



Exploratory factor analysis for the Hirsch index, 17 h-type variants, and some traditional bibliometric indicators

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ABSTRACT

The purpose of this article is to come up with a valid categorization and to examine the performance and properties of a wide range of h-type indices presented recently in the relevant literature. By exploratory factor analysis (EFA) we study the relationship between the h-index, its variants, and some standard bibliometric indicators of 26 physicists compiled from the Science Citation Index in the Web of Science.

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1. Introduction

Hirsch (2005) introduced a new indicator for the assessment of the research performance of scientists. The proposed h-index is intended to measure simultaneously the quality and sustainability of scientific output, as well as, to some extent, the diversity of scientific research. The specific index attracted interest immediately and received great attention in the scientometrics literature. Not only it has found a wide use in a very short time, but also a series of articles were subsequently published either proposing modifications of the original h-index for its improvement, or implementations of the newly proposed index.

The h-index (sometimes called the Hirsch index or the Hirsch number) is based on the distribution of citations received by a given researcher's publications. By definition: "A scientist has index h if h of his N_p papers have at least h citations each, and the other $(N_p - h)$ papers have at most h citations each".

The index is designed to improve simpler measures such as the total number of citations or publications, to distinguish truly influential (in terms of citations) scientists from those who simply publish many papers. Among the advantages of this index is its simplicity, the fact that it encourages researchers to produce high quality work, the fact that it can combine citation impact with publication activity and that is also not affected by single papers that have many citations.

The h-index is robust to the numbers of citations received by the papers belonging to the h-core (i.e. the papers receiving h or more citations). Various h-type indices have been devised in order to overcome this "robustness" [e.g. the g-index (Egghe, 2006), the A-index (Jin, 2006), the R-index (Jin, Liang, Rousseau, & Egghe, 2007), and the h_w -index (Egghe & Rousseau, 2008)]. Many other variants have modified, specified, or generalized the original definition (see, e.g. Egghe, 2006; Jin et al., 2007; Schreiber, 2007b, 2008b; Sidiropoulos, Katsaros, & Manolopoulos 2006; Tol, 2009). However, more and more voices argue against the usefulness of all these measures (see e.g. Adler, Ewing, & Taylor, 2009; Bornmann, Mutz, Daniel, Wallon, &

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Ledin, 2009; Meho, 2007; Schreiber, 2007a; Vinkler, 2007). In the same vein, Van Noorden (2010) states that “many metrics correlate strongly with one another, suggesting that they are capturing much of the same information about the data they describe”.

By performing exploratory factor analysis (EFA) on some of the more important h-type indices, including the h-index, Bornmann, Mutz, and Daniel (2008) conclude that indices can be categorized into two basic groups: those that “describe the most productive core of the output of a scientist and tell us the number of papers in the core” (p. 836) and those that “depict the impact of the papers in the core” (p. 836). According to the authors, h-index and g-index belong to the first category, while A-index and R-index belong to the second. Nevertheless, Bornmann et al. (2008) recommended a more thorough validation of their factor analysis (FA) results by using other data sets, especially from different fields of research.

Schreiber, Malesios, and Psarakis (2011) have shown that the distinction is not so evident for the citation records of 26 physicists, which were previously analyzed (Schreiber, 2008a, 2010b). Specifically, the authors utilized 7 bibliometric indices – similar to the analysis of Bornmann et al. (2008), with the addition of standard indicators of quantity and impact, namely total number of publications n , total number of citations S and average number of citations $\bar{c}(n)$. In particular, the nearly equal factor loadings for g in the EFA of the raw data seemed to verify the assumption (Schreiber, 2010a) that the g-index measures both, the quantity and the impact. However, this was not substantiated by the more comprehensive FA. Conspicuous differences to the findings of Bornmann et al. (2008), Bornmann, Mutz, and Daniel (2009) and Bornmann, Mutz, Daniel, et al. (2009) have also been found. On the other hand, the results were mostly in agreement with those of Costas and Bordons (2007, 2008) and Hendrix (2008).

In the current article, we expand the previous analysis of Schreiber et al. (2011), by once again utilizing EFA using this time an augmented database consisting of a set of 17 indicators – in addition to the h-index – that have been proposed in recent years to improve the h-index, illustrated in detail by Schreiber (2010b). The actual values of these indices and some standard bibliometric indicators can be found in Appendix A and a short description in Appendix B. By this investigation we attempt to clarify the properties and behaviour of the various indices, by coming up with categorizations to latent items provided by the FA. Moreover, we attempt to interpret the categorization of those indices based on previous research and the properties shared by the indices.

In addition we investigate the claim that the g-index can measure both the actual scientific productivity and the scientific impact of a scientist, a property not shared by the majority of the other indices.

With the present manuscript we present a case study in order to add more information to previously published analyses. In the next section we describe the data base, Section 3 gives an overview over the methodology and in Section 4 we describe our results for the various indices. For the discussion in Section 5 the dataset has been expanded by including standard bibliometric measures.

