol. Rev. 22, 635–659.
Merton, R.K., 1973. The Sociology of Science: Theoretical and Empirical Investigations. Chicago University Press.
Moav, O., Neeman, Z., 2012. Saving rates and poverty: the role of conspicuous consumption and human capital. Econ. J. 122, 933–956.
Mokyr, J., 2005. Long term economic growth and the history of technology. In: Aghion, P., Durlauf, S. (Eds.), The Handbook of Economic Growth.
Mokyr, J., 2008. The institutional origins of the industrial revolution. In: Helpman, E. (Ed.), Institutions and Economic Performance. Harvard University Press.
Nelson, R.R., 1959. The simple economics of basic scientific research. J. Polit. Econ. 67, 549–583.
Price de Solla, D., 1963. Little Science, Big Science. Columbia University Press, New York.
Raup, D., 1986. The Nemesis Affair: A Story of the Death of Dinosaurs and the Ways of Science. New York Free Press.
Reinganum, J., 1989. The timing of innovation: research, development and diffusion. In: Willig, R., Schmalensee, R. (Eds.), The Handbook of Industrial Organization, North-Holland, Amsterdam.
Romer, P.M., 1990. Endogenous technological change. J. Polit. Econ. 98, 71–102.
Rosenberg, N., Birdzell Jr., L.E., 1986. How the West Grew Rich: The Economic Transformation of the Industrial World. Basic Books.
Rosenberg, N., Birdzell Jr., L.E., 1990. Science, technology and the western miracle. Sci. Am. 263, 42–54.
Schofer, E., 2004. Cross-national differences in the expansion of science, 1970–1990. Soc. Forces 83, 215–248.
Shell, K., 1967. A model of inventive activity and capital accumulation. In: Shell, K. (Ed.), Essays on the Theory of Optimal Economic Growth. The MIT Press, Cambridge.
Stephan, P.E., 1996. The economics of science. J. Econ. Lit. XXXIV, 1199–1235.
Stephan, P.E., Levin, S.G., 1992. How science is done; why science is done. Striking the Mother Lode in Science: The importance of Era, Place and Time. Oxford University Press, New York, pp. 11–24.
Tuckman, H.P., 1976. Publication, Teaching and the Academic Reward Structure. Lexington Books, Lexington, MA.
Weinberg, A.M., 1967. Reflections on Big Science. Pergamon, London.

Weinberg, B.A., 2011. Developing science: scientific performance and brain drains in the developing world. J. Dev. Econ. 95, 95–104.
Weiss, Y., Fershtman, C., 1998. Social status and economic performance: a survey. Eur. Econ. Rev. 42, 801–820.
Zilsel, E., 1942. The sociological roots of science. Am. J. Sociol. 47 (4), 544–562.
Zuckerman, H., 1992. The proliferation of prizes: Nobel complements and Nobel surrogates in the reward system of science. Theor. Med. 13, 217–231.