2. Data

The citation data for 26 present or former members of the Institute of Physics at Chemnitz University of Technology, including all full and associate professors as well as scientists who have been working as assistants or senior assistants (see Table A1) were collected in the time period of January and February 2007 from the ISI Thomson Scientific Web of Science (WoS) database (Schreiber, 2007a). A large effort has been made in establishing a correct data base, giving particular attention to the precision problem, on the one hand excluding homographs, i.e. to establish that indeed the considered publications have been written by the scientists considered in the sample and on the other hand including possible variants of spelling of the author names. As a consequence with 26 citation records the data base is relatively small, but to establish a larger data base we would have needed to reduce our demands with respect to precision. The aim was not to obtain a homogeneous sample, which is certainly more appropriate for theoretical considerations, but to obtain a reasonable sample for a realistic case study. In our opinion, the current data represent a typical sample of researchers at a more average institute, while many other investigations in the literature have concentrated on prominent scientists or rather homogeneous groups of distinguished professors.

In the current article we utilize 18 Hirsch-type indices, namely w , $h(2)$, h , \tilde{h} , A , f , t , g , \tilde{g} , m , h_w , R , \tilde{h} , π , e , s , h_T and x (Maxprod). In parallel to the h- and g-indices we also utilize the interpolated \tilde{h} and \tilde{g} in compliance with the analysis of Schreiber (2010b). In addition the standard bibliometric indicators n , n_1 , S , c_1 , $\bar{c}(n_\pi)$ and $\bar{c}(n)$ for each dataset are also used. The selection of the indices by Schreiber (2010b) and thus the present selection is not arbitrary, but intends to comprise important variants of the h-index which are directly based on the raw number of citations and do not require modifications like fractionalized counting which could be applied to take multiple authorship into account (Schreiber, 2008b), or reducing the number of citations by subtracting self-citations (Schreiber, 2009).

3. Methodology – overview

The statistical methodology of EFA can be used to examine for latent associations present in a set of observed variables, and reduce dimensionality of the data to a few representative factors. Our aim here is to provide a categorization of the h-index and its variants, by employing EFA. Although the sample size used for the FA can be regarded as relatively small ($N=26$), recent studies based on simulations have shown that when certain conditions exist the small sample size does not play a very important role and reliable FA results can be obtained. Specifically, for a relatively small number of factors,

Table 1
One-sample Kolmogorov–Smirnov test.

	Mean	Median	Std. Dev.	Normal distribution		Student distribution	
				<i>D</i>	<i>p</i> -Value	<i>D</i>	<i>p</i> -Value
<i>w</i>	3.54	3.5	1.84	0.285	0.029*	0.215	n.s.
<i>h</i> (2)	5.00	5	1.60	0.230	n.s.	0.188	n.s.
<i>h</i>	14.88	14	6.92	0.186	n.s.	0.100	n.s.
\tilde{h}	15.05	14	6.89	0.194	n.s.	0.087	n.s.
<i>A</i>	33.55	29.5	17.8	0.217	n.s.	0.096	n.s.
<i>f</i>	19.23	18	9.59	0.196	n.s.	0.096	n.s.
<i>t</i>	20.92	20	10.44	0.192	n.s.	0.120	n.s.
<i>g</i>	23.96	22	11.99	0.202	n.s.	0.094	n.s.
\tilde{g}	24.40	22.4	12.00	0.197	n.s.	0.095	n.s.
<i>m</i>	25.58	23.25	12.95	0.198	n.s.	0.107	n.s.
<i>h_w</i>	19.03	17.75	9.20	0.186	n.s.	0.092	n.s.
<i>R</i>	22.18	20.2	10.82	0.199	n.s.	0.090	n.s.
<i>h</i>	19.80	17.55	10.17	0.247	n.s.	0.246	n.s.
π	4.55	2.95	4.93	0.273	0.041*	0.273	0.041*
<i>e</i>	16.26	14.3	8.69	0.199	n.s.	0.088	n.s.
<i>s</i>	12.60	10.9	6.64	0.252	n.s.	0.252	n.s.
<i>h_T</i>	24.72	22.35	12.32	0.247	n.s.	0.247	n.s.
<i>x</i>	336.7	231	341.3	0.319	0.01*	0.250	n.s.

n.s., non-significant.

* Significant at a 5% significance level.

high communalities mitigate the problem of small sample sizes (Preacher & MacCallum, 2002). The importance of large communalities and a small number of factors for providing valid EFA results even with very small datasets was pointed out in a series of related articles (see e.g. Pennell, 1968; Velicer & Fava, 1998; Velicer, Peacock, & Jackson, 1982). Our analysis is a typical example for this, since communalities are extremely high (way above 0.9 in almost all variables) and the number of factors is very small (2 factors), indicating that the analysis can produce valid and robust results.

Bornmann et al. (2008) have applied a logarithmic transformation to the raw data before utilizing FA, because all variables should be approximately normally distributed in order to apply EFA techniques. For our data, Table 1 presents results of the Kolmogorov–Smirnov test for normality, which indicate that the data are adequately normally distributed – hence can be used for conducting FA – although they are better described by Student's *t*-distribution than by the normal distribution,ⁱ what is not surprising due to the small number of datasets.

However, it is of interest to check if there are any discrepancies in the results between the raw data and the transformed ones, and thus additionally to the raw data *x* the logarithmically transformed shifted data ($\ln(x + 1)$) and the square-root transformed data \sqrt{x} were also utilized. The latter transformation was applied in this context by Costas and Bordons (2008).

The purpose of the FA is to determine the relationship of the observables with an unknown set of factors. For clarification we note that this has nothing to do with relationships among the observed variables which could be linear or not. For example, a strongly non-linear behaviour between *h* and *n* or \tilde{c} was observed by Schubert and Glänzel (2007) who discussed the proportionality between *h* and $n^{1/(\alpha+1)} \tilde{c}^{\alpha/(\alpha+1)}$.

4. Exploratory factor analysis – results

We used a least squares factor extraction procedure since it has been argued that the least squares method performs well when using a small number of datasets in comparison to other factor extraction methods such as maximum likelihood (see Ihara & Okamoto, 1985) and a rotated varimax transformation. The statistical package SPSS 15.0 was utilized for the analysis (SPSS, 1999).

EFA gave a value of 0.828 for the Kaiser–Meyer–Olkin (KMO) measure of model adequacy (Kaiser, 1974), indicating that the 18 indices are suitable for the FA (see Table C1 in Appendix C). Similar values were obtained for the transformed data.

Both the eigenvalue criterion (according to which one drops any factors with an eigenvalue of less than one) and the scree plot criterion indicated the existence of two major latent structures (factors) as the best solution for explaining the variability in the data. The two factors extracted accounted for 97.64%, 96.48% and 97.11% of the total variance in the raw, the log-transformed, and the square-root transformed data, respectively. For the raw data we see that the first factor accounts for the 53.9% of the variance, the second factor for 43.7%.

The matrix of factor loadings for the three models can be found in Table 2. The corresponding communalities shared by the items are presented in Table C2.

ⁱ Explanations on the possible reasons for the differences between the distributions of the citations of our data with those of Bornmann et al. (2008) are presented in Schreiber et al. (2011).

Table 2

Varimax rotated loading matrices (applying least squares extraction and Kaiser normalization) for the 3 EFA models with values above 0.7 given in bold face.

Indices	Raw indices x		$\ln(x+1)$		\sqrt{x}	
	Component 1	Component 2	Component 1	Component 2	Component 1	Component 2
w	0.711	0.629	0.688	0.588	0.702	0.597
$h(2)$	0.736	0.629	0.749	0.615	0.748	0.618
h	0.827	0.553	0.864	0.492	0.848	0.520
\tilde{h}	0.827	0.555	0.866	0.493	0.849	0.521
A	0.499	0.863	0.444	0.895	0.471	0.880
f	0.816	0.572	0.850	0.519	0.835	0.543
t	0.784	0.619	0.809	0.585	0.799	0.599
g	0.685	0.727	0.675	0.735	0.682	0.730
\tilde{g}	0.691	0.722	0.685	0.726	0.690	0.723
m	0.706	0.649	0.650	0.607	0.686	0.624
h_w	0.691	0.721	0.677	0.734	0.686	0.726
R	0.678	0.733	0.675	0.734	0.678	0.733
h	0.798	0.587	0.786	0.599	0.792	0.592
π	0.675	0.704	0.640	0.753	0.659	0.735
e	0.549	0.836	0.494	0.867	0.523	0.852
s	0.831	0.540	0.836	0.531	0.834	0.534
h_T	0.835	0.550	0.851	0.523	0.844	0.534
x	0.770	0.591	0.744	0.619	0.762	0.599
Eigenvalues	9.701	7.873	9.614	7.752	9.717	7.763

A possible interpretation is complicated, when choosing a value of 0.6 as a cut-off threshold for the factor loadings. Then for the raw data 9 items load on both factors. Choosing a threshold level 0.7 leads to a clear separation of all indices to the two dimensions for the raw data. Now, A , e , g , \tilde{g} , h_w , R , π fall into the second category, the others into the first category. This is also true for the transformed data with the exception of m which is no more attributed to any of the factors. In contrast [Bornmann et al. \(2008\)](#) assign h and g to the same factor (measuring quantity of the research output).

We cannot conclude – as [Bornmann et al. \(2008\)](#) did – that the first factor relates to the number of papers in the productive core of the researchers' outputs, because indices like f and h load on that factor, but are based on the number of citations in the core. On the other hand, all the indices loading on the second factor reflect the quality dimension.

A and e load particularly strongly on the second factor. This confirms from another viewpoint the observation of [Schreiber \(2010a, 2010b\)](#) that A and e are closely related. This could be so, because these indices are the only ones solely based on h and total number of h-core citations $S(h)$ (The related index R is based entirely on $S(h)$).

The observation of [Schreiber \(2010b\)](#) that the rank orders for w and $h(2)$ are not very different, is reflected in the FA as both indicators share similar loadings on the two dimensions. Both indices – along with h – are based directly on citation counts for different core sizes. However, in the current analysis, h loads more strongly on the first factor. Thus larger core sizes seem to be favoured in the first dimension.

On the other hand the indices f , t , g , and A which depend on different average citation numbers load the more on the first dimension the smaller their values are. Thus larger averages are favoured in the second dimension. The loadings of g , R and h_w are quite similar but differ from those of the h -index although all of them depend on the square root of the summed number of citations.

In addition to the varimax orthogonal rotation method, we have utilized an oblique rotation method (specifically promax oblique rotation with least squares extraction) which – in contrast to varimax – does not require the factors to be uncorrelated. There are several studies proposing the use of oblique rotation instead of orthogonal rotation methodology (see e.g. [McCroskey & Young, 1979](#)). The value of the promax rotation exponent k was set to 4 since that value provided more interpretable results ([Tataryn, Wood & Gorsuch, 1999](#)).

Applying a threshold value 0.5 the results in [Table 3](#) provide a clear distinction of the indices, in full compliance with the results of varimax rotation (when using the threshold 0.7). Neither the logarithmic nor the square-root transformations lead to any conspicuous differences.

5. Expanded set

In an effort to further categorize h-type variants into indices based on quantity and quality [Bornmann, Mutz, Daniel, et al. \(2009\)](#) have re-run the EFA of [Bornmann et al. \(2008\)](#) including the standard bibliometric measures n and S . Along the same lines, we re-ran our EFA including besides n also other bibliometric measures, as in [Schreiber \(2010b\)](#), namely the number of cited publications n_1 , the average number of citations per article $\bar{c}(n) = S/n$, the highest number of citations c_1 , and the average number of citations $\bar{c}(n_\pi)$ in the elite set defined by [Vinkler \(2009\)](#) as the most cited $n_\pi = \sqrt{n}$ papers.

Table 3

Promax oblique rotated loading matrices for the 3 EFA models with values above 0.5 given in bold face.

Indices	Raw indices x		$\ln(x+1)$		\sqrt{x}	
	Component 1	Component 2	Component 1	Component 2	Component 1	Component 2
w	0.599	0.384	0.595	0.347	0.612	0.343
$h(2)$	0.642	0.361	0.669	0.336	0.669	0.336
h	0.866	0.148	0.963	0.037	0.922	0.085
\tilde{h}	0.864	0.151	0.966	0.037	0.923	0.085
A	0.022	0.978	-0.067	1.055	-0.026	1.020
f	0.829	0.190	0.916	0.093	0.879	0.135
t	0.731	0.298	0.794	0.234	0.769	0.260
g	0.463	0.574	0.445	0.595	0.458	0.581
\tilde{g}	0.478	0.559	0.470	0.571	0.477	0.563
m	0.571	0.424	0.517	0.410	0.561	0.403
h_w	0.479	0.559	0.450	0.590	0.468	0.571
R	0.446	0.591	0.447	0.593	0.449	0.589
\tilde{h}	0.784	0.233	0.744	0.277	0.764	0.255
π	0.468	0.545	0.374	0.654	0.416	0.611
e	0.132	0.885	0.039	0.964	0.085	0.926
s	0.884	0.123	0.883	0.125	0.887	0.120
h_T	0.881	0.135	0.915	0.098	0.902	0.112
x	0.733	0.266	0.659	0.347	0.707	0.294
Eigenvalues	16.462	15.516	16.018	14.998	16.269	15.233

Table 4

Varimax rotated loading matrices for the 3 EFA models (adding 5 bibliometric measures) with values above 0.685 given in bold face.

Indices and measures	Raw indices x		$\ln(x+1)$		\sqrt{x}	
	Component 1	Component 2	Component 1	Component 2	Component 1	Component 2
w	0.685	0.648	0.678	0.591	0.687	0.605
$h(2)$	0.696	0.665	0.730	0.627	0.721	0.640
h	0.767	0.618	0.833	0.521	0.804	0.565
\tilde{h}	0.769	0.617	0.837	0.520	0.807	0.563
A	0.496	0.857	0.473	0.869	0.491	0.859
f	0.763	0.629	0.827	0.539	0.799	0.580
t	0.738	0.665	0.790	0.601	0.769	0.627
g	0.658	0.753	0.682	0.731	0.676	0.737
\tilde{g}	0.662	0.750	0.689	0.725	0.681	0.733
m	0.659	0.692	0.623	0.624	0.658	0.648
h_w	0.666	0.745	0.682	0.730	0.681	0.732
R	0.647	0.763	0.674	0.738	0.666	0.746
\tilde{h}	0.806	0.590	0.821	0.565	0.819	0.569
π	0.674	0.707	0.675	0.720	0.686	0.710
e	0.541	0.835	0.516	0.848	0.535	0.837
s	0.834	0.550	0.862	0.504	0.853	0.519
h_T	0.812	0.580	0.856	0.515	0.840	0.540
x	0.788	0.585	0.781	0.584	0.799	0.569
n_1	0.949	0.195	0.950	0.195	0.951	0.190
n	0.958	0.149	0.966	0.143	0.964	0.142
c_1	0.346	0.843	0.302	0.853	0.316	0.847
$\tilde{c}(n_\pi)$	0.369	0.926	0.377	0.927	0.378	0.926
$\tilde{c}(n)$	0.107	0.938	0.157	0.952	0.139	0.953
Eigenvalues	11.13	10.975	11.742	10.188	11.614	10.423

In this way we intend – similarly to Bornmann, Mutz, Daniel, et al. (2009) – a categorization of the indices to the quantity dimension (expressed by n and n_1) and the impact dimension (expressed by $\tilde{c}(n)$, $\tilde{c}(n_\pi)$ and c_1). The results of the EFA using the least squares extraction method and the varimax rotation with Kaiser normalization are presented in Tables 4, C3 and C4. Once again, the results suggest a factor structure with only two factors having an eigenvalue larger than 1, which both explain 96.1% of the variability in the data.

From Table 4 we see that by selecting a threshold between 0.674 and 0.685, we get a clear distinction of all the raw indices, with the first dimension of the EFA comprising w , $h(2)$, h , \tilde{h} , f , t , \tilde{h} , s , h_T , x , n_1 , n while A , g , \tilde{g} , m , h_w , R , π , e , c_1 , $\tilde{c}(n_\pi)$, $\tilde{c}(n)$ load on the second factor. The high loadings of n and n_1 on the first factor and $\tilde{c}(n)$ and $\tilde{c}(n_\pi)$ on the second factor, mean that by including these standard bibliometric indicators into the analysis we have successfully enforced a separation

Table 5Varimax rotated loading matrices for the raw indices (adding S) with values above 0.685 given in bold face.

Indices and other measures	Raw indices x	
	Component 1	Component 2
w	0.686	0.646
$h(2)$	0.694	0.664
h	0.767	0.616
\tilde{h}	0.769	0.616
A	0.499	0.855
f	0.763	0.627
t	0.739	0.663
g	0.659	0.752
\tilde{g}	0.663	0.748
m	0.661	0.691
h_w	0.668	0.743
R	0.648	0.761
\tilde{h}	0.807	0.588
π	0.681	0.703
e	0.543	0.834
s	0.835	0.548
h_T	0.813	0.579
x	0.794	0.581
n_1	0.951	0.192
n	0.959	0.146
S	0.782	0.581
c_1	0.354	0.840
$\tilde{c}(n_\pi)$	0.372	0.924
$\tilde{c}(n)$	0.106	0.939
Eigenvalues	11.795	11.261

between the quantity and the quality dimension. For the logarithmically transformed data, the distinction is slightly less clear, because both loadings of w and m are below the threshold, while \tilde{g} loads evenly (and strongly) on both factors. In the case of the square-root transformation the two loadings of m are larger, but still below the threshold, while now π loads strongly on both factors.

Results of the promax oblique rotation (with $k=3$ and least squares extraction) in Table C5 show once again a more distinct separation to the two dimensions.

The obtained results suggest that the g -index (accordingly also \tilde{g}) contributes more in measuring the quality dimension, whereas the h -index (and accordingly \tilde{h}) measures mostly the quantity dimension.

To achieve an even clearer categorization of the indices we have performed the analysis including also the total number of citations S , as this specific metric has been also utilized by Bornmann et al. (2008). The contribution of the indices to the two factors shown in Table 5 yields a clear distinction in full agreement with Table 4, if again the threshold value 0.685 is used.

A somewhat surprising result is that S exhibits higher loading on the first factor, rather than on the second factor on which the other indicators that are based on the citations load strongly. That was already observed by Schreiber et al. (2011), and might be explained by the assumption that S correlates more strongly with n than with $\tilde{c}(n)$, since more papers attract more citations. This may also be an indication that S is not the best indicator for measuring quality. The same argument applies to \tilde{h} , because it is proportional to \sqrt{S} . Thus it loads strongly on the first factor just like S .

It has been suggested to us that the heterogeneity of the investigated sample could lead to unbalanced factor loadings and thus be an explanation for S loading high on the first factor. This may be the case, but should not be used as an argument against the FA. Certainly the heterogeneity does not correlate with the different factors, because relatively heterogeneous indicators (as measured by their relative range in terms of the ratio between maximum and minimum values) as well as relatively homogeneous indicators can be found among those loading most strongly on the first factor, like S , x and $h(2)$, t , respectively, as well as among those loading high on the second factor, like the heterogeneous π and c_1 on the one hand and the rather homogeneous $\tilde{c}(n)$, h_w on the other hand.

Most distinctive (except from the standard bibliometric indices) in terms of very high loadings are A and e belonging clearly in the group of indices measuring the impact of the productive core and \tilde{h} , s , x and h_T measuring the number of papers in the productive core.

Schubert and Glänzel (2007) have noted that the h -index is approximately proportional to $n^{1/(\alpha+1)} \tilde{c}^{\alpha/(\alpha+1)}$; this relation is rather well fulfilled by the present sample, too for $\alpha=2$, i.e. h is proportional to $n^{1/3} \tilde{c}^{2/3}$. We have performed an additional EFA including this quantity and obtained nearly the same loadings as in Table 5 with $n^{1/3} \tilde{c}^{2/3}$ loading rather strongly on the second factor (namely with a loading of 0.780), which is not so surprising, because n loads strongly on the first factor but \tilde{c} more strongly on the second. On the other hand, due to the observed proportionality between h and $n^{1/3} \tilde{c}^{2/3}$ one might

expect, that this quantity loads more strongly on the first factor as h does. This is not so, which means that an approximately linear relationship (in this case between h and $n^{1/3}\bar{c}^{2/3}$) can be deceiving.

In order to verify the statistical significance of the parameters of our model, we have performed a confirmatory factor analysis (CFA) (Jöreskog, 1969) for the measures in Table 5. This means that we fit the model structure obtained from EFA. The results in Appendix D confirm that all items are significant at the 5% level. Most R^2 values in Table D2 are higher than 0.9 and even the smallest value of 0.6 for $\bar{c}(n)$ is still rather large, so that the validity of the performed EFA is confirmed.

6. Conclusions

In this paper we have examined the relationship of the h-index with other related indices measuring research performance using EFA. We have shown for our dataset consisting of a wide variety of bibliometric indices, that a distinction is possible to one of the two basic dimensions of scientific performance, namely the quality and quantity of scientific output. In summary, two different groups of indices were identified according to the results of EFA. Generally, there was also indication based on the results of the conducted EFA that most of the indices cannot be fully categorized in any of the two factors. However, for some of the indices there is a stronger tendency to describe the quantity of the productive core. Among these indices are w , $h(2)$, h , \hat{h} , f , t , \hat{h} , s , h_T , and x . By definition of the h-index, both quantity and impact of articles are taken into account, but our analysis suggests that the number of publications plays the more important role. In the same manner, for other indices there is a stronger tendency to describe the impact of the productive core, including A , g , \tilde{g} , m , h_w , R , π and e . These results also confirm the observations of Schreiber (2010a), who based on theoretical arguments suggests that g , A and R belong to the same category of indices, and contrast the different classifications between g and A , R by Bornmann et al. (2008).

Nevertheless, the present investigation adds to the results derived by Schreiber et al. (2011), by generalizing the preliminary findings obtained using a set of 7 indices, this time by including more of the important h-type indices proposed to correct insufficiencies of the Hirsch index. We started our analysis with the h-index and 17 variants and achieved a clear distinction of the indices by means of the EFA. This means that the FA has yielded a relation between our set of 18 h-type indices and a set of only 2 factors, as confirmed by the CFA. Including the standard bibliometric measures, the categorization becomes easier to interpret, because of the high loadings obtained for these indicators. With the expanded dataset we were able to show that the indices loading strongly on the first factor are more related with the productivity measured by the number n of publications, while the indices loading more strongly on the second factor are more correlated with the impact as measured by the average number of citations $\bar{c}(n)$. As n and $\bar{c}(n)$ have led to the highest loadings for the two factors, we are tempted to suggest the use of these two traditional bibliometric indicators rather than a combination of two or more variants of the h-index.

Only if one really insists on measuring the achievements by a single quantity, the use of h or one of the variants appears to be better than one of the traditional bibliometric indicators. But it seems to be unnecessary to utilize one of the more complicated variants, rather than the original h-index or the likewise simple g-index, where the former is closer to n , i.e. the productivity dimension, and the latter is closer to \bar{c} , i.e. the impact dimension. We note that the R-index leads to nearly the same results as the g-index and that it is easier to calculate, because a smaller number of data is necessary which means that the precision problem is smaller. Therefore one might be tempted to prefer the R-index, although its definition is not self-consistent, but requires first the determination of the h-index.

Appendix A. Values of the utilized indices and bibliographic measures

Table A1.

Appendix B. Definition of the discussed indices

Let the papers of a particular scientist be sorted by the number of citations $c(r)$, where r is the rank of the paper in the sorted list. $S(r) = \sum_{i=1}^r c(i)$ is then the total number of citations to the r most cited papers, and $S = S(n)$.

The *A-index* (Jin, 2006) is the average number of citations received by the articles in the h-core.

The *e-index* (Zhang, 2009) is defined by the number of excess citations, $e^2 = S(h) - h^2$ which means counting the more-than-h citations received by each paper in the h-core.

The *f-index* (Tol, 2009) is the highest number of articles that received f or more citations on average, where the average is calculated as the harmonic mean.

The *g-index* is the highest number g of articles that together received g^2 or more citations (Egghe, 2006). This is equivalent to the highest number of articles that received g or more citations on average (Schreiber, 2010a). Here average means the arithmetic mean.

The interpolated \tilde{g} - index (Schreiber, 2008a) is given by $\tilde{g}^2 = S(\tilde{g})$ where $S(x)$ interpolates piecewise between $S(r)$ and $S(r+1)$. This interpolation was proposed for the so-called real g-index g_r by Rousseau (2006) and further studied by Guns and Rousseau (2009).

The *h-index* is the highest number h of articles that each received h or more citations (Hirsch, 2005).

Table A1

Characteristics of the 26 datasets analyzed in the present study. The datasets for each researcher are indexed A, B, C, . . . , Z, reflecting decreasing values of the h-index (Schreiber, 2007a).

Dataset	w	$h(2)$	h	\tilde{h}	A	f	t	g	\tilde{g}	m	h_w	R	h	π	e	s	h_T	x	n	n_1	$\bar{c}(n)$	$\bar{c}(n_\pi)$	c_1	S
A	10	10	39	39	93.9	53	58	67	67.1	72	51.7	60.5	54.8	24.5	46.3	35.3	67.6	1,665	290	250	20.7	144.1	457	5997
B	7	8	27	28	62.6	36	40	45	45.6	47	35.3	41.1	39.9	13.4	31.0	25.7	48.5	938	270	214	11.8	83.5	182	3177
C	5	7	23	23	47.3	31	33	36	36.7	40	28.5	33.0	28.8	7.4	23.6	18.5	37.4	609	126	103	13.2	66.8	129	1661
D	4	6	20	20	35.5	26	27	29	29.8	31	23.6	26.6	32.6	6.7	17.6	21.5	37.5	744	322	259	6.6	37.2	73	2124
E	4	6	19	19	62.4	25	28	37	37.2	38	28.2	34.4	26.8	8.8	28.7	16.4	31.2	522	63	57	22.8	109.5	279	1439
F	4	5	18	18	32.2	23	24	26	26.6	29	20.7	24.1	23.7	4.3	16.0	15.6	30.2	408	131	107	8.6	39.1	53	1127
G	3	5	17	17	28.4	21	22	23	23.9	26	18.3	22.0	18.7	2.8	13.9	12.5	24.9	289	49	47	14.2	40.3	57	697
H	4	6	16	16	35.9	21	23	26	26.2	31	21.4	24.0	19.4	4.0	17.8	12.1	25.3	294	70	47	10.7	50.0	70	749
I	4	6	15	15	46.1	20	23	28	28.8	24	22.3	26.3	21.0	5.5	21.6	12.9	25.4	284	65	53	13.6	68.8	149	885
J	4	5	15	15	32.1	19	20	23	23.6	23	18.1	21.9	16.9	3.3	16.0	10.4	21.9	247	51	32	11.3	46.9	112	574
K	3	5	14	15	27.7	19	20	21	22	27	16.8	19.7	17.3	3.0	13.9	10.9	22.8	228	79	56	7.5	33.6	55	596
L	4	5	14	14	30.6	18	20	22	22.7	23	17.8	20.7	18.5	3.4	15.2	11.8	23.7	234	88	67	7.7	37.4	64	681
M	4	5	14	14	34.0	18	20	24	24.1	21	18.3	21.8	19.1	3.8	16.7	12.2	23.7	221	70	60	10.4	47.1	100	726
N	3	5	14	14	27.7	18	20	22	22.1	26	17.7	19.7	18.5	2.9	13.9	12.2	23.9	261	72	61	9.5	36.0	55	687
O	3	4	13	13	22.8	16	17	19	19.1	18	14.9	17.2	16.6	2.4	11.3	10.9	21.3	203	77	66	7.1	26.3	47	550
P	4	5	13	13	41.5	16	19	24	24.7	27	20.5	23.2	17.8	4.5	19.2	10.7	21.1	245	47	37	13.4	63.7	108	631
Q	2	4	13	13	17.1	15	15	15	15.9	17	13.0	14.9	14.5	1.7	7.3	9.4	19.0	189	86	59	4.9	18.9	24	422
R	4	5	12	12	27.0	15	17	19	19.8	20	15.4	18.0	15.0	2.5	13.4	9.6	19.4	160	46	37	9.8	36.3	53	451
S	3	4	12	12	22.8	15	16	18	18.2	18	13.8	16.6	14.8	2.2	11.4	9.6	19.2	160	61	48	7.2	27.9	40	439
T	2	4	10	11	18.0	13	14	15	15.1	16	11.4	13.4	13.7	1.7	8.9	8.9	17.5	162	78	56	4.8	18.7	31	375
U	2	4	10	11	23.7	14	15	17	17.2	24	13.4	15.4	13.2	2.0	11.7	8.5	17.4	138	44	34	8.0	28.3	41	351
V	3	4	10	10	24.4	13	14	17	17.2	15	13.0	15.6	13.9	2.2	12.0	8.8	17.1	116	60	49	6.5	27.8	79	389
W	2	3	9	9	15.6	11	11	13	13.2	12	10.1	11.8	11.4	1.2	7.7	7.3	14.5	112	53	37	4.9	17.4	42	261
X	1	3	8	8	35.1	10	11	18	18.2	11	14.3	16.8	13.2	2.6	14.7	6.8	13.2	204	35	29	9.9	44.0	204	346
Y	1	3	7	7	11.0	8	9	9	9.5	10	7.9	8.8	7.6	0.6	5.3	4.9	10.2	52	25	19	4.6	12.6	19	116
Z	2	3	5	5	17.0	6	8	10	10	23	8.5	9.2	7.2	0.8	7.7	4.4	8.8	69	15	12	6.9	19.8	25	103

The interpolated \tilde{h} -index (Schreiber, 2008a) is given by $\tilde{h} = \tilde{c}(\tilde{h})$ where $\tilde{c}(x) = c(r) + (x - r)(c(r + 1) - c(r))$ interpolates linearly the rank-frequency function between r and $r + 1$. This interpolation was proposed for the so-called real h-index h_r by Rousseau (2006) and further studied by Guns and Rousseau (2009).

The h_T -index (Anderson, Hankin, & Killworth, 2008) is the sum of the weights

$$w(i, r) = \begin{cases} (2i - 1)^{-1} & r \leq i \\ (2r - 1)^{-1} & r \geq i \end{cases} \text{ of the } i\text{th citation to the } r\text{th paper.}$$

The h_w -index (Egghe & Rousseau, 2008) is the square root of the total number S_w of citations received by the highest number of articles that each received S_w/h or more citations.

The $h(2)$ -index (Kosmulski, 2006) is the highest number $h(2)$ of articles that each received $[h(2)]^2$ or more citations.

The \tilde{h} -index (Miller, 2006) is the square root of half of the total number of citations.

The m -index (Bornmann et al., 2008) is the median number of citations received by the articles in the h-core.

The π -index (Vinkler, 2009) is defined as one hundredth of the total number of citations received by papers in the elite set which comprises the most cited \sqrt{n} papers.

The R -index (Jin et al., 2007) is the square root of the total number of citations received by the articles in the h-core. This is equivalent to $R = \sqrt{Ah}$.

The s -index (Silagadze, 2009) $s = (1/2)\sqrt{S(S/S_0)}$ measures the deviation from a uniform citation record, in terms of $S_0 = \log n$ and $S = \sum_{r=1}^n (c(r)/s)\log(c(r)/s)$.

The t -index (Tol, 2009) is the highest number of articles that received t or more citations on average, where the average is calculated as the geometric mean.

The w -index (Wu, 2010) is the highest number w of articles that each received $10w$ or more citations.

The x -index (Kosmulski, 2007) is defined as the maximum of the product of rank and citation frequency.

Appendix C. Supporting tables for the analysis

Tables C1–C5.

Table C1

KMO test for the 18 indices from Table 2.

	Raw indices x	$\ln(x + 1)$	\sqrt{x}
KMO	0.828	0.822	0.841
p -Value	<0.001	<0.001	<0.001

Table C2

Variance explained by the 3 EFA models in Table 2.

Indices	Raw indices x	$\ln(x + 1)$	\sqrt{x}
w	0.901	0.819	0.849
$h(2)$	0.938	0.939	0.942
h	0.990	0.989	0.989
\tilde{h}	0.993	0.994	0.993
A	0.994	0.998	0.997
f	0.994	0.991	0.993
t	0.997	0.997	0.997
g	0.999	0.996	0.998
\tilde{g}	0.999	0.997	0.998
m	0.919	0.791	0.860
h_w	0.999	0.997	0.998
R	0.998	0.995	0.996
\tilde{h}	0.981	0.976	0.978
π	0.951	0.977	0.975
e	0.999	0.996	0.998
s	0.982	0.981	0.981
h_T	0.999	0.997	0.998
x	0.942	0.937	0.940

Table C3

KMO test for the 23 indices and measures from Table 4.

	Raw indices x	$\ln(x + 1)$	\sqrt{x}
KMO	0.66	0.716	0.657
p -Value	<0.001	<0.001	<0.001

Table C4

Variance explained by the 3 EFA models in Table 4.

Indices	Raw indices x	$\ln(x+1)$	\sqrt{x}
w	0.889	0.809	0.837
$h(2)$	0.926	0.926	0.930
h	0.970	0.965	0.965
\tilde{h}	0.972	0.971	0.969
A	0.981	0.979	0.979
f	0.977	0.974	0.974
t	0.987	0.985	0.986
g	0.999	0.999	0.999
\tilde{g}	0.999	0.999	0.999
m	0.914	0.778	0.852
h_w	0.998	0.998	0.998
R	0.999	0.999	0.999
\tilde{h}	0.998	0.993	0.995
π	0.954	0.974	0.976
e	0.990	0.986	0.987
s	0.998	0.997	0.997
h_T	0.996	0.998	0.997
x	0.963	0.951	0.961
n_1	0.938	0.941	0.940
n	0.940	0.953	0.950
c_1	0.830	0.819	0.817
$\tilde{c}(n_\pi)$	0.993	0.999	0.999
$\tilde{c}(n)$	0.891	0.931	0.927

Table C5

Promax oblique rotated loading matrices for the raw indices and measures from Table 4 with values above 0.54 given in bold face.

Indices and measures	Raw indices x	
	Component 1	Component 2
w	0.548	0.481
$h(2)$	0.554	0.497
h	0.667	0.402
\tilde{h}	0.670	0.401
A	0.206	0.839
f	0.656	0.419
t	0.607	0.477
g	0.462	0.628
\tilde{g}	0.469	0.621
m	0.493	0.550
h_w	0.477	0.613
R	0.443	0.645
\tilde{h}	0.730	0.348
π	0.506	0.561
e	0.273	0.789
s	0.785	0.284
h_T	0.742	0.333
x	0.710	0.350
n_1	1.104	-0.222
n	1.139	-0.284
c_1	0.023	0.895
$\tilde{c}(n_\pi)$	0.012	0.988
$\tilde{c}(n)$	-0.326	1.134
Eigenvalues	17.577	17.441

Appendix D. Results for the CFA, using the indices and other bibliographic measures as in Table 5

Tables D1 and D2.

Table D1
Confirmatory factor analysis matrix calculated with AMOS 7.0 (Arbuckle, 2006).

Indices and measures	x	
	Component	
	1	2
w	0.94	
h(2)	0.96	
h	0.99	
\tilde{h}	0.99	
f	0.99	
t	0.99	
\tilde{h}	0.99	
s	1.00	
h_T	1.00	
x	0.97	
n_1	0.88	
n	0.86	
S	0.96	
A		0.97
g		1.00
\tilde{g}		1.00
m		0.95
h_w		1.00
R		1.00
π		0.97
e		0.98
c_1		0.86
$\tilde{c}(n)$		0.78
$\tilde{c}(n_\pi)$		0.93

Table D2
Summary statistics of the CFA model fit.

Indices and measures	Unstandardized loadings	Standard error	p-Value	R ²
w	1.124	0.107	<0.05	0.879
h(2)	0.889	0.085	<0.05	0.918
h	3.977	0.314	<0.05	0.982
\tilde{h}	3.967	0.310	<0.05	0.985
f	5.523	0.430	<0.05	0.986
t	6.000	0.474	<0.05	0.981
\tilde{h}	5.868	0.452	<0.05	0.989
s	3.839	0.290	<0.05	0.994
h_T	7.152	0.528	<0.05	0.999
x	192.296	17.069	<0.05	0.943
n_1	33.505	4.397	<0.05	0.773
n	39.526	5.539	<0.05	0.738
S	682.675	63.766	<0.05	0.925
A	1.440	0.072	<0.05	0.941
g	0.694	0.035	<0.05	0.999
\tilde{g}	0.695	0.035	<0.05	0.999
m	0.715	0.058	<0.05	0.909
h_w	0.532	0.027	<0.05	0.998
R	0.627	0.032	<0.05	0.999
π	0.278	0.019	<0.05	0.946
e	0.495	0.031	<0.05	0.967
c_1	4.804	0.608	<0.05	0.748
$\tilde{c}(n)$	0.203	0.035	<0.05	0.601
$\tilde{c}(n_\pi)$	1.620	0.147	<0.05	0.874

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